A RESIDUAL-BASED TEST OF THE NULL OF COINTEGRATION IN PANEL DATA

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Abstract

This paper proposes a residual-based Lagrange Multiplier (LM) test for the null of cointegration in panel data. The test is analogous to the locally best unbiased invariant (LBUI) for a moving average (MA) unit root. The asymptotic distribution of the test is derived under the null. Monte Carlo simulations are performed to study the size and power properties of the proposed test.

Overall, the empirical sizes of the LM-FM and LM-DOLS are close to the true size even in small samples. The power is quite good for the panels where $T \geq 50$, and decent with panels for fewer observations in T. In our fixed sample of $N = 50$ and $T = 50$, the presence of a moving average and correlation between the regressor errors and regressors causes the two tests to perform quite differently, complicating the choice of estimation procedures. In general, the LM-DOLS test seems to be better at correcting these effects, although in some cases the LM-FM test is more powerful.

Although much of the non-stationary time series econometrics has been criticized for having more to do with the specific properties of the data set rather than underlying economic models, the recent development of the cointegration literature has allowed for a concrete bridge between economic long run theory and time series methods. Our test now allows for the testing of the null of cointegration in a panel setting and should be of considerable interest to economists in a wide variety of fields.

1 Introduction

Evaluating the statistical properties of data along the time dimension has proven to be very different from analysis in the cross-section dimension. As economists have gained access to better data with more observations across time, understanding these properties has grown increasingly important. An area of particular concern in time series econometrics is the use of non-stationary data. With the desire to study the behavior of a cross-section of observations over time and the increased use of panel data, one new research area is examining the properties of non-stationary time series data in panel form. It is an intriguing question to ask: how exactly does this hybrid style of data combine the statistical elements of traditional cross-sectional analysis and time series analysis? In particular, what is the correct way to analyze non-stationarity, the spurious regression problem, and cointegration in panel data?

There are a few statistical procedures that test for the presence of units in panel data in the literature, e.g., Quah (1994), Breitung and Meyer (1994), Levin and Lin (1993), Im Pesaran and Shin (1995) and Maddala and Wu (1996). Recently, Kao (1997) and Pedroni (1995) have proposed residual-based tests for cointegration in panel data. However, all of these tests work from a null of no cointegration (or no unit root). This approach has been criticized in the time series econometrics literature and tests for the null of cointegration (and unit roots) have been proposed, e.g., Kwiatkowski et al., (1992), Quintos and Phillips (1993), Shin (1994), Choi (1994), Leybourne and McCabe (1993). Yet no tests have been proposed to test for the null of cointegration in panel data.

This paper develops a residual-based test for the null of cointegration in panel data. The test is an extension of the Lagrange Multiplier (LM) test and locally best unbiased invariant (LBUI) test for an MA unit root in the literature. The proposed test belongs to the class of tests proposed by King and Hillier (1985) for testing the variance-covariance matrix of the error term in a linear regression model. This class of tests has been very useful in the pure time series context. Saikkonen and Luukonen (1993), Tanaka (1990), and Kwaitkowski et al. (1992) have tested for stationarity within this framework; Harris and Inder (1994) and Shin (1994) have used the test for cointegration. In particular, the LM test has allowed for the null of stationarity and null of cointegration respectively, which had not previously been available in the literature. Testing the null of cointegration rather than the null of no cointegration can be very appealing in applications where cointegration is predicted a priori by economic theory. As Harris and Inder (1994) and Shin (1994) demonstrate in the pure time series case, the LBUI test in this paper is also free of nuisance parameters.

The plan of the paper is as follows. Section 2 outlines the model under consideration and the assumptions required for construction of the proposed test. The formulation of the test and its limiting distribution are given in Section 3. Some Monte Carlo experiments including the size and power comparison are given in Section 4. Section 5 gives some concluding remarks. All proofs are in the Appendix.

A word on notation. We write integrals like $\int_0^1 W(s)ds$ as $\int W$ when there is no ambiguity over limits. We define $\Omega^{1/2}$ to be any matrix such that $\Omega = \left(\Omega^{1/2}\right)\left(\Omega^{1/2}\right)'$. We use $\Rightarrow$ to denote weak convergence, $\xrightarrow{p}$ to denote convergence in probability, $[x]$ to denote the largest integer $\leq x$, $I(1)$ to signify a time series that is integrated of order one, and $BM\left(\Omega\right)$ to denote Brownian motion with covariance matrix Ω.

2 Model

The model we use to combine time-series and cross-sectional data involves the assumptions that a varying intercept captures differences in behavior over cross-sectional units and that the slope coefficients may also vary across cross-sectional series. This model, which allows for varying slopes, is used in other panel settings such as Pedroni (1996) and Im et al. (1995). Let $\{y_{it}, x_{it}\}$ be $I(1)$ processes for all i and consider the following varying coefficient regression

$$y_{it} = \alpha_i + x_{it}^{'}\beta_i + v_{it},\ i = 1, ..., N,\ t = 1, ..., T, \tag{1}$$

$$x_{it} = x_{it-1} + \varepsilon_{it}, \tag{2}$$

$$v_{it} = \gamma_{it} + u_{it}, \tag{3}$$

$$\gamma_{it} = \gamma_{it-1} + \theta u_{it},$$

where $\{u_{it}\}$ are $i.i.d.$ $N(0,\, \sigma_u^2)$.

Our null hypothesis of cointegration is $H_0 : \theta = 0$ against $H_A : |\theta| \neq 0$. By backward substitution of (3) we write model (1) as

$$\begin{aligned} y_{it} &= \alpha_i + x_{it}^{'}\beta_i + \theta \textstyle\sum_{j=1}^{t} u_{ij} + u_{it}, \\ &= \alpha_i + x_{it}^{'}\beta_i + e_{it}, \end{aligned} \tag{4}$$

where $e_{it} = \theta \sum_{j=1}^{t} u_{ij} + u_{it}$. We note that

$$E\left(e_{it}\right) = 0$$

and

$$\begin{aligned} E\left(e_{it}e_{js}\right) &= \min\left(t,s\right)\theta^2\sigma_u^2 + 2\theta\sigma_u^2 + \sigma_u^2 & i = j, t = s, \\ &= \min(t,s)\theta^2\sigma_u^2 + \theta\sigma_u^2 & i = j, t \neq s, \\ &= 0 & i \neq j, t = s, \\ &= 0 & i \neq j, t \neq s. \end{aligned} \tag{5}$$

Then (1) can be written in matrix form,

$$y_i = \mathbf{X}_i\boldsymbol{\beta}_i + e_i, \text{ for } i = 1, ..., N \tag{6}$$

where

$$y_i^{'} = \left(y_{i1}, \cdots, y_{iT}\right),$$

$$e_i^{'} = \left(e_{i1}, \cdots, e_{iT}\right),$$

and

$$\mathbf{X}_i = \left(l_T, X_i\right),$$

where $\boldsymbol{\beta}_i = \left(\alpha_i, \beta_i^{'}\right)^{'}$ is a $(k+1) \times 1$ vector of parameter, X_i is the $T \times k$ vector of x's and l_T is a $T \times 1$ vector of ones. Equation (6) can also be written as

$$\begin{aligned} y &= \begin{bmatrix} \mathbf{X}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{X}_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}_N \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_N \end{bmatrix} + e \qquad (7) \\ &= \mathbf{X}\boldsymbol{\beta} + e, \end{aligned}$$

where

$$y^{'} = (y_{11}, \cdots, y_{iT}, y_{21}, \cdots, y_{2T}, \cdots, y_{N1}, \cdots, y_{NT}),$$

$$e^{'} = (e_{11}, \cdots, e_{1T}, e_{21}, \cdots, e_{2T}, \cdots, e_{N1}, \cdots, e_{NT}),$$

$$\mathbf{X} = \mathbf{X}^* C,$$

and

$$\boldsymbol{\beta}^{'} = \left(\boldsymbol{\beta}_1^{'}, \boldsymbol{\beta}_2^{'}, ..., \boldsymbol{\beta}_N^{'}\right),$$

where $X^* = \left(X_1^{'}, ..., X_N^{'}\right)$, $C = (\mathbf{l}_N \otimes I_{k^*})$ is an $Nk^* \times k^*$ matrix with $k^* = k+1$, I_{k^*} is the $k^* \times k^*$ identity matrix, and $\mathbf{l}_N$ is a $N \times 1$ vector of ones. It follows that $E\left(ee^{'}\right) = \sigma_u^2 Q(\theta)$ with

$$Q(\theta) = I_N \otimes \left(\theta^2 A_T + \theta\left(J_T + I_T\right) + I_T\right) \qquad (8)$$

where J_T is a $T \times T$ matrix of ones and A_T is a $T \times T$ matrix with ij^{th} element $min(i, j)$

$$A_T = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & \cdots & T \end{bmatrix}.$$

Note that if $\theta = 0$ then $e_{it} = u_{it}$ and is stationary, whereas if $\theta \neq 0$ then e_{it} is nonstationary. Also θ can be interpreted as the size of the effects a random shock would have on the random walk and stationary component.

This shows that the testing problem fits the general framework of King and Hillier (1985), which can be used to obtain locally optimal invariant tests for H_0. The LBUI statistic in (8) can be shown to be an LM statistic (e.g., Tanaka, 1996, p. 326):

$$LM = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} S_{it}^2}{s^2}, \tag{9}$$

where S_{it} is partial sum process of the residuals,

$$S_{it} = \sum_{j=1}^{t} \widehat{e}_{ij},$$

and s^2 is a consistent estimator of σ_u^2 under H_0, where

$$s^2 = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \widehat{e}_{it}^2. \tag{10}$$

Remark 1 *The statistic in (9) is still an LM statistic even if u_{it} is nonnormal and serially correlated. Also the iid assumption for u_{it} will be relaxed in the next section.*

3 Asymptotics

In this section, we will derive a residual-based test of the null of cointegration, which is a panel data version of the locally best invariant (LBUI) test of Shin (1994) and Harris and Inder (1994). First, we allow $\left(u_{it}, \varepsilon_{it}^{'}\right)^{'}$ for general weakly dependent and heterogeneously distributed innovations. Define $w_t = \left(u_{it}, \varepsilon_{it}^{'}\right)^{'}$. The long-run covariance matrix of w_{it} is given by

$$\Omega = \lim_{T\to\infty} \frac{1}{T} E\left(\sum_{t=1}^{T} w_{it}\right)\left(\sum_{t=1}^{T} w_{it}\right)^{'} = \Sigma + \Gamma + \Gamma^{'} \equiv \begin{bmatrix} \varpi_1^2 & \varpi_{12} \\ \varpi_{21} & \Omega_{22} \end{bmatrix}, \tag{11}$$

where

$$\Gamma = \lim_{T\to\infty} \frac{1}{T} \sum_{k=1}^{T-1} \sum_{t=k+1}^{T} E\left(w_{it} w_{it-k}^{'}\right) \equiv \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \tag{12}$$

and

$$\Sigma = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E\left(w_{it} w_{it}^{'}\right) \equiv \begin{bmatrix} \sigma_1^2 & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}. \tag{13}$$

Also we define

$$\begin{aligned}\Pi &= \Sigma + \Gamma \\ &= \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \\ &= \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}.\end{aligned}$$

Assumption 1 *Here we assume* Ω_{22} *to be positive definite, i.e., there are no cointegrating relationships among the regressors* x_{it}*.*

Assumption 2 *(e.g., Phillips, 1986)*

1. $E\left[w_{it}\right] = 0$,
2. $\sup_t E\left|w_{it}\right|^p$ *for some* $2 \leq p < \infty$,
3. $\{w_{it}\}_1^\infty$ *are strong mixing with mixing number* η_m *satisfying* $\sum_{m=1}^\infty \eta_m^{1-2/p} < \infty$.

Then w_{it} satisfies the invariance principle so that

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{[Tr]} w_{it} \Rightarrow B(r) \equiv BM\left(\Omega\right) \text{ as } T \to \infty, \tag{14}$$

where

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

Define

$$\varpi_{1.2}^2 = \varpi_1^2 - \varpi_{12}\Omega_{22}^{-1}\varpi_{21}. \tag{15}$$

Then B can be rewritten as

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} \varpi_{1.2} & \varpi_{12}\Omega_{22}^{-1/2} \\ 0 & \Omega_{22}^{1/2} \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$$

where $\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = BM\left(I\right)$ is a standardized Brownian motion.

If we allow for correlation in the error processes, a major problem arises. Although we could still estimate our regression equation with OLS and construct the test statistic in equation (9), given the serial correlation and endogenous x_{it}, this estimation is asymptotically biased. Unbiased estimation requires the use of an optimal estimator such as the fully modified (FM) estimator (e.g., Phillips and Hansen, 1990) or the dynamic ordinary least squares (DOLS) estimator of Saikkonen (1991) to correct for the problems.

Remark 2 *Recently, Kao and Chiang (1997) examined the limiting distribution of OLS for (1) with common β across i and they showed the limiting distribution involved various nuisance parameters even asymptotically. Kao and Chiang (1997) and Pedroni (1996) proposed an FM estimator, and Kao and Chiang (1997) proposed a DOLS for (1) with common β across i. Kao and Chiang (1997) and Pedroni (1996) showed the FM and DOLS estimators are free of nuisance parameters.*

Define,

$$u_{it}^{+} = u_{it} - \widehat{\varpi}_{12}\widehat{\Omega}_{22}^{-1}\varepsilon_{it}$$

and

$$y_{it}^{+} = y_{it} - \widehat{\varpi}_{12}\widehat{\Omega}_{22}^{-1}\varepsilon_{it}.$$

The FM estimator is given by

$$\hat{\beta}_{iFM} = (X_i'X_i)^{-1}(X_i'y_i^{+} - e_k T\hat{\delta}^{+}) \tag{16}$$

where $e_k = [0, I_k]'$ and

$$\begin{aligned}
\hat{\delta}^{+} &= \widehat{\Pi}_2 \begin{bmatrix} 1 \\ -\widehat{\Omega}_{22}^{-1}\widehat{\varpi}_{21} \end{bmatrix} \\
&= \begin{bmatrix} \widehat{\Pi}_{21} & \widehat{\Pi}_{22} \end{bmatrix} \begin{bmatrix} 1 \\ -\widehat{\Omega}_{22}^{-1}\widehat{\varpi}_{21} \end{bmatrix} \\
&= \widehat{\Pi}_{21} - \widehat{\Pi}_{22}\widehat{\Omega}_{22}^{-1}\widehat{\varpi}_{21},
\end{aligned}$$

where $\left(\widehat{\varpi}_{12}, \widehat{\Omega}_{22}^{-1}, \widehat{\Pi}_{21}, \widehat{\Pi}_{22}\right)$ are consistent estimators of $\left(\varpi_{12}, \Omega_{22}^{-1}, \Pi_{21}, \Pi_{22}\right)$.

Remark 3 *From Phillips and Hansen (1990), we know that the FM estimator has a compound normal limit distribution and is free of nuisance parameters.*

First, we define a modified test statistic based on the FM estimator:

$$LM^{+} = \frac{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^{2}}\sum_{t=1}^{T}S_{it}^{+2}}{\widehat{\varpi}_{1.2}^{2}}, \tag{17}$$

where

$$S_{it} = \sum_{j=1}^{t}\widehat{e}_{ij}^{+},$$

and

$$\widehat{e}_{ij}^{+} = y_{it}^{+} - \alpha_i - x_{it}\hat{\beta}_i^{+},$$

where $\widehat{\varpi}_{1.2}^{2}$ is a consistent estimator of $\varpi_{1.2}^{2}$.

Remark 4 *LM^{+} can also be written as*

$$LM^{+} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\frac{1}{T^{2}}\sum_{t=1}^{T}S_{it}^{+2}}{\widehat{\varpi}_{1.2}^{2}}\right). \tag{18}$$

Thus, LM^{+} can be seen as the average of the LBUI test statistics across the cross-sectional units. Note that $\widehat{\varpi}_{1.2}^{2}$ is the estimate of $\varpi_{1.2}^{2}$ over all N and T. Since the error terms are i.i.d. across i it is possible to obtain $\widehat{\varpi}_{1.2}^{2}$ from individual time series residuals. In practice we recommend estimating $\varpi_{1.2}^{2}$ for each i because LM^{+} can be obtained by taking the sample average of $\frac{\frac{1}{T^{2}}\sum_{t=1}^{T}S_{it}^{+2}}{\widehat{\varpi}_{1.2}^{2}}$. Also $\frac{\frac{1}{T^{2}}\sum_{t=1}^{T}S_{it}^{+2}}{\widehat{\varpi}_{1.2}^{2}}$ can be found easily by econometrics packages, e.g., COINT 2.0.

Remark 5 *From (18) we note that we actually do not need to assume constant variances for $w_t = \left(u_{it}, \varepsilon_{it}^{'}\right)^{'}$. In fact, we could allow $\varpi_{1.2}^{2}$ be different for different i, i.e., $\varpi_{i1.2}^{2} \neq \varpi_{j1.2}^{2}$ for $i \neq j$. It implies that LM^{+} in (18) is robust with respect to heteroskedasticity. However, if we assume $\varpi_{i1.2}^{2} \neq \varpi_{j1.2}^{2}$, as pointed out by a referee, equation (4) becomes a seemingly unrelated regression (SUR) model. Further, the assumption of independence across i will make SUR and single equation least squares be identical. In this sense, LM^{+} in (18) cannot improve anything in comparison with LBUI tests for each individual series.*

Before going into the first theorem of the paper, we need to consider some preliminary results. All limits in Lemma 1 and Theorem 1 use sequential asymptotics in which $T \to \infty$ followed by $N \to \infty$ (e.g., Phillips and Moon, 1997).

Lemma 1 *If Assumptions 1 and 2 hold, then*

$$LM^{+} \xrightarrow{p} \mu_v,$$

where $\mu_v \equiv E\left[\int V^2\right]$ *and* V *is defined as in Harris and Inder (1994) and Shin (1994).*

We are now in a position to state the following theorem:

Theorem 1 *Assume Assumptions 1-3 hold. Then, the test statistics in (17) for the null of cointegration have the following limiting distributions:*

$$\sqrt{N}\left(LM^{+} - \mu_v\right) \Rightarrow N\left(0, \sigma_v^2\right),$$

where $\sigma_v^2 = Var\left(\int V^2\right).$

Remark 6 $\mu_v \equiv E\left[\int V^2\right]$ *and* $\sigma_v^2 \equiv Var\left(\int V^2\right)$ *can be found by simulation as Im et al. (1995) and Harris and Inder (1994) suggested. The estimation of* μ_v *and* σ_v^2 *is discussed in Appendix C. We note from Theorem 1 that the asymptotic distribution of* LM^{+} *is free of nuisance parameters.*

Remark 7 *If we estimate* β_i *with* $\widehat{\beta}_{iD}$, *the DOLS of* β_i, *by running the following regression:*

$$y_{it} = \alpha_i + x_{it}^{'}\beta_i + \sum_{j=-q}^{q} c_{ij} \triangle x_{it+j} + v_{it}^{*}. \tag{19}$$

for some q, *then we can construct the LM statistics similar to (17) and (18). Since the limiting distributions of* $\widehat{\beta}_{iD}$ *and* $\hat{\beta}_{iFM}$ *are the same, it is straightforward to show that the LM statistic of the null of cointegration using* $\widehat{\beta}_{iD}$ *has the same limiting distribution given in Theorem 1.*

4 Monte Carlo Simulations

To examine the finite sample properties of the proposed tests, we conducted Monte Carlo experiments similar to the design of Harris and Inder (1993) and Phillips and Loretan (1991). The data generating process (DGP) was

$$y_{it} = \alpha_i + \beta_i x_{it} + \theta \sum_{j=1}^{t} u_{ij} + u_{it},$$

$$u_{it} = \phi_{it} + \pi\phi_{it-1},$$

and

$$x_{it} = x_{it-1} + \varepsilon_{it},$$

for $i = 1, ..., N$, $t = 1, ...T$, where

$$\begin{pmatrix} \phi_{it} \\ \varepsilon_{it} \end{pmatrix} \overset{iid}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \delta\sigma \\ \delta\sigma & \sigma^2 \end{bmatrix} \right).$$

The RNDN procedure in GAUSS was used to generate the random numbers. The data were generated by creating $T + 1,000$ observations and discarding the first $1,000$ observations to remove the effect of the initial conditions. The results we report are based on $10,000$ replications. We generated α_i and β_i from uniform distributions $U[0, 10]$ and $U[0, 2]$ respectively. $\{u_{it}\}$ and $\{\varepsilon_{it}\}$ were constructed with $u_{i0} = 0$ and $\varepsilon_{i0} = 0$. The FM estimator and

$$\widehat{\varpi}_{1.2}^2 = \widehat{\varpi}_1^2 - \widehat{\varpi}_{12}\widehat{\Omega}_{22}^{-1}\widehat{\varpi}_{21}$$

were obtained by using the procedure FM in COINT 2.0 with a Bartlett window of lag length 5. The DOLS estimator was obtained using the procedure PS in COINT 2.0 with one lag and one lead.

For the purposes of this paper, we consider the following values for the parameters in the DGP: $\delta = (-0.5, 0, .05)$, $\theta = (0, 0.05, 0.15, 0.25)$, and $\pi = (-0.8, 0, 0.8)$. Choosing different values for σ did not affect the results and the results are reported only for $\sigma = 1$. When $\theta \neq 0$, y_{it} and x_{it} are not cointegrated, and when $\theta = 0$ they are cointegrated. We obtained test results using both the FM and DOLS estimation: LM-FM and LM-DOLS test statistics. Both methods have their own advantages to recommend their use: the LM-DOLS test statistic uses the asymptotic moments for all values of T and does not require nonparametric correction; the LM-FM test statistic does not depend as crucially on the proper specification of a lag and lead order.

Table I presents the Monte Carlo mean, $\mu_v \equiv E\left[\int V^2\right]$ and variance, $\sigma_v^2 \equiv Var\left(\int V^2\right)$. These were generated setting $T = 4,000$ and approximating the functional of Brownian motion of the limiting distribution (See Appendix C). In Table I these are given under the heading $T = *$ and are generated allowing the number of regressors to vary from 1 to 5. For use of the test with small sample sizes, however, these moments may not be applicable and we therefore generated finite sample moments of the test setting $\theta = 0, \delta = 0$, and $\sigma = 1$. These were generated using $50,000$ replications of the test where $N = 1$ and T is given. These are approximations of the finite sample moments to be used in applications of the test. Again, the regressors were allowed to vary from 1 to 5. With the LM-FM test statistic, adding regressors increases the demands on the data for the non-parametric correction. Thus, as the number of regressors increases, the sample size appropriate for the test increases. With the LM-DOLS test statistic, the moments for $T = 4,000$ were used in all cases.

Figures 1 and 2 show a comparison of the simulated LM-FM moments using three and five lags and LM-DOLS moments using both one lead and one lag and two leads and two lags. For these simulations k, the number of regressors, is equal to one. It is clear that in almost all cases the LM-DOLS moments are closer than the LM-FM to the simulated Brownian motion moments. In particular, the simulated LM-FM variances are quite far away from the asymptotic variance for small T. Thus, while using the asymptotic moments for the LM-DOLS test does not significantly bias the results, using the asymptotic moments with the LM-FM test does. Also, it can be seen that increasing the number of lags and leads in the LM-DOLS test does not significantly affect the moments. Finally, both tests do have moments that approach the asymptotic moments as T increases.

Table II presents the estimates of the empirical size of LM-FM and LM-DOLS at the 5% level when the asymptotic critical value of 1.645 is used. The

TABLE I

Means and Variances for given T and k

	k=1	k=2	k=3	k=4	k=5		k=1	k=2	k=3	k=4	k=5
T=15	.3214					T=28	.2136	.2327	.2515		
	.0789						.0320	.0420	.0448		
T=16	.3077					T=29	.2049	.2274	.2437		
	.0720						.0307	.0407	.0423		
T=17	.2948					T=30	.2027	.2209	.2380		
	.0680						.0301	.0384	.0412		
T=18	.2826					T=35	.1863	.1981	.2102	.2200	
	.0625						.0238	.0294	.0315	.0307	
T=19	.2732					T=40	.1763	.1821	.1908	.1973	.2052
	.0581						.0208	.0242	.0258	.0258	.0254
T=20	.2622					T=45	.1688	.1704	.1768	.1831	.1873
	.0543						.0187	.0214	.0220	.0232	.0217
T=21	.2522					T=50	.1621	.1622	.1660	.1692	.1757
	.0494						.0168	.0188	.0195	.0192	.0202
T=22	.2451					T=60	.1528	.1481	.1486	.1514	.1547
	.0460						.0147	.0151	.0155	.0153	.0155
T=23	.2370					T=70	.1479	.1388	.1374	.1389	.1418
	.0415						.0137	.0128	.0133	.0132	.0134
T=24	.2329					T=80	.1435	.1318	.1286	.1300	.1317
	.0438						.0131	.0118	.0115	.0116	.0117
T=25	.2243	.2545	.2770			T=90	.1395	.1271	.1220	.1223	.1244
	.0360	.0500	.0541				.0127	.0107	.0099	.0102	.0106
T=26	.2201	.2464	.2694			T=100	.1372	.1219	.1220	.1163	.1179
	.0362	.0450	.0520				.0123	.0099	.0091	.0093	.0098
T=27	.2136	.2371	.2592			T=*	.1162	.0850	.0658	.0533	.0440
	.0330	.0421	.0490				.0109	.0055	.0028	.0016	.0009

Note:
(a) The number of replications = 50,000.
(b) $\alpha_i \sim U[0, 10]$ and $\beta_i \sim U[0, 2]$.
(c) The lag length of the Bartlett windows = 5.
(d) T=* was done with T=4000 using Brownian motion approximation.

TABLE II

Empirical Size of the LM-FM and LM-DOLS Tests

	N = 1		N = 15		N = 25		N = 50		N = 100	
	LM-FM	LM-DOLS	LM-FM	LM-DOLS	LM-FM	LM-DOLS	LM-FM	LM-DOLS	LM-FM	LM-DOLS
$T = 15$	.059	.017	.071	.002	.073	.001	.071	.000	.072	.000
	1.851	.574	1.942	-0.913	1.892	-1.495	1.896	-2.705	1.896	-4.392
$T = 25$	.057	.021	.064	.002	.068	.001	.067	.000	.070	.000
	1.792	.874	1.840	-0.176	1.889	-0.633	1.831	-1.444	1.863	-2.594
$T = 50$	.059	.039	.062	.010	.064	.006	.062	.001	.059	.000
	1.844	1.395	1.796	.709	1.810	0.488	1.783	.043	1.745	-.057
$T = 100$	.069	.049	.060	.028	.057	.021	.055	.009	.057	.003
	1.922	1.631	1.768	1.273	1.727	1.120	1.696	.815	1.731	.496

Note:
(a) The number of replications = 10,000.
(b) $\alpha_i \sim U[0, 10]$ and $\beta_i \sim U[0, 2]$.
(c) The lag length of the Bartlett windows = 5 for LM-FM test.
(d) The 1 lag and 1 lead are used for the LM-DOLS test.
(e) Size based on one-sided N(0,1) critical value 1.645.
(f) 5 percent critical values are listed underneath size values.
(g) $\pi = 0, \delta = 0$.

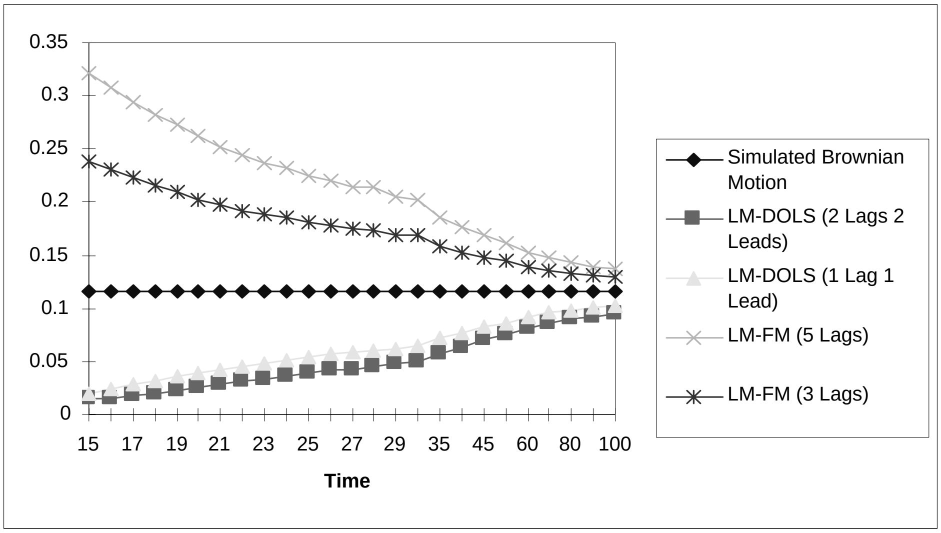

Figure 1: Simulated Means with k = 1

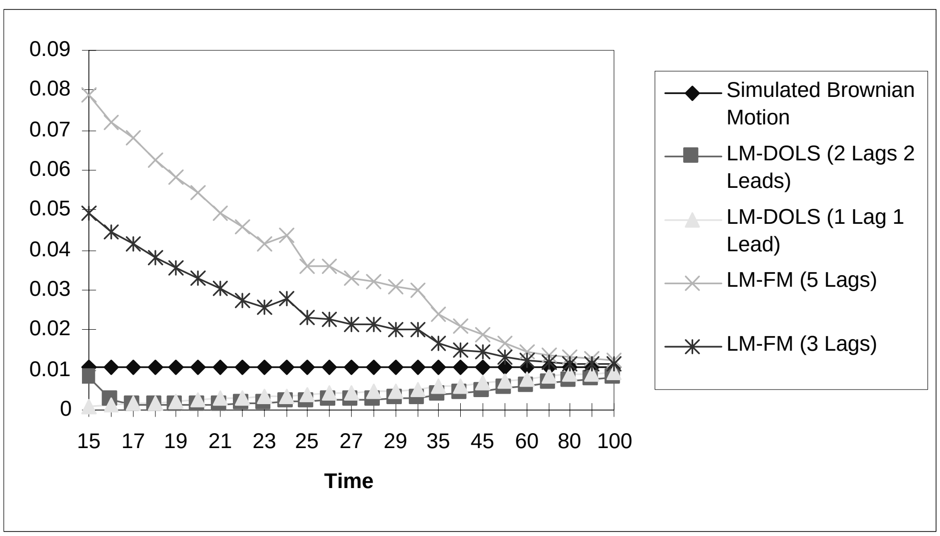

Figure 2: Simulated Variances with k = 1

tests are considered with no moving average component ($\pi = 0$) and with strong exogeneity ($\delta = 0$). Under this specification, it is notable that for the LM-FM test statistic even in small sample sizes the empirical size of the test is reasonably close to .05. In fact, when $T = 15$ and $N = 15$ the empirical size is .0706. For the largest sample recorded, $T = 100$ and $N = 100$, there is a slight improvement to a size of .0569. In general, this test performs well under the null hypothesis. The results using LM-DOLS are somewhat different. The LM-DOLS test has a much greater propensity to under- rather than over-reject the null hypothesis. For all N, increasing the observations in T does result in critical values closer to 1.645.

Table III presents the size-adjusted power of LM-FM and LM-DOLS. Again $\pi = 0$ and $\delta = 0$. Three specifications of θ are tested: $0.25, 0.15$ and 0.05. The power is adjusted using the critical values from Table II. The first observation is that the value of θ influences the power of the test. As θ approaches zero, the power of the test diminishes in almost all cases, although even with $\theta = .05$ and large enough T and N the power is approximately 1. With $\theta = .15$ the power is even stronger, and when $T = 50$ and $N = 50$ the power is .9864 for the LM-FM and .9697 for the LM-DOLS. The second observation is that in general for all θ the T dimension is more important than the N dimension in raising the power of the test. For example, with the LM-FM test statistic at $\theta = .15$, if we fix T at 25 and increase N from 15 to 100 the power only increases from .1515 to .4107. However, if we fix N at 25 and increase T from 15 to 100 the power increases from .0835 to .9999. This test, therefore, would not be as powerful with data sets with large cross-section dimensions and small time dimensions, especially for θ close to 0. In the pure time series case, $N = 1$, the maximum power reached for the test was .5170 for the LM-FM test and .5562 for the LM-DOLS test when $T = 100$ and $\theta = .25$.

The greatest difference between the two tests is the performance when $T \leq 25$. In general, the LM-FM test has greater power. Further, increasing the size on N has a greater impact on the power of the LM-FM test than the LM-DOLS test with small T. For example, with $T = 25$ and $\theta = .25$, increasing N from 15

TABLE III

Size-Corrected Power of the LM-FM and LM-DOLS Tests

		$\theta = .05$		$\theta = .15$		$\theta = .25$	
		LM-FM	LM-DOLS	LM-FM	LM-DOLS	LM-FM	LM-DOLS
$N = 1$							
	$T = 15$	.0496	.0451	.0557	.0378	.0642	.0312
	$T = 25$	.0553	.0479	.0767	.0564	.0983	.0645
	$T = 50$	.0761	.0675	.1824	.1670	.2559	.2303
	$T = 100$	.1723	.1604	.4190	.4415	.5170	.5562
$N = 15$							
	$T = 15$	.0529	.0373	.0732	.0207	.1076	.0114
	$T = 25$	.0584	.0406	.1515	.0491	.2847	.0576
	$T = 50$	.1421	.1218	.7057	.6599	.9178	.8827
	$T = 100$	.6720	.6465	.9988	.9989	.9999	.9999
$N = 25$							
	$T = 15$	.0552	.0332	.0835	.0152	.1290	.0068
	$T = 25$	.0618	.0395	.1793	.0515	.3622	.0607
	$T = 50$	.1750	.1419	.8659	.8248	.9836	.9713
	$T = 100$	.8287	.8080	.9999	.9999	.9999	.9999
$N = 50$							
	$T = 15$	.0541	.0308	.0963	.0108	.1731	.0045
	$T = 25$	.0708	.0355	.2731	.0506	.5770	.0613
	$T = 50$	.2519	.1890	.9864	.9697	.9999	.9997
	$T = 100$	.9733	.9659	.9999	.9999	.9999	.9999
$N = 100$							
	$T = 15$	.0548	.0261	.1158	.0070	.2574	.0021
	$T = 25$	.0791	.0288	.4107	.0467	.8118	.0684
	$T = 50$	.4022	.2736	.9999	.9995	.9999	.9999
	$T = 100$	.9999	.9994	.9999	.9999	.9999	.9999

Note:
(a) The number of replications = 10,000.
(b) $\alpha_i \sim U[0, 10]$ and $\beta_i \sim U[0, 2]$.
(c) Power is corrected with critical values from Table 2.
(d) The lag length of the Bartlett windows = 5 for LM-FM test.
(e) The 1 lag and 1 lead are used for the LM-DOLS test.
(f) $\pi = 0, \delta = 0$.

to 50 increases the power of the LM-FM test from .2847 to .5770, while the power of the LM-DOLS test scarcely improves, moving only from .0576 to .0613. Most significantly, perhaps, is the result that for $T = 15$ increasing θ does not improve the power for the LM-DOLS. In general, the low power for small T for DOLS-FM might be partially explained by the range in θ. One common representation of a non-stationary variable is the random walk where the variable remembers each past error "exactly." In this study, non-stationarity is defined as "partial but permanent" memory of past errors. With small T, the LM-DOLS test seems unable to distinguish between the "partial but permanent" memory and "finite memory" of past errors.

To examine the robustness of our specification, we examine size and size-adjusted power for a variety of different values of π and δ in Tables IV and V. Table IV contains estimates of the size of tests at the 5% level when the asymptotic critical value, 1.645, from the $N(0, 1)$ distribution was used. Sample size is fixed at $N = 50$ and $T = 50$. When $\pi < 0$, a negative moving average component, the test does not reject the null hypothesis as often as the theory would predict for both estimation procedures. In fact, in all cases presented with a negative moving average component, the test never rejects the null hypothesis. When $\pi > 0$ and $\delta = 0$, the test over-rejects the null hypothesis with the LM-FM, the empirical size is .1835, and under-rejects the null with the LM-DOLS, empirical size of .0282. The endogeneity parameter, δ, also has a significant impact on the results. When $\pi < 0$ and $\delta \neq 0$, the negative moving average seems to dominate the results. However, when $\pi > 0$ and $\delta \neq 0$, the endogeneity and positive moving average seemingly reinforce each other and the test over-rejects the null for both estimation procedures. In particular, when $\delta \neq 0$ and $\pi > 0$, the test almost always rejects the null hypothesis with empirical sizes ranging from .9984 to .9997 for the LM-FM test statistic and from .6480 to .6484 for the LM-DOLS test statistic. The most obvious difference in the performance of the two methods comes when $\pi \geq 0$ and $\delta \neq 0$. In this case, the LM-DOLS test has empirical sizes much lower than the LM-FM test. This seems somewhat counter to the theory of the LM-FM test statistic, which should correct for δ

TABLE IV

Empirical Size for Different DGP Specifications of the LM-FM and LM-DOLS Tests

		LM-FM	LM-DOLS
$\delta = -0.5$			
	$\pi = -0.8$	.0000	.0000
	$\pi = 0.0$	.9989	.1878
	$\pi = 0.8$	.9997	.6484
$\delta = 0.0$			
	$\pi = -0.8$	.0000	.0000
	$\pi = 0.0$	.0620	.0010
	$\pi = 0.8$	.1835	.0282
$\delta = 0.5$			
	$\pi = -0.8$	.0000	.0000
	$\pi = 0.0$	.9984	.1884
	$\pi = 0.8$	.9993	.6480

Note:
(a) The number of replications = 10,000.
(b) $\alpha_i \sim U[0, 10]$ and $\beta_i \sim U[0, 2]$.
(c) The lag length of the Bartlett windows = 5 for LM-FM test.
(d) The 1 lag and 1 lead are used for the LM-DOLS test.
(e) N=50 and T=50.

TABLE V

Size-Corrected Power for Different DGP Specifications of the LM-FM and LM-DOLS Tests

		LM-FM	LM-DOLS
$\delta = -0.5$			
	$\pi = -0.8$	.9999	.6065
	$\pi = 0.0$	.3775	.9996
	$\pi = 0.8$	.4607	.8857
$\delta = 0.0$			
	$\pi = -0.8$	.9999	.6319
	$\pi = 0.0$	.9999	.9997
	$\pi = 0.8$	.9994	.8855
$\delta = 0.5$			
	$\pi = -0.8$	.9999	.6253
	$\pi = 0.0$	.3629	.9999
	$\pi = 0.8$	.4484	.8833

Note:
(a) The number of replications = 10,000.
(b) $\alpha_i \sim U[0, 10]$ and $\beta_i \sim U[0, 2]$.
(c) The lag length of the Bartlett windows = 5 for LM-FM test.
(d) The 1 lag and 1 lead are used for the LM-DOLS test.
(e) N=50 and T=50.
(f) $\theta = .25$.

asymptotically. In this fixed sample size, however, $\delta \neq 0$ significantly impacts the results of the test. In the sample sizes considered, it seems that the LM-DOLS test does a better job of correcting for the two effects. In fact, for the LM-FM the finite sample moments calculated in Table I may not be good approximations when endogeneity of error terms is present.

Table V contains estimates of the size-adjusted power of the LM-FM and LM-DOLS tests at the 5% level. Sample size is fixed at $N = 50$ and $T = 50$. The case where $\theta = .25$ is considered. The performance of the tests here is quite different. The power of the LM-FM test statistic is greatest for a negative moving average component, $\pi < 0$, while for the LM-DOLS test this is exactly the case where the power is lowest. For the LM-FM test with a negative moving average component, the power is as great as when no moving average is present. Also, in the case of a positive moving average and no endogeneity, $\pi > 0$ and $\delta = 0$, the LM-FM test performs quite well. Similar to the performance documented in Table 4, the test does not perform well when $\pi \geq 0$ and $\delta \neq 0$, with power ranging in that case from .3629 to .4607. It is exactly under these conditions that the LM-DOLS test has the highest power, with values ranging from .8855 to .9999.

The choice of the lag length 5 for a Barlett window could be too big for $T = 15$ or 25 for the results in Tables II-V, as pointed out by a referee. In Tables VI and VII, we extend the experiments by changing the lag length from 5 to 3 for a Barlett window. Overall, the results show that chaiging the lag length from 5 to 3 does not lead to substantial changes in the size and power of the LM-FM test.

5 Conclusion

In this paper, we propose a test for the null hypothesis of cointegration in panel data and derive asymptotic distributions for each test. This test provides an alternative approach to testing for cointegration in panel data. The test proposed is directly derived from the test for the variance co-variance matrix of the error terms of a panel of cross-sectional variables observed over time which

TABLE VI

Empirical Size of the LM-FM Test with Different Lag Lengths for the Bartlett Window

	N = 1		N = 15		N = 25		N = 50		N = 100	
	LM-FM(5)	LM-FM(3)	LM-FM(5)	LM-FM(3)	LM-FM(5)	LM-FM(3)	LM-FM(5)	LM-FM(3)	LM-FM(5)	LM-FM(3)
T=15	.0586	.0542	.0706	.0692	.0727	.0706	.0706	.0699	.0720	.0690
	1.8510	1.7443	1.9417	1.9080	1.8922	1.929	1.8962	1.8974	1.8964	1.8395
T=25	.0568	.0562	.0636	.0643	.0676	.0667	.0672	.0679	.0700	.0672
	1.7917	1.7937	1.8402	1.8039	1.8889	1.8506	1.8311	1.8562	1.8633	1.8237
T=50	.0597	.0671	.0623	.0624	.0640	.0612	.0620	.0573	.0590	.0560
	1.8441	1.9309	1.7959	1.8199	1.8101	1.7838	1.7832	1.7145	1.7451	1.7106
T=100	.0690	.0678	.0601	.0615	.0574	.0579	.0550	.0456	.0569	.0591
	1.9221	1.9899	1.7681	1.7866	1.7269	1.7480	1.6964	1.6965	1.7312	1.7407

Note:
(a) The number of replications = 10,000.
(b) $\alpha_i \sim U[0, 10]$ and $\beta_i \sim U[0, 2]$.
(c) The lag length of the Bartlett windows = 5 for LM-FM(5) and =3 for LM-FM(3) tests.
(d) Size based on one-sided N(0,1) critical value 1.645.
(e) 5 percent critical values are listed underneath size values.
(f) $\pi = 0, \delta = 0$.

TABLE VII

Size-Corrected Power of the LM-FM Test with Different Lag Lengths for the Bartlett Window

		$\theta = .05$		$\theta = .15$		$\theta = .25$	
		LM-FM(5)	LM-FM(3)	LM-FM(5)	LM-FM(3)	LM-FM(5)	LM-FM(3)
N=1							
	T=15	.0496	.0488	.0557	.0568	.0642	.0635
	T=25	.0553	.0520	.0767	.0853	.0983	.1222
	T=50	.0761	.0824	.1824	.2153	.2559	.3100
	T=100	.1723	.1798	.4190	.4682	.5170	.6019
N=15							
	T=15	.0529	.0537	.0732	.0797	.1076	.1184
	T=25	.0584	.0646	.1515	.1914	.2847	.3947
	T=50	.1421	.1711	.7057	.8377	.9178	.9797
	T=100	.6720	.7278	.9988	.9997	.9999	.9999
N=25							
	T=15	.0552	.0527	.0835	.0830	.1290	.1368
	T=25	.0618	.0664	.1793	.2343	.3622	.5212
	T=50	.1750	.2281	.8659	.9547	.9836	.9983
	T=100	.8287	.8080	.9999	.9999	.9999	.9999
N=50							
	T=15	.0541	.0544	.0963	.1009	.1731	.1939
	T=25	.0708	.0727	.2731	.3537	.5770	.7590
	T=50	.2519	.3439	.9864	.9985	.9999	.9999
	T=100	.9733	.9875	.9999	.9999	.9999	.9999
N=100							
	T=15	.0548	.0596	.1158	.1376	.2574	.3168
	T=25	.0791	.0882	.4107	.5730	.8118	.9543
	T=50	.4022	.5263	.9999	.9999	.9999	.9999
	T=100	.9999	.9999	.9999	.9999	.9999	.9999

Note:
(a) The number of replications = 10,000.
(b) $\alpha_i \sim U[0,10]$ and $\beta_i \sim U[0,2]$.
(c) Power is corrected with critical values from Table 2.
(d) The lag length of the Bartlett windows = 5 for LM-FM(5) and =3 for LM-FM(3) tests.
(e) $\pi = 0, \delta = 0$.

is a LBUI for the panel. This test is free of nuisance parameters asymptotically and can allow for heteroskedascity.

We then demonstrate the small sample properties of the proposed test using two test statistics: LM-FM and LM-DOLS. Overall, the empirical sizes of the LM-FM and LM-DOLS are close to .05 even in small samples. The power was quite good for the panels where $T \geq 50$, and decent with panels for fewer observations in T. In our fixed sample of $N = 50$ and $T = 50$, the presence of a moving average and endogeneity of error terms did cause the two tests to perform quite differently, complicating the choice of estimation procedures. In general, the LM-DOLS test seemed to be better at correcting these effects, although in some cases the LM-FM test resulted in a more powerful test.

Although much of the non-stationary time series econometrics has been criticized for having more to do with the specific properties of the data set than with underlying economic models, the recent development of the cointegration literature has allowed for a concrete bridge between economic long run theory and time series methods. Our test now allows for the testing of the null of cointegration in a panel setting and should be of considerable interest to economists in a wide variety of fields.

Appendix

A Proof of Lemma 1

Proof. First, we note that

$$\begin{aligned} \frac{1}{T^2}\sum_{t=1}^{T} S_{it}^{+2} &= \frac{1}{T^2}\sum_{t=1}^{T}\left(\sum_{j=1}^{t}\widehat{e}_{ij}^{+}\right)^2 \\ &\Rightarrow \left(\varpi_{1.2}^2 \int V^2\right) \end{aligned}$$

from Harris and Inder (1994), as $T \to \infty$ for fixed N, where V is a functional of Brownian motions. Then if we appy the law of large numbers to

$\frac{1}{N}\sum_{i=1}^{N}\left(\varpi_{1.2}^{2}\int V^{2}\right)$, we have as $N\rightarrow\infty$,

$$\frac{1}{N}\sum_{i=1}^{N}\left(\varpi_{1.2}^{2}\int V^{2}\right)\overset{p}{\rightarrow}E\left[\varpi_{1.2}^{2}\int V^{2}\right]=\varpi_{1.2}^{2}E\left[\int V^{2}\right].$$

Hence, we have shown that

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^{2}}\sum_{t=1}^{T}S_{it}^{+2}\overset{p}{\rightarrow}E\left[\varpi_{1.2}^{2}\int V^{2}\right]=\varpi_{1.2}^{2}E\left[\int V^{2}\right].$$

as $T\rightarrow\infty$ first and then $N\rightarrow\infty$ by the sequential limit theory. It follows that

$$\begin{aligned} LM^{+} &= \frac{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^{2}}\sum_{t=1}^{T}S_{it}^{+2}}{\widehat{\varpi}_{1.2}^{2}} \\ &\overset{p}{\rightarrow} \frac{\varpi_{1.2}^{2}E\left[\int V^{2}\right]}{\varpi_{1.2}^{2}} \\ &= E\left[\int V^{2}\right]=\mu_{v} \end{aligned}$$

establishing Lemma 1 since $\widehat{\varpi}_{1.2}^{2}\overset{p}{\rightarrow}\varpi_{1.2}^{2}$. ■

B Proof of Theorem 1

Proof. By Lemma 1 we know that

$$LM^{+}=\frac{1}{N}\sum_{i=1}^{N}\left\{\frac{\frac{1}{T^{2}}\sum_{t=1}^{T}S_{it}^{+2}}{\widehat{\varpi}_{1.2}^{2}}\right\}\overset{p}{\rightarrow}\mu_{v}.$$

The next step is to find an appropriate normalization of LM^{+} to make sure it converges to a proper random variable. Note that for the fixed N as $T\rightarrow\infty$ we have

$$\sqrt{N}\left(\frac{1}{N}\sum_{i=1}^{N}\left\{\frac{\frac{1}{T^{2}}\sum_{t=1}^{T}S_{it}^{+2}}{\widehat{\varpi}_{1.2}^{2}}\right\}-\mu_{v}\right)\Rightarrow\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\int V^{2}-\mu_{v}\right)$$

which will converge to a normal variable by the Lindberg-Levy central limit theorem as $N\rightarrow\infty$:

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\int V^{2}-\mu_{v}\right)\Rightarrow N\left(0,\sigma_{v}^{2}\right),$$

where $\sigma_v^2 \equiv Var\left(\int V^2\right)$. Then by the sequential limit theory we have the following

$$\sqrt{N}\left(LM^{+} - \mu_v\right) \Rightarrow N\left(0, \sigma_v^2\right)$$

as $T \to \infty$ and then $N \to \infty$ proving Theorem 1.

■

C Computing $\int V^2$, $E\left[\int V^2\right]$, and $Var\left[\int V^2\right]$

We thank David Harris for the discussion of the computation and sharing his GAUSS code with us. One way to approach this problem is to think of a Brownian motion that can be approximated by a large standardized normal random walk. This is just the reverse of doing the asymptotics, where we let the random walk be approximated by a Brownian motion (BM). Let the random walk be

$$x_t = x_{t-1} + \varepsilon_t,$$

where ε_t is *i.i.d.* $N(0,1)$, $x_0 = 0$, $t = 1, ..., T$. A large T is chosen (say $4,000$) to try to replicate an infinite sample as closely as possible. From the functional central limit theorem we know that

$$T^{-1/2} \sum_{t=1}^{[Tr]} \varepsilon_t \Rightarrow W(r),$$

where $W(r)$ is a standard BM and $0 \leq r \leq 1$. For a given r, an approximate single realization from a BM is therefore

$$W(r) \approx T^{-1/2} \sum_{t=1}^{[Tr]} \varepsilon_t.$$

Then think of the integral as a sample mean, i.e., replace $\int_0^1$ with $\frac{1}{T}\sum_{t=1}^{T}$. Again, this is just the opposite of the asymptotics. Thus, to simulate $\int_0^1 W(r)dr$ replace $W(r)$ by $T^{-1/2}\sum_{t=1}^{[Tr]} x_t$ and $\int_0^1$ by $\frac{1}{T}\sum_{t=1}^{T}$, so a single realization is

$$\int_0^1 W(r)dr \approx T^{-3/2} \sum_{t=1}^{[Tr]} x_t.$$

Similarly

$$\int_0^1 W^2(r)dr \approx T^{-2}\sum_{t=1}^{[Tr]} x_t^2.$$

The main idea is to look at the asymptotics and work backwards, i.e., replace the limit quantity with its sample counterpart artificially generated from with a large sample.

The other way to proceed is to simply simulate the test statistic under ideal conditions with a large sample.

Following the paper of Harris and Inder (1994) we simulate the mean and variance of $V(r)^2$, where

$$V(r) = W_{1.2}(r) - P^{-1}\mathbf{Q}r - (\int_0^r W_2(s)'ds)(\mathbf{S} - \mathbf{R}P^{-1}\mathbf{Q}),$$

with

$$\begin{aligned} P = &\quad 1 - \textstyle\int W_2^{'}(\int W_2 W_2^{'})^{-1}\int W_2, \\ P = &\quad 1 - \textstyle\int W_2'(\int W_2 W_2')^{-1}\int W_2, \\ \mathbf{Q} = &\quad W_{1.2}(1)^{'} - \textstyle\int W_2^{'}(\int W_2 W_2^{'})^{-1}\int W_2 dW_{1.2}, \\ \mathbf{R} = &\quad (\textstyle\int W_2 W_2^{'})^{-1}\int W_2, \\ \mathbf{S} = &\quad (\textstyle\int W_2 W_2^{'})^{-1}\int W_2 dW_{1.2}, \end{aligned}$$

and W_2 and W_{12} are independent standard Brownian motion. Note that $V(r)$ is independent of nuisance parameters and depends only on the dimension of k, the number of regressors. ■

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