Bounded Influence Estimation and Outlier Detection for GARCH Models With an Application to Foreign Exchange Rates

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Abstract

In this paper, we propose a bounded influence estimation (BIE) and outlier detection procedure for GARCH models. Previous studies show that maximum likelihood estimates of GARCH models are sensitive to outliers and financial time series present a heavy tail due to outliers. The proposed BIE limits the influence of a small subset of the data and is asymptotically normal. Its robustness against outliers and model misspecification is examined and supported. We further use BIE with GARCH models to develop a method for detection of additive outliers. An application to the exchange rates of major currencies is provided.

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1. INTRODUCTION

The purpose of this paper is twofold. First, we propose a bounded influence estimator (BIE) for GARCH models. Second, we propose an outlier detection procedure with an application to foreign exchange rates.

Knowledge of the distribution of exchange rates has important implications for theories of international finance and their applications. It is also of importance to varying issues related to foreign exchange. For instance, options pricing on foreign currencies relies on the right specification of the stochastic processes of exchange rates. In testing exchange market efficiency, the information of the statistical properties of exchange rate distribution is essential. Moreover, the volatility of exchange rates itself is a major risk component in international investing. Hence, clear understanding of the behavior and variance of exchange rates is important both for portfolio selection and for the evaluation of the performance of international asset portfolios.

Empirical evidence on the distribution of exchange rates, however, has been far from conclusive. While most previous studies have recognized that the rate of change in a foreign currency is not normally distributed, there is a lack of consensus on what type of distribution is most appropriate for describing the behavior of exchange rates. Examples of alternative statistical distributions, which have been commonly suggested in describing changes in exchange rates, include the symmetric stable Paretian, the Student t, the mixture of normal distributions, and the normal distribution with time-varying parameters (e.g., Friedman and Vandersteel, 1982; Booth and Glassman, 1987; Tucker and Scott, 1987; Canova, 1993; and Lye et al., 1998). Nevertheless, none of these well-documented alternatives has gained general acceptance.

An alternative approach to the issues of exchange rates is ARCH/GARCH models (see Engle, 1982; Bollerslev et al., 1992, 1995, for a survey). This model is intuitively appealing because of the observed volatility clustering of exchange rates, i.e., periods of high volatility tend to follow periods of high volatility. Hsieh (1988, 1989a) and Baillie and Bollerslev (1989) applied the ARCH model to daily exchange-rate series. Diebold and Nerlove (1989) estimated the ARCH model for weekly spot-exchange rates. Recently, Andersen and Bollerslev (1998) captured the volatility persistence (ARCH) of intraday exchange rates[1]. Overall, the findings of ARCH in exchange rates are important. First, ARCH models are consistent with unconditional leptokurtosis in the changes of exchange rates (e.g., see Westerfield, 1977; Boothe and Glassman, 1987; Bollerslev et al., 1993, 1995). Second, ARCH models may prove to provide particularly helpful tools in future analyses and enhance the understanding of currency-option pricing with stochastic volatilities models (e.g., Hull and White, 1987; Melino and Turnbull, 1990)[2].

Among all assumptions of ARCH models, a very strong one is the conditional normal distribution of the disturbance. However, numerous studies (e.g., Hsieh, 1988, 1989; Baillie and Bollerslev, 1989; Andersen et al., 2001) showed that the distribution of the changes in exchange rates is, unconditionally as well as conditionally, far from being normal. In fact, leptokurtosis and skewness are frequently present. Hence, the normality assumption seems to be inadequate and often leads to spurious or inefficient inferences. This is mainly due to the fact that exchange rates are contaminated by some outliers or extreme values so that the conditional distribution looks heavy-tailed.

To account for heavy tails of the conditional distribution, student-t, among other alternatives, is often adopted instead of normal (e.g., Engle and Bollerslev, 1986; and Bollerslev, 1987). However, estimated residuals from GARCH models are still frequently observed with excess kurtosis, even

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[1] Drost and Nijman (1993) and Drost and Werker (1996) provided some theoretic results regarding temporal aggregation of ARCH.

[2] An alternative approach is to study the implied standard deviation (ISDs) derived from currency and exchange rate options. Previous studies indicated that ISDs were either biased forecasts for future volatility, or less efficient in predicting than historical time-series (Lamoureux and Lastrapes, 1993; Canina and Figlewski, 1993; Jorion, 1995). Andersen et al. (2001) constructed model-free estimates of daily exchange rate volatility.

when conditional student-t errors are allowed (Franses and Ghijsels, 1999). Alternatively, Hsieh (1989b) and Nelson (1991) used the generalized error distribution (GED), which encompasses the normal, exponential, and uniform distributions. Nelson (1991), nevertheless, noted that the GED has only one parameter to control the shape of the conditional distribution, which may not be flexible enough in the presence of many outliers.

This paper proposes a BIE that is robust against departure from normality (of the conditional distribution) to describe the behavior of exchange rate changes. BIE limits the influence of any small subset of the data and is asymptotically normal (Krasker and Welsh, 1982). By construction, BIE provides a mechanism to detect over influential observations and limit their impact on the parameter estimation. In this paper, the BIE is used with GARCH to identify additive outliers (AO) and other outliers caused by abnormal information arrivals that may be triggered by changes in domestic policies and international shocks. The identification of outliers allows us to analyze major economic and political factors that are not described by the statistical model but directly contribute to dramatic changes in exchange rates.

Balke and Fomby (1994) and Dijk et al. (1999) found that neglecting AOs can erroneously suggest misspecification or inadequate descriptive models for financial (particularly ARCH) modeling. Franses and Ghijsels (1999) documented that neglected AOs substantially dampen the forecasting properties of GARCH models. The proposed BIE brings robustness to GARCH models against outliers including AOs. Previous studies attempted to detect and remove an outlier, in order to obtain "true" estimators of GARCH models. However, one concern arises that no observation can be regarded as an outlier with 100% assurance. Hence, an observation may be deleted by error. In addition, it is widely contended that an outlier may contain important information indeed, though its influence should not be assigned as high as its nominal magnitude suggests. The BIE identifies abnormal deviations from normality and downweights the influence of these observations accordingly. Thus, it achieves efficiency and robustness simultaneously. In this paper, the performance of the BIE will be compared with the maximum likelihood estimate (MLE), and a semiparametric estimator (SP) (Engle and Gonzalez-Rivera, 1991). Issues related to the assumption of the distribution such as non-normality, leptokurtosis, and outlying observations will also be addressed.

The remainder of this paper is organized as follows. Section 2 provides the background on the proposed BIE and places it in context with related work. Section 3 describes the BIE in details. Section 4 gives the estimation results from an application of the foreign exchange rates. Finally, Section 5 provides a summary.

2. The MODEL: ARCH/GARCH

Consider the ARCH model suggested by Geweke (1986),

$$y_t \mid \psi_{t-1} \sim N(0, \sigma_t^2) \qquad (1)$$
$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2,$$

where y_t is the rate of change for the foreign exchange spot rate, ψ_{t-1} is the information set available at time t - 1 and σ_t^2 is the conditional variance. Note that the conditional variance σ_t^2 is positive for all values of α. Equation (1) is sometimes referred as the log-ARCH model. The log-likelihood function is

$$\ln L = - \sum_{t=1}^{T} (\log s_t^2 + \frac{y_t^2}{s_t^2}) . \qquad (2)$$

The MLE either maximizes equation (2) or solves the following first order condition

$$\frac{\partial \ln l}{\partial \theta} = 0, \qquad (3)$$

where $\theta = (\alpha_1, \alpha_2)$. Note that if $\alpha_2 = 0$, the changes in exchange rates reduce to a random walk.

Hsieh (1989), Engle and Bollerslev (1986), and Baillie and Bollerslev (1989) have found that the MLE of ARCH is sensitive to distributional assumptions. One explanation is that the observations are contaminated by outliers and/or extreme values that make the conditional distribution look heavy tailed. Consequently, the outliers may not be helpful in predicting future variances, and the estimates in the variance function may be unduly influenced by a few extreme observations. These arguments strongly suggest the need of constructing robust-resistant ARCH parameter estimates and use these robust estimates to detect outliers.

Note that equation (1) can be written as

$$\log y_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2 + v_t, \qquad (4)$$

where $v_t = \log y_t^2 - \log\sigma_t^2$ are uncorrelated for t = 1,2,..., T. Thus equation (1) can be rewritten as an autoregressive model of order 1 (AR(1)) for $\log y_t^2$. Hence, the process $\log y_t^2$ has the same correlation structure as that of an AR(1) process with AR parameter α_2.

The generalized ARCH (GARCH) allows the conditional variance to depend not only on past residuals, but also on its own past realizations. The GARCH(1,1) extension of the log-ARCH model is

$$y_t \mid \psi_{t-1} \sim N(0, \sigma_t^2)$$
$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2 + \alpha_3 \log \sigma_{t-1}^2. \qquad (5)$$

Equation (5) can also be written as

$$\log y_t^2 = \alpha_1 + (\alpha_2 + \alpha_3)\log y_{t-1}^2 + \alpha_3 v_{t-1} + v_t, \qquad (6)$$

where $v_t = \log y_t^2 - \log\sigma_t^2$. This reveals that $\log y_t^2$ in equation (5) follows an autoregressive and moving average model (ARMA(1, 1)) with serially uncorrelated v_t.

Outlier detection is important for achieving the right statistical inference and has been an intriguing topic of numerous studies (see, e.g., Chen and Liu, 1993; Davies and Gather, 1993; Hadi and Simonoff, 1993; Ljung, 1993; Rocke and Woodruff, 1996; Penny, 1996). However, little attention has been given to outlying observations in the standard GARCH. Among a few others, Dijk et al. (1999) and Franses and Ghijsels (1999) found that neglecting AOs dampens the tests and estimates of ARCH effects. Jorion (1988) has a model that is very similar to the AO model. In his model, Jorion allows the mean of the exchange rate to follow a jump process, while the variance of the exchange rate follows an ARCH process. However, the present study considers the AO in the ARCH process to the variance but not the mean.

Now we take a more careful look at the outlying observations on ARCH models. Before assessing the effects of outliers on the ARCH models, we define what we mean by outliers in the time series models. Two major types of outliers have been defined by Fox (1972): One is called the additive effect outliers (AO) model; the other is referred as the innovation outlier (IO) model[3]. An IO represents an extraordinary shock at time t influencing y_t, y_{t+1}, ..., through the dynamic system described by equation (1).

In the IO model, occasional innovations have larger variance than the majority and therefore, can appear as outliers. In the AO model, on the other hand, the isolated outlier has an additive transient character that is unrelated to the time series model. Thus the AO is also called a gross error, since only the level of t^{th} observation is affected. In fact, IO-type outliers transmit their effect through to later observations while AO-type outliers do not. We also note that IO model will create a heavy-tailed distribution and ARCH model is heavy-tailed. ARCH model, therefore, are able to capture IOs by construction. Assume that the observations are generated from

$$z_t = x_t + e_t \qquad (7)$$

[3] Chen and Liu (1993) conducted a joint examination on the effects of IO, AO, and two other types of outlier, a level shift (LS) and a temporary change (TC), in time series.

where $x_t = \log y_t^2$ follows an AR(1) model in equation (4), and e_t is an independent sequence of variables, independent of the sequence of x_t. The variable e_t has distribution H, given by

$$H = (1 - \varepsilon)\delta_0 + \varepsilon G,$$

where δ_0 is the distribution that assigns probability 1 to the origin and G is an arbitrary distribution. Therefore, with probability $1 - \varepsilon$, the AR(1) process x_t itself is observed, and with probability ε the observation is the AR(1) process x_t plus an error with distribution G. Further insights into the effects of AOs to the ARCH model can be seen as follows: Let

$$z_t = x_t + e_t$$
$$x_t = \alpha_1 + \alpha_2 x_{t-1} + v_t$$
$$e_t \sim (1 - \varepsilon)\delta_0 + \varepsilon G.$$

Making the autoregressive transformation of z_t, we have

$$z_t - \alpha_2 z_{t-1} = x_t - \alpha_2 x_{t-1} + e_t - \alpha_2 e_{t-1}. \quad (8)$$

Note that the sum of the two uncorrelated moving average (MA(1)) processes on the RHS of equation (8) is MA(1). Hence equation (8) represents an ARMA(1, 1) process. That is, the AR(1) model with AOs becomes an ARMA(1, 1) model in equation (8). In other words, the ARCH(1) model with AOs will become a GARCH(1, 1). Hence, GARCH (1, 1) model in equation (5) is able to capture AOs.

In equations (6) and (8), the AO hypothesis appears to imply a testable restriction on the parameters of GARCH (1,1) model. In particular, the AO hypothesis implies that from equation (8) the estimated AR parameter will be equal to the estimated MA parameter in a GARCH (1,1) model. This AO hypothesis will be tested in a later paper.

3. BOUNDED INFLUENCE ESTIMATION

The foregoing analysis shows that the MLE of ARCH models may be sensitive to AO-type outliers. Consequently, the detection of outlying observations calls for a robust estimation. The motivation for BIE arises from studies such as Krasker and Welsh (1982), Kao and Dutkowsky (1989), Peracchi (1990a, 1990b, 1991), Naranjo and Hettmansperger (1994), Heritier and Ronchetti (1994), and Shen (1995).

The BIE proposed here is an iteratively reweighting technique where the weights decrease as some norms of the score function increases. The BIE for θ, denoted by $\hat{\theta}$, solves

$$\sum_{t=1}^{T} w(y_t, \hat{\theta}) s(y_t, \hat{\theta}) = 0, \tag{9}$$

where w(.) is a nonnegative weight function, θ is a K by 1 vector of parameters to be estimated and s(.) is the score function. The optimal weights are obtained as

$$w(x, \theta) = \min\left\{1, \frac{bK^{1/2}}{\left[s^T(x,q)A^{-1}s(x,q)\right]^{1/2}}\right\}, \tag{10}$$

where

$$A = E[w^2(y, \theta)s(y, \theta)s^T(y, \theta)]. \tag{11}$$

The influence bound b is specified prior to estimation. Krasker and Welsch (1982) demonstrated that b has lower bound of unity.

The problem of selecting the optimal influence bound has not been conclusively resolved (see Samarov 1985; Powell, 1990). A criterion requires a predetermined level of asymptotic efficiency relative to the MLE at the "ideal" model. Hampel et al. (1986, p. 252) pointed out, however, that such an approach may lead to estimators with very low robustness. They suggested choosing the influence bound near 1. Carroll and Ruppert (1987) and Kao and Dutkowsky (1989) used bounds ranging from 1.1 to 1.7 in their empirical studies. Peracchi (1990a) suggested that b is chosen so as to obtain an average weight of about 95%.

The BIE falls within the class of weighted MLE, as shown in Equation (9). It modifies the score function and finds the roots of the resulting likelihood functions. Equation (10) describes the choice of observation weights based on a Mahalanobis-type distance of $s(y_t, \theta)$ from the centroid of $\{s(y_t, \theta): t = 1,2, ..., T\}$. An observation is downweighted only if its influence exceeds the maximum allowable influence $bK^{1/2}$. Observations with influence below this bound receive a weight of unity. In this way the BIE resembles the MLE while, at the same time, the estimator protects against highly influential observations. From Equation (11) we see that A is a robust version of the second-moment matrix of $s(y, \theta)$.

The influence function (IF) of the BIE is

$$IF(y, \theta) = B^{-1} w(y, \theta)s(y, \theta), \tag{12}$$

where

$$B = -E\{\partial[w(y, \theta)s(y, \theta)]/\partial\theta\}. \tag{13}$$

Note that the influence function (IF) (see, e.g., Hampel, 1986; Peracchi, 1990b) measures the effect, on the asymptotic bias of an estimator, of an arbitrarily small contamination of the assumed statistical model.

The corresponding asymptotic covariance matrix of the BIE, denoted by V, is then

$$V = B^{-1}A\, B^{-1}. \tag{14}$$

Since the IF is a K x 1 vector, there is no natural ordering for influence. Obtaining a scalar in measuring of influence requires the application of appropriate norm for IF(y, θ). This norm maps the IF into R^1, combining the influence of a given observation over each parameter in θ to compute an overall measure of this influence. The Euclidean norm cannot be used here since it depends heavily upon the scaling of independent variables. A more suitable measure that is independent of particular parameterization is the self-standardized gross-error sensitivity (e.g., Krasker and Welsch, 1982),

$$\gamma = \max\{s^T(y,\theta)\, A^{-1}\, s(y,\theta)\}^{1/2}. \qquad (15)$$

The γ in equation (15) measures the worst effect that a small amount of contamination by gross-error can have on the bias of the BIE. The construction of the weights in equation (15) implies that $\gamma < b$ for suitable choices of the influence bound. Therefore, the foregoing estimator achieves bounded influence. Bounding the gross-error sensitivity ensures robustness, with greater robustness produced by smaller bounds. The details of the computational algorithm can be found in Carroll et al. (1987), Kao et al. (1989), and Peracchi (1990a, 1990b).

Note that bounded influence weights, w(.), provide useful diagnostic information for outliers and influential observations, in particular, and identifying potential sources of model failure. Recently, several nonparametric and semiparametric estimators for GARCH have been discussed in the literature (e.g., Robinson, 1988; Pagan and Ullah, 1988; Diebold and Nason, 1990; Pagan and Schwert, 1990). Gallant et al. (1990) used a semi-nonparametric method where the conditional density is estimated with a polynomial expansion using ARCH as a leading term. Engle and Gonzalez-Rivera (1991) estimated the conditional distribution using a nonparametric penalized likelihood density estimation of Tapia and Thompson (1978). Weiss (1986) and Bollerslev and Woodridge (1988) proposed a quasi-maximum likelihood (QMLE) for GARCH. These estimators have certain robustness properties (such as consistency), but can be very inefficient, for they disregard entirely the information contained in the parametric assumptions. For example, Engle and Gonzalez-Rivera (1991) showed that the loss of efficiency of the QMLE could go up to 84% due to misspecification of the density. The BIE, on the other hand, provides a compromise between efficiency and robustness, since they take parametric assumptions into account.

4. DATA AND EMPIRICAL RESULTS

The data set consists of daily spot rates of foreign exchange rates (in terms of U.S. dollars)[4]. A ten-year sample of five major currencies are selected: the British Pound (BP), Canadian Dollar (CD), Deutsche Mark (DM), French Franc (FF) and Japanese Yen (JY). There are 2599 daily observations from January 2, 1991 to April 30, 2001. The analyzed series is the first differences of the logarithms of the spot price of a specific currency in terms of U.S. dollars. Hence, the data represent the continuously compounded percentage rate of return for holding the particular currency one day. Table 1 reports the descriptive statistics of the data. The return skewness of each currency is negative except the British Pound. Meanwhile, British Pound, Canadian Dollar and Japanese Yen observe excess kurtosis, while Deutsche Mark and French Franc present less kurtosis.

4.1 The Estimation Results

Table 2 reports the MLE and BIE of the parameters of the ARCH(1) processes. A DUMINF subroutine of the IMSL libraries is used to compute the maximum likelihood estimators. The algorithm of computing BIE is written in FORTRAN and (9) is solved by subroutine DNEQNF in the IMSL libraries. For a given currency, the leading two rows display the parameter estimates of ARCH(1) process. Standard errors appear in the parentheses. The MLE of α_1 and α_2 are significantly different from zero.

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[4] Federal Reserve Bank of Chicago provided the daily foreign exchange rate data.

As mentioned earlier, there are outliers in the daily exchange rate data that may not be representative of the true exchange rate process. Including these non-representative data may cause spurious parameter estimation. To assess the effect of outlying observations on parameter estimation, the ARCH(1) process is re-estimated with the BIE. Column three to column six in Table 2 report the estimates of the BIE for the ARCH(1) process. Different values of bounds are set (1.1 to 1.7) in the estimation. The smaller the bounds, the more the data were down-weighted. Table 2 shows that all the MLE and BIE parameter estimates are significant at the 1% level. However, the MLE estimates are very sensitive to outliers. The BIE estimates are stable with respect to different bounds and obtained estimates of α_1 close to that of MLE. On the contrary, the estimates of α_2 of the five currencies increased about 22% to 300% for the BIE compared to the MLE. If the BIE represents the true parameter estimates of the population, then the ARCH effect of exchange rates has been underestimated by a substantial amount when the MLE is used as in most of previous studies. The ARCH model cannot inference the true autocorrelation of the conditional variance, with the presence of outliers.

The preceding results show that using BIE-ARCH(1) leads to significant difference with respect to ARCH(1). This is due to the fact that BIE is less sensitive than the MLE to local violations of the model assumptions. By construction, both BIE-GARCH(1,1) and GARCH(1,1) accommodate IO and AO-type outliers, which tend to produce close inferences. However, BIE-GARCH(1,1) is more robust than GARCH(1,1) since it also captures outliers other than IO and AO. Table 3 reports the MLE and BIE for the GARCH(1, 1) model. It shows that the BIE estimates are stable with respect to different bounds. The MLE of GARCH(1,1) tends to underestimate the ARCH effect of the exchange rates (α_2). Among the five currencies, only JY observed a bigger α_2 in MLE than in BIE. JY disassociated itself with the group again after it presented a significantly larger kurtosis (4.5088) in Table 1. Nevertheless, the extent of the underestimate looks less than that of ARCH(1), due to the accommodation of AOs in GARCH. The MLE estimates of the GARCH effect (α_2) are close to the BIE estimates. We conduct the likelihood ratio test for the GARCH model vs. the ARCH model. Denote θ and θ_1 the parameter sets of GARCH and GARCH, respectively. θ_1 is a subset of θ, since the ARCH is a restricted version of GARCH. The likelihood ratio $\Lambda=2(\ln l(\theta)-\ln l(\theta_1))$ follows $c^2(1)$ distribution. The likelihood ratio and p-value (in parentheses) for each currency are reported in Table 3. The GARCH model demonstrated a better goodness-of-fit compared to the ARCH model.

Table 3 also presents the estimation results of the various currencies using the semiparametric GARCH proposed by Engle and Gonzalex-Rivera (1990) (see Engle and Gonzalez-Rivera for details on the computations). The semi-parametric and MLE estimates are shown to be close. It seems that using semi-parametric GARCH does not lead to significant difference with respect to MLE. Hence, EG's semi-parametric ARCH is not robust with respect to outliers, which is not surprising (see Huber, 1981, p. 6). For example, the sample mean is a nonparametric estimator of the population mean, but it is highly sensitive to outliers and therefore very non-robust. Since the semi-parametric GARCH compasses all distribution types of the conditional residuals, it suggests that using alternative distribution types such as student t and GED for GARCH may not prevent influence sensitivity to outliers.

4.2 Robustness Test

To test the robustness of the models with normal conditional residuals, we run the foregoing analysis on the two half-samples of British Pound. The first half-sample covers a period from January 2, 1991 to December 30, 1995, while the second covers from January 2, 1996 to April 2001. The finding is consistent with that of the full sample. Table 4 reports the robustness test. The BIE estimates are stable with respect to different bounds, for the log-ARCH and log-GARCH analyses, respectively. For both models in each half-sample, MLE is sensitive to outliers compared to BIE. In both log-ARCH and log-GARCH, MLE tends to underestimate the ARCH effect of the BP exchange rates substantially. The semi-parametric estimates are close to the MLE for the log-GARCH model, which is also sensitive to outliers.

The previous results show that the bounded inference estimator provides a robust estimation of GARCH models with normal conditional residuals. As mentioned earlier, other distribution models, such as student-t, have been employed by earlier studies. Here we examine the bounded inference estimator of ARCH and GARCH with student-t conditional residuals. We modify models (1) and (5) by replacing the normal distribution with the student-t distribution of δ degree of freedom as

$$y_t \mid \psi_{t-1} \sim \sigma_t \cdot t(\delta) \qquad (1')$$

$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2,$$

and

$$y_t \mid \psi_{t-1} \sim \sigma_t \cdot t(\delta)$$

$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2 + \alpha_3 \log \sigma_{t-1}^2. \qquad (5')$$

We report the estimation of (1') and (5') for the British Pound in Table 5. The ARCH effect estimates are improved compared to the models with conditional normal residuals, due to the heavy tail included in t distribution. However, these ARCH effect is still underestimated in both ARCH and GARCH models relative to the bounded inference estimator with various bounds. The degree of freedom estimates are small, 4.052 for ARCH and 4.624 for GARCH, which indicate that the conditional normal distribution doesn't fit the real data relative to the t distribution. To examine this issue, we conduct likelihood ratio test for the restriction δ is infinitely large, which leads the t distribution to be equivalent to the standard normal distribution. The likelihood ratio Λ, which follows $c^2(1)$, and p-value (in parentheses) are reported for ARCH and GARCH. In either model, the t-distribution significantly improves the estimation relative to the normal distribution, though the underestimation still presents.

The BIE estimates of ARCH and GARCH effects are stable and close to those reported in Tables 2 and 3, and the degree of freedom estimates are large relative to the standard MLE in Table 4. As matter of fact, the degree of freedom estimates increase as the bound gets tighter. Since the BIE limits the inference of outliers, it is not sensitive to the distribution models employed and hence, is robust to model misspecification.

4.4 Detection of AO

The BIE-GARCH(1, 1) identified two groups of abnormal data in the foreign exchange rates. The first group includes the "shocks" that cannot be explained by the ARCH(1) and GARCH(1, 1) process. As shown in Table 6, these are large fluctuations in foreign exchange associated with important political and economic events. The second group includes the AO-type outliers that are also captured by the GARCH(1, 1) process. The procedure of identifying these AO outliers is as follows. Using BIE we fitted the exchange rate data to the BIE-ARCH(1) and the BIE-GARCH(1, 1) process. We found some observations are down-weighted substantially for the ARCH(1) process but are either not down-weighted or down-weighted slightly for the GARCH(1, 1). This means that these observations do not fit the ARCH(1) process well but fit fairly well to the GARCH(1, 1) process. Since the only difference between these two models is the inclusion of a moving average component in the GARCH(1, 1) process, these observations must be associated with the AO-type outliers. In this way, we identify the AO effects of economic and political changes that cause the jumps in exchange rate movements.

Table 6 reports the data points that were substantially down-weighted by the BIE for the British Pound for the purpose of demonstration. As shown in the table, most of the observations down-weighted in the BIE-GARCH(1, 1) process are also down-weighted in the BIE-ARCH(1) process. The AO-type outliers are listed in Table 6. The unexplained outliers in Table 6 may be due to the level-shift (LS) type outliers or structural change in Lastrapes (1989), Diebold and Pauly (1988), Chen and Tiao (1990), Lamoureux and Lastrapes (1990), and Chen and Liu (1993). Further work is needed for explaining GARCH with LS-type outliers (e.g., Gourieroux and Monfort, 1990; Chu, 1991; McCulloch and Tsay; 1993).

4.5 Common Factor in Foreign Exchange Markets

BIE downweights the observations that exceed a given bound. The preceding analysis shows that BIE estimates are less sensitive to outliers, compared to the MLE and semi-parametric estimates of GARCH. An intriguing question thus arises: What observations were down-weighted? Let’s first look at the commonality in foreign exchange rates. Information related to the general economy and market leads to a common trend in returns, price volatility, and liquidity in equity markets, in both intraday and daily levels (e.g., see Karolyi and Stulz, 1996; Chordia et al., 2000; and Hasbrouck and Seppi, 2001). Similarly, changes of the U.S. dollar value and shocks to the global economy contribute to the widely observed commonality in foreign exchange rates. We conduct principle component analysis on the five currencies and report the results in Table 7. Panel A shows that the first component explains 56% of the variation of exchange returns of the five currencies, which suggests the presence of a strong common factor. The rest four components are negligible. If the five currencies were perfectly independent, each component should explain 20% of the variation. Panel B reports the principal component analysis on the weighted returns of the five currencies, using the weights from the BIE-GARCH with bound 1.5. After down-weighting, the first component explains 52% of the total variation, slightly less than that of Panel A. This suggests that, some down-weighted observations (outliers) contributed to the co-movement of currencies, which should be shocks related to the intrinsic value of U.S. dollar or the global economy. Panel C presents the principal component analysis on the five series of weights from the BIE-GARCH. The first component explains 47% of the total variation. This suggests that, the independently searched optimal weights of the five currencies share a common factor. We conduct the same test on the raw and weighted return volatilities (absolute returns) of the five currencies, and get similar and consistent results. Hence, a significant portion of the outliers were shocks related to the fundamental of U.S. currency and the global economy. Expectedly, major policy changes from one major country and international turbulences trigger abnormal jumps or fluctuations major currencies.

5. CONCLUSION

This paper proposes BIE to estimate the conditional heteroskedasticity of foreign exchange rates, which are robust against outliers and model misspecification. It extends the current literature on the distribution of exchange rate changes in a number of ways. First, the distribution was estimated with a robust parametric model BIE. The preceding results show that exchange rate changes estimated from the same set of data can differ significantly depending on the choice of the model and estimation technique. In particular, the ARCH(1) can differ significantly from BIE as a consequence of the presence of only a small fraction of extreme observations. This BIE estimation procedure offers an efficient mechanism to down-weight outlying observation and therefore, provides more accurate estimates for the parameters of the exchange rate changes distribution. Second, the proposed BIE-ARCH and BIE-GARCH are able to detect additive outliers by construction. Third, the down-weighting technique of BIE produces stable estimates with respect to varying bounds.

We found a clustering of outliers among currencies. That is, the causal events of outliers in one currency are permutational, which are likely to influence other currencies. This analysis provides policy makers very valuable information on the sensitivity of exchange rate to policy shifts and economic events.

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Table 1: Statistics of the Data

The data set consists of five major currencies in terms of U.S. dollars for a period from January 2, 1991 to April 30, 2001.

Currency	British Pound	Canadian Dollar	Deutsche Mark	French Franc	Japanese Yen
Mean Exchange Rate	0.7385	1.6083	1.7039	5.7754	115.25
Daily Returns (%)					
Mean	-0.0109	-0.0118	0.0151	0.0145	-0.0033
Standard Deviation	0.3054	0.5882	0.6771	0.6506	0.7362
Skewness	0.0393	-0.2445	-0.0717	-0.1230	-0.5495
Kurtosis	2.3975	2.8137	1.5998	1.5250	4.5088

Table 2: MLE and BIE for the log-ARCH process

$$y_t \mid y_{t-1} \sim N(0, \sigma_t^2)$$
$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2$$

Currency	MLE	BIE(1.7)	BIE(1.5)	BIE(1.3)
British Pound				
α_1	-9.415	-9.244	-9.306	-9.407
	(0.060)	(0.068)	(0.066)	(0.642)
$\alpha_2 \times 10^2$	7.322	12.836	12.985	12.985
	(0.454)	(0.531)	(0.517)	(0.502)
Log-likelihood	9679			
Canadian Dollar				
α_1	-10.572	-10.460	-10.513	-10.615
	(0.088)	(0.086)	(0.084)	(0.083)
$\alpha_2 \times 10^2$	7.813	12.310	12.380	12.260
	(0.660)	(0.624)	(0.616)	(0.606)
Log-likelihood	11381			
Deutsche Mark				
α_1	-9.593	-9.196	-9.258	-9.416
	(0.070)	(0.070)	(0.069)	(0.067)
$\alpha_2 \times 10^2$	3.480	10.714	10.740	10.639
	(0.563)	(0.546)	(0.544)	(0.527)
Log-likelihood	9292			
French Franc				
α_1	-9.584	-9.269	-9.332	-9.503
	(0.077)	(0.082)	(0.081)	(0.077)
$\alpha_2 \times 10^2$	4.224	10.351	10.432	10.358
	(0.619)	(0.660)	(0.650)	(0.620)
Log-likelihood	9397			
Japanese Yen				
α_1	-8.997	-9.393	-9.457	-9.529
	(0.059)	(0.065)	(0.062)	(0.061)
$\alpha_2 \times 10^2$	7.334	9.003	9.809	9.697
	(0.525)	(0.550)	(0.518)	(0.510)
Log-likelihood	9098			

Note: Asymptotic standard errors are in parentheses below each coefficient. All the parameter estimates are significant at the 1% level. BIE(1.5) is the BIE with bound to be 1.5. Sample period is from January 2, 1991 to April 30, 2001.

Table 3: MLE, BIE and SP for the log-GARCH Process

Currency	MLE	BIE(1.7)	BIE(1.5)	BIE(1.3)	SP
British Pound					
α_1	-0.066 (0.015)	-0.106 (0.019)	-0.107 (0.019)	-0.112 (0.017)	-0.065 (0.015)
α_2 x 10^2	1.958 (0.168)	2.611 (0.196)	2.643 (0.192)	3.269 (0.179)	2.088 (0.169)
α_3	0.971 (0.003)	0.961 (0.003)	0.961 (0.003)	0.955 (0.003)	0.970 (0.003)
Log-likelihood	9794				9890
Λ	230 (0.000)				
Canadian Dollar					
α_1	-0.101 (0.025)	-0.022† (0.017)	-0.025† (0.017)	-0.058 (0.018)	-0.109 (0.025)
α_2 x 10^2	3.019 (0.196)	3.322 (0.196)	3.336 (0.193)	3.477 (0.192)	3.047 (0.194)
α_3	0.957 (0.004)	0.962 (0.003)	0.962 (0.002)	0.958 (0.003)	0.956 (0.004)
Log-likelihood	11504				11572
Λ	246 (0.000)				
Deutsche Mark					
α_1	-0.105 (0.028)	-0.092 (0.025)	-0.085 (0.022)	-0.087 (0.021)	-0.133 (0.033)
α_2 x 10^2	2.080 (0.261)	2.398 (0.256)	2.365 (0.236)	2.365 (0.231)	2.415 (0.288)
α_3	0.966 (0.005)	0.965 (0.004)	0.966 (0.004)	0.966 (0.004)	0.960 (0.006)
Log-likelihood	9371				9426
Λ	158 (0.000)				
French Franc					
α_1	-0.118 (0.032)	-0.109 (0.029)	-0.112 (0.028)	-0.128 (0.028)	-0.096 (0.026)
α_2 x 10^2	1.914 (0.250)	2.803 (0.286)	2.846 (0.279)	2.911 (0.260)	1.715 (0.214)
α_3	0.966 (0.006)	0.959 (0.005)	0.958 (0.005)	0.957 (0.005)	0.971 (0.005)
Log-likelihood	9468				9527
Λ	142 (0.000)				
Japanese Yen					
α_1	-0.108 (0.021)	-0.095 (0.019)	-0.090 (0.018)	-0.096 (0.017)	-0.095 (0.019)
α_2 x 10^2	2.841 (0.225)	2.292 (0.194)	2.435 (0.185)	2.767 (0.180)	2.947 (0.219)
α_3	0.956 (0.004)	0.965 (0.003)	0.965 (0.003)	0.961 (0.003)	0.956 (0.004)
Log-likelihood	9218				9345
Λ	240 (0.000)				

Note: Asymptotic standard errors are in the parentheses. All the parameter estimates are significant at the 1% level, except those marked with symbol † . BIE(1.5) is the BIE with bound to be 1.5. Sample period is from January 2, 1991 to April 30, 2001.

Table 4: Robustness Test on British Pound

This test runs the log-ARCH and log-GARCH analysis on two subperiods of British Pound, respectively.

Currency	MLE	BIE(1.7)	BIE(1.5)	BIE(1.3)	SP
Log- ARCH analysis on data period from January 2, 1991- December 30, 1995					
α_1	-9.091	-8.805	-8.874	-8.982	
	(0.085)	(0.092)	(0.091)	(0.089)	
α_2 x 10^2	7.619	14.102	14.159	14.072	
	(0.641)	(0.723)	(0.717)	(0.704)	
Log- ARCH analysis on data period from January 2, 1996- April 30, 2001					
α_1	-10.347	-10.050	-10.126	-10.532	
	(0.141)	(0.130)	(0.128)	(0.117)	
α_2 x 10^2	2.852*	8.906	8.809	7.661	
	(1.112)	(1.004)	(0.989)	(0.916)	
Log-GARCH analysis on data period from January 2, 1991- December 30, 1995					
α_1	-0.159	-0.041*	-0.018†	-0.020†	-0.123
	(0.041)	(0.017)	(0.015)	(0.015)	(0.032)
α_2 x 10^2	2.525	3.011	3.164	3.794	2.309
	(0.313)	(0.224)	(0.206)	(0.207)	(0.266)
α_3	0.955	0.963	0.964	0.958	0.961
	(0.007)	(0.003)	(0.003)	(0.003)	(0.005)
Log-likelihood	4524				4575
Log-ARCH analysis on data period from January 2, 1996- April 30, 2001					
α_1	-0.338*	-0.254	-0.289	-0.383	-0.396*
	(0.136)	(0.082)	(0.084)	(0.093)	(0.156)
α_2 x 10^2	1.869	3.130	3.316	3.711	2.016
	(0.414)	(0.395)	(0.391)	(0.396)	(0.434)
α_3	0.947	0.942	0.938	0.927	0.940
	(0.015)	(0.010)	(0.010)	(0.011)	(0.017)
Log-likelihood	5274				5311

Note: Asymptotic standard errors are in the parentheses. All the parameter estimates are significant at the 1% level, unless otherwise noted.

† : Not significant at the 5% level.

* : Significant at the 5% level.

BIE(1.5) is the BIE with bound to be 1.5.

Table 5: Robustness Test on British Pound with Student-t Distribution

This test runs the log-ARCH and log-GARCH analysis on the full sample of British Pound using t distribution.

Currency	MLE	BIE(1.7)	BIE(1.5)	BIE(1.3)
		Log- ARCH Analysis		
α_1	-9.851	-9.419	-9.468	-9.669
	(0.161)	(0.111)	(0.102)	(0.095)
α_2 x 10^2	8.866	13.325	14.025	14.111
	(1.293)	(0.901)	(0.825)	(0.770)
δ	4.052	25.055	43.860	60.071
	(0.389)	(4.226)	(9.211)	(13.420)
Log-likelihood	9812			
Λ	266 (0.000)			
		Log-GARCH Analysis		
α_1	-0.057	-0.086	-0.092	-0.100
	(0.026)	(0.024)	(0.024)	(0.024)
α_2 x 10^2	2.156	2.776	3.130	3.759
	(0.329)	(0.274)	(0.275)	(0.283)
α_3	0.971	0.962	0.958	0.952
	(0.005)	(0.004)	(0.004)	(0.004)
δ	4.624	29.525	46.497	65.078
	(0.503)	(6.220)	(11.797)	(17.685)
Log-likelihood	9891			
Λ	194 (0.000)			

Note: Asymptotic standard errors are in the parentheses. All the parameter estimates are significant at the 1% level. BIE(1.5) is the BIE with bound to be 1.5.

Table 6: Selected Downweighted Cases from the BIE(1.5): The Case of British Pound

Date	BIE-ARCH Weights	BIE-GARCH Weights	Outliers Type	Exchange Rate Returns (%)
3/15/1991	0.05	0.25	AO	-1.357
3/18/1991	0.08	0.11		-2.182
4/19/1991	0.05	0.06		-2.980
4/30/1991	0.06	0.07		2.707
5/17/1991	0.09	0.17		-1.850
7/12/1991	0.06	0.06		2.671
8/16/1991	0.04	0.15		-1.552
8/21/1991	0.09	0.08		2.342
11/27/1991	0.04	0.03		-1.728
1/9/1992	0.00	0.06		-2.161
1/10/1992	0.08	0.09		-2.153
5/14/1992	0.09	1.00	AO	0.495
6/29/1992	0.03	0.38	AO	0.843
8/24/1992	0.09	0.09		2.385
9/11/1992	0.02	0.07		-2.693
9/16/1992	0.04	0.04		-3.286
9/18/1992	0.06	0.07		-2.530
9/29/1992	0.08	0.10		2.258
10/1/1992	0.07	0.08		-2.562
10/16/1992	0.07	0.07		-2.653
11/16/1992	0.09	0.14		-2.022
1/5/1993	0.05	0.04		2.889
1/29/1993	0.07	0.11		-2.065
2/1/1993	0.08	0.11		-2.143
2/16/1993	0.03	0.11		2.094
5/10/1993	0.04	0.08		-2.350
7/13/1993	0.05	0.50	AO	1.003
8/26/1994	0.01	0.03		-1.165
3/10/1995	0.09	0.05		-2.191
11/13/1995	0.08	0.06		-1.525
11/13/1996	0.09	0.82	AO	0.485
12/3/1996	0.04	0.02		-2.526
12/19/1996	0.04	0.17		-1.185
1/22/1998	0.07	0.15		1.429
6/16/1998	0.01	0.05		1.279
8/28/1998	0.08	0.06		1.902
2/19/1999	0.08	0.64	AO	-0.509
2/8/2000	0.05	0.31	AO	1.174
7/20/2000	0.09	0.09		1.012
9/22/2000	0.10	0.07		1.997
12/1/2000	0.04	0.27	AO	1.141
2/1/2001	0.06	0.29	AO	1.103

Additive outliers are detected if the BIE-ARCH weights are small ($\leq$10%) and BIE-GARCH weights are large ($\geq$25%).

Table 7: Common Factor Among Exchange Rates

Panel A: Principal Component Analysis on the Returns of the Five Currencies

Component	Eigenvalue	Difference	Proportion	Cumulative
1	2.8030	1.8011	0.5606	0.5606
2	1.0019	0.2296	0.2004	0.7610
3	0.7723	0.3785	0.1545	0.9154
4	0.3938	0.3648	0.0788	0.9942
5	0.0290		0.0058	1.0000

Panel B: Principal Component Analysis on the Weighted (BIE 1.5) Returns of the Five Currencies

Component	Eigenvalue	Difference	Proportion	Cumulative
1	2.6169	1.6112	0.5234	0.5234
2	1.0057	0.1742	0.2011	0.7245
3	0.8315	0.3363	0.1663	0.8908
4	0.4952	0.4445	0.0990	0.9899
5	0.0507		0.0101	1.0000

Panel C: Principal Component Analysis on the Weights (BIE 1.5) of the Five Currencies

Component	Eigenvalue	Difference	Proportion	Cumulative
1	2.3438	1.3607	0.4688	0.4688
2	0.9831	0.0792	0.1966	0.6654
3	0.9039	0.2386	0.1808	0.8462
4	0.6653	0.5614	0.1331	0.9792
5	0.1039		0.0208	1.0000