ECN 510 Exam 1

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1.

1.

$$\begin{aligned} \text{Portfolio} =& x(30+T) + (1-x)(180-T) \\ E[T] = & 70, \sigma = 10 \end{aligned}$$

First we find the expected value:

$$\begin{aligned} E[x(30+T) + (1-x)(180-T)] =& x(30+70) + (1-x)(180-70) \\ =& 100x + 110(1-x) \\ =& 110 - 10x \end{aligned}$$

Now we derive the variance and standard deviation:

$$\begin{aligned} V[x(30+T) + (1-x)(180-T)] =& V[x(30+T)] + V[(1-x)(180-T)] \\ &+ 2cov[x(30+T), (1-x)(180-T)] \end{aligned}$$

the components become:

$$\begin{aligned} V[x(30+T)] =& 100x^2 \\ V[(1-x)(180-T)] =& 100(-1x)^2 \\ & \text{by } V[aX+b] = a^2V[X] \\ cov[x(30+T), (1-x)(180-T)] & \\ =& E[x(30+T-30-70)(1-x)(180-T-180-70)] \\ =& E[x(1-x)(T-70)^2] \\ =& -x(1-x)E[(T-70)^2] \\ =& -x(1-x)100 \\ & \text{by } E[(T-70)^2] = V[T] = \sigma^2 \end{aligned}$$

so:

$$\begin{aligned} V[x(30+T) + (1-x)(180-T)] =& 100x^2 + 100(1-x)^2 - 2x(1-x)100 \\ =& 100[x^2 + (1-x)^2 - 2x(1-x)] \\ =& 100[x + x^2 - 2x + 1 + 2x^2 - 2x] \\ =& 100[4x^2 - 4x + 1] = 100(2x-1)^2 \\ \sigma^2 =& 100(2x-1)^2 \\ \sigma =& \sqrt{100(2x-1)^2} \end{aligned}$$

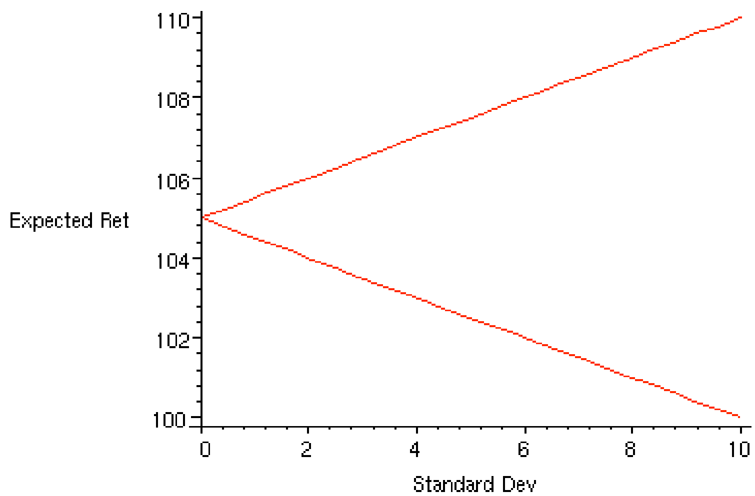

Figure 1: Portfolio σ vs. expected value

2. Our safest investment is where $\sigma = 0$. Solving for x, we get $x = 0.5$. Substituting this in to our expected value equation we get $110 - 10(0.5) = 105$. On our graph we can see this is where the curve touches the x-axis, at 105.

3. Our most profitable value would be putting all of our money into firm B. We can see this by simply looking at the equation for expected return; it is maximized when $x = 0$, so $110 - 10(0) = 110$. It is also apparent by looking at the graph: the highest value for expected return is 110.

2.

1.

$$U(C_S, C_{NS}) = 0.5ln(C_S) + 0.5ln(C_{NS})$$

$$\begin{aligned} C_S = \$100,000 + K - 0.6K &= \$100,000 + 0.4K \\ C_{NS} &= \$200,000 - 0.6K \\ K &= \frac{C_S - \$100,000}{0.4} \end{aligned}$$

$$\begin{aligned} C_{NS} &= \$200,000 - 0.6\left(\frac{C_S - \$100,000}{0.4}\right) \\ \frac{2}{3}C_{NS} &= \$1333,333.33 - C_S + \$100,000 \\ C_S + \frac{2}{3}C_{NS} &= \$233,333.33 \\ \text{slope} &= -\frac{3}{2} \end{aligned}$$

2.

$$MRS = -\frac{MC_S}{MC_{NS}} = -\frac{0.5C_S^{-1}}{0.5C_{NS}^{-1}} = -\frac{C_S^{-1}}{C_{NS}^{-1}} = -\frac{C_S}{C_{NS}}$$

Set MRS equal to slope:

$$\begin{aligned} -\frac{C_{NS}}{C_S} &= -\frac{3}{2} \\ C_{NS} &= \frac{3}{2}C_S \end{aligned}$$

Solve for (C_S, C_{NS}):

$$\begin{aligned} C_S + \frac{2}{3}(\frac{3}{2}C_S) &= \$233,333.33 \\ 2C_S &= \$233,333.33, \qquad C_S = \$116,666.67 \\ & \qquad C_{NS} = \$174,999.99 \end{aligned}$$

Tom's bundle is $(C_S, C_{NS}) = (\$116,666.67, \$174,999.99)$.

3. Tom will want to buy $K = \frac{\$116,666.67 - \$100,000}{0.4} = \$41,666.68$ of insurance (he will pay a premium of $\$41,666.68(0.6) = \$25,000.01$).

3.

1.

$$PV = \frac{\$22}{(1 + 0.1)} = \$20$$

2. We want the sum of values until the return of holding the asset (wine) drops below the interest rate (then we sell since we could be making more by having the money in the bank). This occurs at the 1st time period, as $\frac{1}{22} = 0.0455$. So, we would pay the same as in question 3.1, \$20.