

Spillovers Across Labor Markets in Search Models: Effects of “Animal Spirits” and Offshoring¹

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August, 2009

Abstract

We study the effects of additional distortions or shocks, such as a fair-wage constraint and/or the possibility of offshoring unskilled jobs, in a two-factor general equilibrium model of unemployment with search frictions. While the direct point of impact of such shocks is on unskilled workers, we also find interesting indirect spillover effects on skilled workers. A binding fair-wage constraint increases the unskilled unemployment rate and can at the same time lead to a higher unemployment rate and a lower wage for skilled workers, as compared to an equilibrium where fairness considerations are absent or non-binding. Introducing offshoring of unskilled jobs in our search model always leads to an increase in skilled wage, a decrease in skilled unemployment and an increase in unskilled unemployment. Also, offshoring makes it more likely that the fair-wage constraint binds.

Key words: Fair wages, unemployment, strategic effect, offshoring

JEL Classification Codes: E24, F16, F41

¹We would like to thank seminar participants at UC-Riverside, University of Melbourne and University of New South Wales and conference participants at the Dallas Fed-SMU Conference on Globalization, Political Economy and Trade Policy held at Southern Methodist University for useful comments and suggestions. We are also indebted to Carl Davidson and John Leahy for very helpful comments on an earlier version. The standard disclaimer applies.

1 Introduction

Over the last three decades, there has emerged a vast and well-developed literature on the theory of search unemployment both in closed and open economies.² Empirical support for the existence of substantial search frictions and their importance in generating unemployment has made the introduction of such frictions into economic models of labor markets the most standard and accepted way of modeling unemployment. This does not mean that there do not exist other additional distortions or other possible shocks that can interact with search frictions to magnify or reduce their impact. The objective of our paper is to study such interactions and their spillover from the point of impact to the rest of the economy. While this is primarily a theoretical paper, we focus on distortions and shocks that have empirical relevance.

The first distortion we introduce is a fair-wage constraint. Akerlof and Shiller (2009) argue that fairness is an important aspect of “animal spirits” or “the thought patterns that animate people’s ideas and feelings,” whose study is crucial in understanding how economies behave. Akerlof and Yellen (1990) actually focus on such fairness or fair-wage considerations to explain the existence of unemployment. Surveys of managers and workers, sociological studies of work environments, firm-level studies of pay structures, experiments, personnel management textbooks etc. provide a wealth of evidence supporting the assumption or idea of a fair-wage.³

The second shock we introduce in the model is the offshoring of unskilled jobs. The subject of offshoring has attracted a lot of attention from the public, politicians and the media, especially because developed-country firms have been shifting operations overseas or are contracting out manufacturing of their production and service inputs to businesses in developing countries to cut costs. The alarming, though questionable, estimates by Bardhan and Kroll (2003) and McKinsey (2005), that 11 percent of our jobs are potentially at risk of being offshored, further adds to the importance of the study of offshoring.

By introducing fair-wage concerns into a model of search unemployment with two types of labor,

²See Pissarides (2000) for an excellent and comprehensive treatment of search unemployment in a closed economy. Davidson, Martin and Matusz (1999), Davidson and Matusz (2004), Moore and Ranjan (2005), Helpman and Itskhoki (2007) are examples of models of open-economy models of search unemployment.

³See for instance Akerlof and Yellen (1990), Bewley (2005) and Howitt (2002) for a survey of the evidence.

namely skilled and unskilled, we obtain several new results. One of the key results, which we find quite interesting, is that introducing fairness considerations in a search model leads not only to an increase in the unemployment of unskilled workers, a group for whom the fair-wage constraint is binding, but also leads to a change in the unemployment of skilled workers, a group for whom the fair-wage constraint is not binding. That is, there could be a positive or negative spillover effect on the skilled labor market. Whether the spillover effect is positive or negative depends on the strength of the “strategic effect” in the wage and employment decision of firms, first identified by Stole and Zweibel (1996).⁴ Since wages are determined through Nash bargaining with workers in the second stage conditional on the employment choice in the first stage, the “strategic effect” refers to a firm’s incentive to over or under hire workers in the first stage to keep the wages lower in the second stage.⁵

Abstracting from the "strategic effect," it is shown that the introduction of the "fair-wage" constraint leads to an increase in the unemployment of skilled workers. Intuitively, a binding fair-wage constraint makes it more expensive to hire unskilled workers and hence raises the cost of producing output for competitive firms. The ensuing losses lead to a reduction in industry output and consequently losses of both skilled and unskilled labor jobs resulting in increased unemployment and lower wages of skilled labor as well. Increased skilled unemployment and lower skilled wage offsets the effect of fair wage constraint on the cost and hence profitability of competitive firms. The presence of the “strategic effect” makes the effect of the fair-wage constraint ambiguous on skilled unemployment, which now can go down for high enough values of the elasticity of substitution between skilled and unskilled labor in the firm’s production function.

The above results are consistent with the empirical fact that unemployment rates are higher for Europe relative to the US for all educational attainment categories of workers. According to the

⁴See Cahuc and Wasmer (2001) and Cahuc, Marque and Wasmer (2008) for comprehensive analyses of the role and importance of strategic effects in closed-economy models of search unemployment.

⁵In the absence of the fair wage constraint, the “strategic effects” of hiring of any one type of worker on the two wages (skilled and unskilled) offset each other. Interestingly, introducing a binding fairness (“fair-wage”) constraint results in a net “strategic effect” because the unskilled wage equals the fair wage which is taken as given by the firms. The strength of the "strategic effect" in our analysis depends on the elasticity of substitution between skilled and unskilled labor in the production function.

OECD Employment Outlook (2007), while the unemployment rate in the US in 2006 for people with less than secondary education was roughly 9 percent, it was 5 percent and 2.6 percent for people with upper secondary and tertiary education, respectively. The EU unemployment rates for the same year for the same three categories were 13, 7 and 4 percent respectively. At the same time, based on survey evidence of attitudes to poverty and income, one can argue that social norms of fairness are stronger in Europe than in the US. According to calculations by Alesina and Glaeser (2005), based on data from the World Values Survey for the years 1983-97, while only 29 percent of the responders from the US believe that the poor are trapped in poverty, about 60 percent of the European responders believe this to be the case. Furthermore, only 30 percent in the US believe that luck determines income, while 54 percent in the EU believe in luck being a determinant. Alesina and Glaeser also find that 60 percent of the Americans surveyed believe that the poor are lazy, while only 26 percent of the EU nationals surveyed believe so.⁶ Finally, using the International Social Survey Program (ISSP) surveys of public opinion, Osberg and Smeeding (2006) find in the case of the US "less concern for leveling up at the bottom of the distribution than in other nations." That is, there is less concern for raising the income of the poor relative to the mean income in the US than in other countries.

Importantly, for intermediate relative factor endowments, we also find that introducing fairness considerations in a search model of unemployment can lead to the possibility of multiple equilibria, which, as shown later in the paper, is very clearly driven by our "strategic effect." The implication here is that, in this intermediate range of skill abundance, countries with identical preference for fairness and with identical relative factor endowments can have different wages and unemployment rates.

The second major shock we introduce in our model of search frictions is the offshoring of unskilled jobs. In addition to directly affecting the unskilled labor market, it also has a spillover effect on the skilled labor market. In the absence of fairness considerations, offshoring has a

⁶Alesina and Angeletos (2005) point to the fact that while the pre-tax inequality is much higher in the US than in Europe (Gini coefficient of 38.5 as opposed to 29.1), the redistributive policies are much more extensive and the tax structure much more progressive in the latter. They argue that "the difference in political support for redistribution appears, rather, to reflect a difference in social perceptions regarding the fairness of market outcomes and the underlying sources of income inequality."

positive spillover effect on the skilled labor market in the form of higher skilled wage and lower skilled unemployment. However, the wage of unskilled workers goes down and their unemployment goes up. The reasons here have to do with the substitutability between domestic unskilled labor and the imported, offshored input and the complementarity of the two with skilled labor. For the same reasons, starting from an offshoring equilibrium, a decrease in the cost of the offshored input has the same impact on the two labor markets as a move from autarky to offshoring.

When fairness considerations and offshoring are introduced simultaneously, we get some additional results. Now, most importantly, there is a complementarity in both directions between offshoring and the fairness constraint in the following sense. While offshoring raises the likelihood that the fair-wage constraint binds, the introduction of the fair-wage constraint or an increase in the society's preference for fairness, through the impact on the cost of hiring domestic unskilled labor, increases the likelihood and magnitude of offshoring. Unlike in the autarky case, in the presence of offshoring taking place, the introduction of the fair-wage constraint or an increase in the society's preference for fairness leaves the skilled labor market totally unaffected. The reason is that the price of the imported input totally fixes the skilled labor market tightness. But this insensitivity of the skilled labor market can result in a more adverse effect on unskilled unemployment. The presence of fairness considerations makes the impact of offshoring on unskilled wage ambiguous because in addition to the direct downward pressure from the substitutable foreign input produced by foreign labor, there is an indirect upward pressure on unskilled wages coming from the fact that offshoring can convert a non-binding fairness constraint into a binding one.

In the next section, we will review the related literature on fair wages, offshoring and unemployment. This is followed by a presentation of the benchmark pure search model in section 3. We then introduce the fair-wage constraint and analyze its impact in section 4. We present in section 5 the impact of offshoring. We then discuss possible extensions in section 6 and conclude in section 7.

2 Related Literature

In this section, we discuss the related literature on fair wages, offshoring and unemployment. We start with Kreickemeier and Nelson (2006) who extend the Akerlof and Yellen (1990) model to a two-

sector setting to study the impact of international trade and technology shocks on unemployment and relative wages.⁷ They show that trade between US and Europe, with the preference for fairness being greater in the latter, leads to reduction in wage inequality and unemployment in the US and increase in wage inequality and unemployment in Europe. They also analyze how trade with the newly industrializing countries (NICs) affects wage inequality and unemployment differently in these two regions.

Cahuc and Zylberberg (2004) (in subsection 2.6 of Chapter 10 of their book), model the European labor market as one that is subject to search frictions and, at the same time, faces an endogenous minimum wage for unskilled workers that is proportional to the skilled wage. In that sense, it is similar to a "fair wage." However, their focus is limited to an evaluation of the effects of skill-biased technical change (and comparing those effects with the ones that arise in a pure search framework). By separating the two types of workers to work in different firms in different sectors, they abstract from cross-factor as well as own "strategic effects."

Another recent related paper is Grossman and Helpman (2008). In that paper, the utility derived by a worker is increasing in her own wage but decreasing in the average wage of the firm. This utility has to be above a threshold for the participation constraint of the worker to be satisfied. Thus, under certain conditions, Grossman and Helpman are able to get an equilibrium where in some firms, a large number of unskilled workers (relative to the number of skilled workers) work at a very low wage (firm-level average wage is low) and in others, a small number of unskilled workers work at very high wages (firm-level average wage is high). Since the effect of own wage and average firm wage are in opposite directions in the worker's utility function, the two types of firms result in the same level of worker utility. There is no unemployment in the Grossman-Helpman model since workers can be employed at very low wages as long as the average wage in the firm is low. In order to get rid of inefficiencies caused by jealousies or fairness considerations, firms offshore the work of unskilled workers in their model. The assumption driving this result is that for a multinational firm, only the average firm wage for operations carried out within the domestic boundaries of a country enters the domestic worker's utility function. Again, there is no unemployment of any kind in this case.

We should also mention here the two recent papers that investigate the impact of offshoring

⁷See also Agell and Lundborg (1995) and Egger and Kreickemeier (2009).

within a search framework. In their model of job search, Davidson, Matusz and Shevchenko (2006) study the impact of offshoring of high-tech jobs on low and high-skilled workers' wages, and on overall welfare. While their emphasis is on the effect of offshoring on relative wages, they also briefly discuss the impact on unemployment. Since job prospects for domestic high-skilled workers do not look as promising upon offshoring some of their jobs, they are willing to accept low-skill jobs and in turn increase the competition for such jobs among workers. Therefore, in the short run, unemployment goes up. In the long-run, however, there is a confounding factor, namely the entry of new firms arising out of an increase in profitability.

Mitra and Ranjan (2009) analyze the impact of offshoring on search unemployment. While there is only one factor, namely labor in that paper, the model allows for varying degrees of intersectoral labor mobility. At high degrees of labor mobility, the unemployment rate of workers in all sectors and activities goes down. As the degree of labor mobility decreases, it becomes more likely that unemployment increases in the sector where jobs are offshored. Offshoring always has the spillover effect of reducing unemployment in the other sector. These results are similar in spirit to results on offshoring in the current paper where unskilled labor, whose jobs are offshored, experiences an increase in unemployment and skilled workers, whose jobs are not offshored, always experience a reduction in unemployment.⁸

Finally, an interesting recent fair-wage model by Egger and Kreckemeier (2008) in an open-economy context analyzes the impact of fragmentation (or offshoring) on the unemployment of unskilled labor. While the authors of that paper abstract from search unemployment in general and skilled unemployment in particular, they derive some interesting results. One of the key results in that paper is that in a sufficiently skill-abundant country, fragmenting the production of a relatively unskilled labor-intensive good into two activities, one more skill-intensive than the other, gives that country an opportunity of getting partially involved in the production of that good along with the production of a more skill-intensive good (that it would have produced in any

⁸Two other papers looking at the impact of offshoring on the labor market (not unemployment per se) need to be mentioned here. Karabay and McLaren (2006) study the effects of free trade and offshore outsourcing on wage volatility and worker welfare in a model where risk sharing takes place through employment relationships. Grossman and Rossi-Hansberg (2008) model offshoring as "trading in tasks" and show that even factors of production whose tasks are offshored can benefit from offshoring due to its productivity enhancing effect.

event). This raises overall demand for unskilled labor and reduces its unemployment.

3 The model

3.1 The goods market

The economy comprises three types of agents: S skilled workers, L unskilled workers, and a large number of entrepreneurs. Entrepreneurs have access to technology to produce a final good, Z , using skilled and unskilled labor. The production function for the final good Z , which is constant returns to scale, is given by

$$Z = F(s, l) \tag{1}$$

where s and l , respectively, are the numbers of skilled and unskilled workers employed. We also assume that the final good, Z , is the numeraire.

3.2 The labor market

Our description of labor market is a static version of Pissarides (2000) along the lines of Helpman and Itskhoki (2007).⁹ Entrepreneurs must post vacancies to hire skilled and unskilled workers to undertake production. Once a vacancy is matched with a worker, she is hired to work for that firm (entrepreneur).

The labor markets for both skilled and unskilled are characterized by a matching technology that depends on the number of searchers (size of the labor force) and the number of job vacancies. Pissarides (2000) describes the empirical support for a constant returns to scale matching function, which is what we use in this paper.

Let u_i denote the unemployment rate of factor i , θ_i the vacancy rate (i.e., the number of vacancies divided by the labor force), S the economy's endowment of skilled labor, and L the endowment of unskilled labor. Since the model is static where all workers search for a job, and a fraction $1 - u_i$ of workers of type- i is matched, θ_i is also the measure of market tightness. Then,

⁹In a recent technical note, Helpman and Itskhoki (2009) develop a dynamic model along similar lines to show that its steady state looks similar to their static model.

we write the number of matches for each factor as constant-returns-to-scale functions as follows:

$$M(\theta_s S, S) = M(\theta_s, 1)S; \quad M(\theta_l L, L) = M(\theta_l, 1)L \quad (2)$$

Define $m_s \equiv \frac{M(\theta_s S, S)}{S} = M(\theta_s, 1)$ and $m_l \equiv \frac{M(\theta_l L, L)}{L} = M(\theta_l, 1)$ as the matching rates for the two factors, where $m'_i(\theta_i) > 0$. Define $q(\theta_i) \equiv \frac{m_i}{\theta_i}$. The constant returns to matching implies $q'(\theta_i) < 0$. With this notation, the probability of finding a job for a searcher of type- i is $\theta_i q(\theta_i)$, and the probability of filling up a vacant job of type- i is $q(\theta_i)$. The former is an increasing function of market tightness, and the latter is a decreasing function of market tightness. The number of vacancies that a firm needs to create for it to expect to create one job at the end of the matching process is $\frac{1}{q(\theta_i)}$. For a large firm, by the law of large numbers the actual ratio of vacancies to jobs (matches) created will be $\frac{1}{q(\theta_i)}$.

We will restrict attention to the case where skilled workers would never prefer to search for an unskilled job, even if that were possible.¹⁰ Note once again that the model is static (one-period). The unemployment rate for each factor is given by:

$$u_i = 1 - m_i = 1 - \theta_i q(\theta_i) \quad (3)$$

An entrepreneur posting vacancies must pay a recruitment cost of $c_i (i = s, l)$ units of the final good per vacancy posted. Since a firm needs to post $\frac{1}{q(\theta_i)}$ vacancies to create one job, the vacancy cost per worker employed equals $\frac{c_i}{q(\theta_i)}$. Once a job is filled, the entrepreneur receives the value of the marginal product of that factor less the factorial wage, w_i , where the wage is denoted in units of the final good.

We solve the entrepreneur's problem in two stages. In the first stage, employment and the number of vacancies are chosen, anticipating the wages as functions of skilled and unskilled employment (determined through bargaining in the second stage) correctly. Then given the employment levels chosen in the first stage, the wage rate is determined by a process of bargaining between the entrepreneur and the worker, along the lines of Stole and Zweibel (1996). A worker and her employer bargain with each other taking into account the impact of the worker's possible exit on wages of other employees. In other words, we allow the possibility of renegotiation of the employer with

¹⁰This can be done by imposing reasonable restrictions on the parameters of production and matching functions and on the relative factor endowments of skilled and unskilled labor.

other employees if bargaining fails with any employee, and this feature is completely factored into the bargaining process.¹¹

The discussion above implies that our overall equilibrium concept is one of subgame perfect equilibrium which is solved using backward induction. That is, taking as given the employment chosen in the first stage, in the second stage the wages are determined through a process of simultaneous Stole-Zweibel bargaining between the firm and the workers. Anticipating the second stage wage as a function of employment, the firm optimally chooses employment in the first stage. The entrepreneur solves the following problem in the first stage taking into account the impact of employment choice on the wages paid in the second stage.

$$\underset{s,l}{Max} F(s, l) - w_s(s, l)s - w_l(s, l)l - \frac{c_s}{q(\theta_s)}s - \frac{c_l}{q(\theta_l)}l \quad (4)$$

The first-order conditions for the optimal choices of s and l are given by

$$F_1(s, l) - w_s - s \frac{\partial w_s}{\partial s} - l \frac{\partial w_l}{\partial s} = \frac{c_s}{q(\theta_s)} \quad (5)$$

$$F_2(s, l) - w_l - s \frac{\partial w_s}{\partial l} - l \frac{\partial w_l}{\partial l} = \frac{c_l}{q(\theta_l)} \quad (6)$$

where subscripts “1” and “2” denote partial derivatives of the production function with respect to the first and second arguments, respectively.

3.3 Second Stage Wage determination

Denote the expressions on the *l.h.s* in the two equations (5) and (6) by J_i , $i = s, l$, where J_i is the surplus of the firm from hiring the marginal worker of type- i . Assuming unemployment benefit to be zero, the bargaining weight of a worker to be β , the bargained wage for a worker of type- i is obtained as follows.

$$w_i^b = \arg \max_{w_i} w_i^\beta J_i^{1-\beta} \quad (7)$$

As mentioned earlier, a worker and her employer bargain with each other taking into account the impact of the worker’s possible exit on wages of other employees. In other words, we allow the possibility of renegotiation of the employer with other employees if bargaining fails with any

¹¹A special case of this, where the worker and employer have equal bargaining weights, exactly boils down to the Shapley value solution to a cooperative, multilateral bargaining problem.

employee, and this feature is completely factored into the bargaining process. Using (5), (6), the first-order conditions of the above maximization problem yields the following expressions for wages for the two types of workers.

$$w_s = \beta[F_1(s, l) - s \frac{\partial w_s}{\partial s} - l \frac{\partial w_l}{\partial s}] \quad (8)$$

$$w_l = \beta[F_2(s, l) - s \frac{\partial w_s}{\partial l} - l \frac{\partial w_l}{\partial l}] \quad (9)$$

The above is a system of differential equations, where each worker's bargained wage is a fraction of the surplus she creates in the form of her marginal product plus the reduction in the wage bill (or minus the increase in the wage bill) of the existing workers through her employment (relative to the situation where she exits and wages with other workers are renegotiated). As seen from the above differential equations, there are own as well as cross effects of skilled and unskilled employment on wages.

It is important to note here that if w_s and w_l are homogeneous of degree zero in s and l , the two first-order conditions (5) and (6) result in the zero-profit condition for the firm being satisfied.¹² Taking this as an important hint in finding the solution to the above set of differential equations, we write w_s and w_l as functions of s/l . Denote s/l by t . Given that $F(s, l)$ is CRS, we can write $F(s, l) = lF(t; 1)$. Denote $F(t; 1)$ by $f(t)$ and it follows that $F_1(s, l) = f'(t)$ and $F_2(s, l) = f(t) - tf'(t)$.

In the appendix we show that the solutions to (8) and (9), namely bargained wage rates in the second (bargaining) stage for any relative skilled employment, t set in the first (prior) stage, are given by

$$\text{Lemma 1: } w_s(t) = \beta f'(t); w_l(t) = \beta(f(t) - tf'(t))$$

That is, the wages are simply a fraction β of the respective marginal products of labor. This is despite the presence of the ‘‘strategic effect’’ mentioned in the introduction which are captured by terms $i \frac{\partial w_i}{\partial j}$, for $i, j = s, l$ in the first-order conditions (5) and (6). For example, $s \frac{\partial w_s}{\partial s}$ captures the effect of hiring an additional skilled worker on skilled wage. If hiring an additional skilled worker

¹²Multiplying (5) by s and (6) by l and adding we get $F(s, l) = (w_s + \frac{c_s}{q_s(\theta_s)})s + (w_l + \frac{c_l}{q_l(\theta_l)})l + s(s \frac{\partial w_s}{\partial s} + l \frac{\partial w_s}{\partial l}) + l(l \frac{\partial w_l}{\partial l} + s \frac{\partial w_l}{\partial s})$. Therefore, the zero profit condition holds if $s \frac{\partial w_s}{\partial s} + l \frac{\partial w_s}{\partial l} = l \frac{\partial w_l}{\partial l} + s \frac{\partial w_l}{\partial s} = 0$, which always holds if w_s and w_l are homogeneous of degree zero in s and l .

lowers their marginal product it will reduce the Nash bargained wage that firms have to pay to skilled workers. Therefore, the value of a skilled job to the firm would exceed the marginal product of skilled labor due to this effect. However, an additional skilled worker also increases the marginal product of unskilled workers which would lead to an increase in the unskilled wage ($\frac{\partial w_l}{\partial s} > 0$), leading to a reduced value of a skilled job for the firm. For a constant returns to scale production function with the same bargaining weights for a skilled and an unskilled worker relative to the employer, these two “strategic effects” cancel out in the determination of the wage bill. Thus, the surplus that is shared between the worker and the firm is the worker’s marginal product and it is shared according to their bargaining weights.

3.4 Equilibrium

Having determined the wage in the second stage as a function of employment choice in the first stage, the factor demands can be obtained from the two first order conditions (5) and (6). It is shown in the appendix that the first-order conditions (5), (6), and lemma 1 imply

$$\text{Lemma 2: } w_s(t) = \beta f'(t) = \frac{\beta}{1-\beta} \frac{c_s}{q(\theta_s)}; w_l(t) = \beta(f(t) - t f'(t)) = \frac{\beta}{1-\beta} \frac{c_l}{q(\theta_l)}.$$

The above relationships are determined by the fact that the surplus from a job- i in equilibrium equals the hiring cost of $\frac{c_i}{q(\theta_i)}$. Since the firms take θ_i as given, lemma 2 provides us with two expressions for the relative demand $t^d = \frac{s^d}{l^d}$ for a firm, both of which must be true in equilibrium. To derive relative labor demand and supply functions in terms of relative market tightness, we assume that the matching function is also of Cobb-Douglas form given as follows.

$$\textbf{Assumption 1: } m(\theta_i) = \mu \theta_i^\delta \text{ and hence } q(\theta_i) = \mu \theta_i^{\delta-1}.$$

Under the above assumption, the relative demand for skilled labor as a function of $\frac{\theta_s}{\theta_l}$ can be obtained from lemma 2 as follows.

$$\frac{f'(t)}{f(t) - t f'(t)} = \frac{c_s q(\theta_l)}{c_l q(\theta_s)} = \frac{c_s}{c_l} \left(\frac{\theta_s}{\theta_l} \right)^{1-\delta} \quad (10)$$

Since $f''(t) < 0$, the relative demand is decreasing in $\frac{\theta_s}{\theta_l}$. This is shown using a downward sloping curve denoted by RLD^u in Figure 1 in $(\frac{s}{l}, \frac{\theta_s}{\theta_l})$ space. This is intuitive as the relative cost of employing skilled labor (relative to unskilled) is increasing in its relative market tightness. The

superscript u stands for the *relatively* “undistorted” case (later referred to as the "unconstrained case").

Having obtained an expression for relative demand, next we derive an expression for the relative supply of two types of labor. Denoting the economy’s endowments of skilled and unskilled labor by S and L respectively, the relative supply (available for employment) is given by $\frac{S(1-u_s)}{L(1-u_l)}$, which using (3) and assumption 1 becomes

$$t^s = \frac{S\theta_s q(\theta_s)}{L\theta_l q(\theta_l)} = \frac{S}{L} \left(\frac{\theta_s}{\theta_l} \right)^\delta \quad (11)$$

The above is clearly increasing in $\frac{\theta_s}{\theta_l}$. In other words, because the relative employment rate is increasing in the relative market tightness of skilled labor, the relative supply of skilled labor available for employment is also increasing in its relative market tightness. This is shown using upward sloping lines denoted by RLS in Figure 1.

The intersection of the downward sloping relative demand with the upward sloping relative supply determines the autarky equilibrium in the unconstrained case as shown in Figure 1. Having obtained the equilibrium t from the intersection of RLD^u and RLS in Figure 1, the corresponding equilibrium values of w_s , w_l , θ_s , and θ_l can be obtained from the expressions in lemma 2. Denote the unconstrained equilibrium wage and market tightness variables by $w_l^u, w_s^u, \theta_l^u, \theta_s^u$. Again, the superscript u stands for the relatively “undistorted” case (later referred to as the "unconstrained case"). As mentioned earlier, when the two first order conditions (5) and (6) are satisfied the representative firm makes zero profits. Denote the unit cost of a representative firm in the unconstrained case as a function of θ_s and θ_l by $\kappa^u(\theta_s, \theta_l)$. Since the price of the final good is 1, the zero profit condition can be written as

$$\kappa^u(\theta_s^u, \theta_l^u) = 1 \quad (12)$$

The zero profit condition above is useful in deriving the results in the case of offshoring.

4 Introduction of a fair-wage constraint

Next, we introduce a distortion in the search model in the form of a floor on the wage that firms can pay to the unskilled worker. Denote this wage by w_l^* . Here w_l^* could either be a policy determined minimum wage or it could be a fair wage, our favored interpretation, in which case it is going to

be endogenously determined.¹³ With the fair-wage interpretation, even though paying w_l^* is not mandatory, firms treat this as a constraint on the wage they can pay to the unskilled workers. There can be many reasons for why firms want to pay at least a fair wage to workers. As discussed by Akerlof and Yellen (1990), workers exert less than full effort if they are not paid a fair wage (which, driven by “animal spirits,” workers themselves may view or justify as pay back for the unfair wage being paid). Paying at least a fair-wage ensures that workers work at their maximum efficiency, which can offset or more than offset the extra cost incurred in hiring each worker.¹⁴ Alternatively, paying a fair wage may be part of corporate social responsibility or a norm in society.

4.1 The effect of the fair-wage constraint on wage determination

Now, the entrepreneur’s problem effectively becomes:

$$\underset{s,l}{Max} F(s, l) - w_s(s, l)s - w_l(s, l)l - \frac{c_s}{q(\theta_s)}s - \frac{c_l}{q(\theta_l)}l \text{ subject to } w_l \geq w_l^* \quad (13)$$

Assuming that w_l^* , determined in general equilibrium, is taken by the firm parametrically (as given), for a firm anticipating a binding fair-wage constraint in the second stage, the first-order conditions for the employment choice in the first stage become

$$f'(t) - w_s - s \frac{\partial w_s}{\partial s} = \frac{c_s}{q(\theta_s)} \quad (14)$$

$$f(t) - tf'(t) - w_l^* - s \frac{\partial w_s}{\partial l} = \frac{c_l}{q(\theta_l)} \quad (15)$$

The value of an extra skilled worker for a firm is given by the *l.h.s* of (14). As in the unconstrained case, if w_s is homogeneous of degree zero in s and l , the two first-order conditions above result in

¹³Even a minimum wage can, in certain cases, be endogenously determined. For example, an inequality-averse government may set the minimum wage to make sure wage inequality does not rise above a certain maximum acceptable level (See, for instance, Cahuc and Zylberberg (2004)).

¹⁴The need for paying at least a fair wage may be more pressing in scenarios such as ours, where there is a recruitment cost per worker in addition to the wage paid. When the efficiency loss per worker is proportional to the amount by which wage paid is below fair wage, as in Akerlof and Yellen (1990), paying less than fair wage will result in waste of resources in recruitment of extra workers for the firm to hire any given number of total efficiency units (and given wage bill). Paying at least a fair wage eliminates this waste.

the zero-profit condition for the firm being satisfied. We thus write w_s as a function of $t = s/l$. Using Nash bargaining, as in the unconstrained case, the wage of a skilled worker is given by the following differential equation

$$w_s(t) = \beta[f'(t) - tw'_s(t)] \quad (16)$$

The solution to the above differential equation is given by

$$w_s(t) = t^{-\frac{1}{\beta}} \int_0^t x^{\frac{1-\beta}{\beta}} f'(x) dx \quad (17)$$

4.2 Solving for the search equilibrium in the presence of a fair-wage constraint

To solve the model we need to specify how the fair wage w_l^* is determined. Akerlof and Yellen (1990) model fair wage for type- i as a linear combination of the wage of the other type and the market clearing wage for type- i . Given the search friction and wage bargaining, there is no market clearing wage in our framework. In principle, we could use the unconstrained bargained wage in place of the market clearing wage, however, to simplify the exposition considerably, we assume that our fair wage, w_l^* , takes the following simple form:¹⁵

Assumption 2: $w_l^* = \tau w_s$

Now, there are two possibilities: either the fair-wage constraint does not bind ($w_l^u > \tau w_s^u$) or it does bind ($w_l^u < \tau w_s^u$ and $w_l^c = w_l^* = \tau w_s^c$, where w_s^c is the equilibrium skilled wage in the case where the wage of unskilled workers is constrained to be equal to its fair wage).

¹⁵Kreickemier and Nelson (2006) and Egger and Kreickemier (2008) modify the Akerlof and Yellen (1990) specification to allow the fair wage of worker type- i to be negatively related to the unemployment rate of type- i . In these papers there is no search friction and the focus is exclusively on unskilled unemployment driven by fair-wage considerations. Since we are adding the fair-wage constraint to a more complicated framework that has search frictions with both skilled and unskilled unemployment, we decided, for the sake of clarity in exposition, to abstract a little here by simplifying the fair-wage constraint. We will later relax this assumption (in our section on possible extensions) to make τ a decreasing function of the unskilled unemployment rate and show that results remain qualitatively unchanged.

When the fair-wage constraint binds, the factor demand equations (14) and (15) can be written as

$$f'(t) - w_s(t) - tw'_s(t) = \frac{c_s}{q(\theta_s)} \quad (18)$$

$$f(t) - tf'(t) + t^2w'_s(t) - w_l^* = \frac{c_l}{q(\theta_l)} \quad (19)$$

In the above expressions $w_s(t)$ is given by (16). To obtain a simplified expression for relative demand, we use the following lemma which is obtained from (14) and (16).

Lemma 3: $w_s(t) = \frac{\beta}{1-\beta} \frac{c_s}{q(\theta_s)}$ holds even in the constrained case.

The reason that the expression for the skilled wage as function of θ_s is unchanged from the expression in lemma 2 for the unconstrained case is the following. Nash bargaining implies that the skilled wage, w_s equals $\frac{\beta}{1-\beta} J_s$, where J_s is the surplus of the firm from hiring the marginal skilled worker. Note from (5) and (14) that, for any given market tightness that the firms and workers take as given, the interaction between a firm and its workers it decides to hire (through its employment decision and wage bargaining) always results in an outcome in which J_s must equal the recruitment cost of hiring an additional worker, $\frac{c_s}{q(\theta_s)}$.

Using lemma 3, (18), (19), and assumption 2, the expression for the relative demand for skilled labor as a function of $\frac{\theta_s}{\theta_l}$ is obtained from the expression below.

$$\frac{w_s(t)}{f(t) - tf'(t) + t^2w'_s(t) - \tau w_s(t)} = \frac{\beta}{1-\beta} \frac{c_s}{c_l} \left(\frac{\theta_s}{\theta_l} \right)^{1-\delta} \quad (20)$$

Next, we prove the following useful lemma in the appendix.

Lemma 4: (a) $w'_s(t) < 0$; (b) $\frac{d[f(t) - tf'(t) + t^2w'_s(t)]}{dt} > 0$

Using lemma 4 above, it can be verified that the relative demand for skilled labor given in (20) is decreasing in $\frac{\theta_s}{\theta_l}$ because the numerator is decreasing in t while the denominator is increasing in t . Therefore, we again get a unique solution for t and $\frac{\theta_s}{\theta_l}$. The corresponding θ_s and θ_l can be obtained from (18) and (19) upon using the expression for skilled wage in (17). Just as was shown in the unconstrained case, if the two first order conditions (18) and (19) are satisfied, the firms make zero profits. Denoting the unit cost of a constrained firm as a function of θ_s and θ_l by $\kappa^c(\theta_s, \theta_l)$, the zero profit condition in the constrained equilibrium is given by

$$\kappa^c(\theta_s^c, \theta_l^c) = 1 \quad (21)$$

4.3 Analyzing the impact of the fairness distortion on the search equilibrium

We can compare the constrained and unconstrained equilibria by deriving the constrained relative demand relative to the unconstrained relative demand for skilled labor. Recall from lemma 2 that for any $\frac{\theta_s}{\theta_l}$ the unconstrained $\frac{w_l}{w_s}$ is given by $\frac{c_l}{c_s} \left(\frac{\theta_s}{\theta_l} \right)^{\delta-1}$. Given the form of the fair wage constraint specified in assumption 2, it binds if $\tau > \frac{w_l}{w_s}$ or if

$$\left(\frac{\theta_s}{\theta_l} \right) > \left(\frac{c_l}{\tau c_s} \right)^{\frac{1}{1-\delta}} \quad (22)$$

Therefore, for any τ there exists a $\theta(\tau) \equiv \left(\frac{c_l}{\tau c_s} \right)^{\frac{1}{1-\delta}}$ such that for $\frac{\theta_s}{\theta_l} > \theta(\tau)$ the constraint binds while for $\frac{\theta_s}{\theta_l} < \theta(\tau)$ it is non-binding. Therefore, for $\frac{\theta_s}{\theta_l} < \theta(\tau)$ the constrained relative demand for skilled labor coincides with the unconstrained relative demand. It is shown in the appendix that the constrained relative demand jumps to the right at $\frac{\theta_s}{\theta_l} = \theta(\tau)$ and lies to the right of the unconstrained relative demand for $\frac{\theta_s}{\theta_l} > \theta(\tau)$. Thus, the relative demand curve with the possibility of the fair-wage constraint becomes the one denoted by RLD^c in Figure 1 which has two segments, one for values of $\frac{\theta_s}{\theta_l}$ greater than $\theta(\tau)$, and the other for values less than $\theta(\tau)$ which corresponds to the unconstrained relative demand curve.

It is easy to understand why the constrained relative demand curve lies to the right of the unconstrained one. Since unskilled labor becomes more expensive, firms substitute skilled labor for unskilled labor and hence the relative demand for skilled labor increases. But, why is there a jump at $\frac{\theta_s}{\theta_l} = \theta(\tau)$? This has to do with the “strategic effect” in wage setting. Now, hiring an additional skilled worker reduces the wage of skilled workers at the bargaining stage, but does not increase the wage of unskilled workers because the latter’s wage is taken as given by the firms. Therefore, firms have an incentive to hire more skilled workers. This is what leads to a jump in the relative demand for skilled workers when the fair wage constraint becomes just binding.

Let us compare the autarky equilibrium in the absence of fairness considerations given by point e_1 in Figure 1 with a constrained autarky equilibrium. To make the comparison of constrained and unconstrained equilibria precise, we do the comparison when the constraint is just binding, that is $\tau = \frac{c_l}{c_s} \left(\frac{\theta_s^u}{\theta_l^u} \right)^{\delta-1}$. The new equilibrium where the constraint binds is obtained by the intersection of RLD^c with RLS and is at point e_2 in Figure 1. That is, both $\frac{\theta_s}{\theta_l}$ and t are higher in the constrained equilibrium. Next, we prove the following lemma in the appendix.

Lemma 5: $\theta_i^c < \theta_i^u$

That is, an increase in $\frac{\theta_s}{\theta_l}$ compared to the unconstrained case implies a lower θ_l , but the impact on θ_s is ambiguous. From lemma 3 we infer that the change in the skilled wage is in the same direction as the change in θ_s . Numerical simulations confirm that the impact on θ_s depends on the strength of the “strategic effect” determined by the elasticity of substitution between skilled and unskilled labor denoted by σ . There is a cut off value of elasticity of substitution σ^* such that for $\sigma < \sigma^*$, θ_s decreases while for $\sigma > \sigma^*$, θ_s increases. Below is a numerical example.

Numerical Example: $c_l = c_s = 1$, $L = 2$, $S = 1$, $\beta = \delta = .5$, $\mu = 1.25$; $F(s, l) = \frac{(\gamma s^\rho + (1-\gamma)l^\rho)^{\frac{1}{\rho}}}{(\gamma^\sigma + (1-\gamma)^\sigma)^{\frac{1}{\sigma-1}}}$ is the CES production function with $\sigma = \frac{1}{1-\rho}$ being the elasticity of substitution between s and l . When $\gamma = .5$, the cutoff value of σ turns out to be 0.48. That is, for $\sigma = .48$ the unconstrained equilibrium θ_s is .606 and the ratio of unskilled to skilled wage is .626. Now, if we set $\tau = .626$, the constrained equilibrium θ_s equals .606. For $\sigma < .48$, the constrained equilibrium θ_s is lower than the unconstrained one, but for $\sigma > .48$ the constrained equilibrium θ_s is higher than the unconstrained one.

Since the unemployment rate u_i is inversely related to θ_i , the result on the comparison between unconstrained and constrained equilibria can be summarized as follows.

Proposition 1: *In a constrained equilibrium where the constraint is just binding, unskilled unemployment is higher and skilled unemployment (and skilled wage) may be higher or lower compared to an unconstrained equilibrium. If the “strategic effect” in employment choice is weak (low elasticity of substitution between skilled and unskilled labor), then skilled unemployment is higher as well and while skilled wage is lower, the unskilled wage is higher.*

To clearly see the role played by the “strategic effect” we can compare the unconstrained and constrained equilibria in the absence of (or when firms ignore it) the “strategic effect” to contrast it with the above result. The lemma below is proved in the appendix.

Lemma 6: In the absence of the “strategic effect”, the constrained equilibrium is identical to the unconstrained one for $\tau = \frac{c_l}{c_s} \left(\frac{\theta_s^u}{\theta_l^u} \right)^{\delta-1}$, and leads to increases in both skilled and unskilled unemployment for $\tau > \frac{c_l}{c_s} \left(\frac{\theta_s^u}{\theta_l^u} \right)^{\delta-1}$.

As mentioned in the introduction, in short, a binding fairness constraint raises the cost of hiring unskilled labor causing losses for competitive firms. The ensuing reduction in output leads to losses of both skilled and unskilled jobs. As job losses reduce the market tightness, they reduce the recruitment costs as well as the wage paid to workers, thereby restoring the zero profit condition of competitive firms.

A more detailed explanation of intuition is as follows. A binding fair-wage constraint, by making unskilled workers more expensive, leads to an increase in the relative demand for skilled labor at given market tightness parameters. This is the partial equilibrium substitution effect. At the unconstrained equilibrium relative market tightness for skilled workers, there is excess relative demand for skilled workers. Also, at the unconstrained equilibrium market tightness parameters, the fair-wage constraint leads to an increase in the unit cost of output and therefore to losses. To prevent losses, at least market tightness of one of the factors has to fall, while equilibrium in the labor markets will require relative market tightness of skilled workers to rise (to close the relative excess demand). This implies that at the very least the market tightness for unskilled workers falls. As the relative market tightness for skilled workers increases, the relative number of skilled workers immediately available for employment (and actually employed) upon search increases. This reduces the marginal product or the marginal value of a skilled worker at the aggregate level because each skilled worker has fewer unskilled workers to work with. This general equilibrium complementarity effect reduces the incentive to create skilled jobs. Therefore, the market tightness for skilled workers falls, and skilled unemployment rises. This is what happens if we ignore the “strategic effect”.

The emergence of the “strategic effect”, when the fair wage constraint is binding, reduces the effective cost of hiring skilled labor because now an extra worker has a negative impact on the skilled wage set in the second stage and no impact on the constrained unskilled wage (recall that firms take the fair wage as given). This added incentive to hire skilled labor can reduce their unemployment if the “strategic effect” is sufficiently strong.

4.3.1 The possibility of multiple equilibria

Given the shape of the relative demand curve with fair-wage constraint, it is possible to get multiple equilibria. To see this, suppose the relative supply curve is one denoted by RLS' in Figure 1. Now, if the society did not have any concern for fairness, then the equilibrium would be at e_3 . However,

fairness concerns cause a stepward shift in the relative demand curve at $\theta(\tau)$ making e_4 a candidate for equilibrium as well. That is, both e_3 and e_4 are possible equilibria when fairness considerations are present.

The intuition for “multiple equilibria” here is the following. Since the relative cost of labor (wage plus recruitment cost) is positively related to the relative market tightness, the fair-wage constraint binds if $\frac{\theta_s}{\theta_l}$ is high. If firms expect $\frac{\theta_s}{\theta_l}$ to be high and therefore the fair-wage constraint to bind, they will end up hiring more skilled workers relative to unskilled workers due to the “strategic effect” discussed earlier. This, in turn, will make the relative market tightness for skilled workers higher creating a “self-fulfilling prophecy.” Similarly, if $\frac{\theta_s}{\theta_l}$ is expected to be low, the fair-wage constraint is then not expected to bind, the relative demand for skilled labor is lower, which in turn leads to a low $\frac{\theta_s}{\theta_l}$ and thus, an effectively unconstrained outcome.

As seen from Figure 1, we have multiple possible expectations regarding $\frac{\theta_s}{\theta_l}$, when relative supply is in the intermediate range, which happens only when the relative endowment of skilled labor is in the intermediate range. In other words, if the relative endowment of skilled labor is very high (low), which the firms know, they will expect the relative market tightness of skilled labor to be always low (high).

4.4 Comparative statics with respect to τ

Let us assume that the economy is at a constrained equilibrium, i.e., the relative supply curve in Figure 1 intersects the fair-wage relative demand curve in its upper right-hand downward sloping part (that lies above the cut-off value of $\frac{\theta_s}{\theta_l}$ at which the fairness constraint binds). It is easy to see from (20) that an increase in τ implies an increase in the relative demand for skilled labor for each $\frac{\theta_s}{\theta_l}$. Therefore, the relative demand curve shifts to the right in the $(\frac{s}{l}, \frac{\theta_s}{\theta_l})$ space. This implies an increase in the equilibrium t and $\frac{\theta_s}{\theta_l}$. From lemmas 3 and 4 it implies a decrease in θ_s . Since $\frac{\theta_s}{\theta_l}$ has increased while θ_s has decreased, θ_l must decrease even more. Also, it is obvious from lemma 4(a) that skilled wage decreases. An increase in t also implies from lemma 4(b) and the first order condition with respect to unskilled labor (19) the following result which is useful in discussing offshoring.

Lemma 7: An increase in τ implies an increase in the hiring cost of unskilled labor defined as $\omega(\tau) \equiv \tau w_s(\tau) + \frac{c_l}{q(\theta_l(\tau))}$. That is, $\omega'(\tau) > 0$.

Next, note that since θ_l decreases, in order for $\omega(\tau)$ to increase, the unskilled wage, given by $\tau w_s(t)$, must increase. Therefore, we get the following result on the impact of an increase in τ .

Proposition 2: *Starting from a constrained equilibrium, an increase in the fairness parameter τ leads to increases in both skilled and unskilled unemployment. Skilled wage falls and unskilled wage rises.*

While a comparison of unconstrained and constrained equilibria yields ambiguous results on the skilled unemployment as mentioned in proposition 1, proposition 2 shows that any increase in τ , starting from a constrained equilibrium, leads to an unambiguous increase in skilled unemployment. As noted earlier, the reason for the ambiguity in proposition 1 is the “strategic effect” in the hiring of skilled labor. Starting from a constrained equilibrium, for an incremental increase in τ the “strategic effect” (or rather a change in it) is of the second order (the effect is present in the initial and final equilibrium, both of which are constrained and so get canceled out when looking at the difference between the two situations), and hence skilled unemployment increases.

Propositions 1 and 2 together provide the following corollary on the relationship between the fairness parameter τ and the skilled unemployment.

Corollary 1: *As long as $\tau < \frac{c_l}{c_s} \left(\frac{\theta_s^u}{\theta_l^u} \right)^{\delta-1}$, any increase in τ has no impact on skilled unemployment. At $\tau = \frac{c_l}{c_s} \left(\frac{\theta_s^u}{\theta_l^u} \right)^{\delta-1}$, there is a downward (upward) jump in skilled unemployment for sufficiently high (low) elasticity of substitution between skilled and unskilled labor. For $\tau > \frac{c_l}{c_s} \left(\frac{\theta_s^u}{\theta_l^u} \right)^{\delta-1}$, skilled unemployment increases with τ irrespective of the elasticity of substitution.*

Having looked at the impact of fairness considerations in a search model of unemployment, we next look at the impact of offshoring of jobs done by unskilled labor on the unemployment rates of skilled and unskilled workers.

5 Impact of offshoring

For offshoring to be possible, firms have to be able to fragment their production in such a way that semi-finished output, whose production only requires the application of unskilled labor, can

be produced in another country (South) and then imported back to be combined with skilled labor at home to produce the final product. To keep things simple, we assume that one unit of this semi-finished good is a perfect substitute for a unit of domestic unskilled labor. We assume the price of imported input inclusive of the search and trade costs as fixed, which is equivalent to a small country assumption. (This assumption is reasonable if we believe that the South has large quantities of unskilled labor and that fixed labor productivity in a large subsistence, numeraire sector there fixes their unskilled wage.) Suppose the unit cost (faced at home) of offshored input is p_m .

Let the amount of this input imported be denoted by m . We assume that the quantity of offshored input is chosen in the first stage along with skilled labor. This assumption may be closer in spirit to the offshoring of *services*, while the assumption of freely adjustable input will be closer to the case of imported intermediate *good*. Later we will note the implication of allowing this input to be imported freely at any stage.

We now start with the case where there are no labor-market frictions other than search frictions. This case isolates the impact of offshoring on unemployment that is caused only by the presence of search frictions and fairness considerations are absent. Hence, this serves as a benchmark for considering the additional offshoring-related issues arising out of fairness considerations.

5.1 Impact of offshoring when there is no distortion other than search frictions

A firm that decides to offshore and not hire unskilled workers domestically, solves the following problem:

$$\underset{s,m}{Max} F(s, m) - w_s(s, m)s - p_m m - \frac{c_s}{q(\theta_s)} s$$

The first order conditions for the above maximization are given by

$$F_1(s, m) - w_s - s \frac{\partial w_s}{\partial s} = \frac{c_s}{q(\theta_s)} \quad (23)$$

$$F_2(s, m) - s \frac{\partial w_s}{\partial m} - p_m = 0 \quad (24)$$

Note from above that the problem of offshoring firms is similar to that of a firm in autarky in the presence of a binding fairness constraint. Therefore, following the same procedure as in the constrained maximization case, the wage of skilled workers in the second stage is given by the

following differential equation

$$w_s(t) = \beta[f'(t) - tw'_s(t)] \quad (25)$$

where t equals s/m in the offshoring case.

To analyze the impact of offshoring on wage and unemployment we make use of the zero profit conditions as follows. Denote the autarky equilibrium values of θ_s and θ_l by θ_s^a and θ_l^a , respectively, where the superscript a stands for autarky. Since firms make zero profits in equilibrium, the unit cost of a firm in autarky equilibrium, denoted by $\kappa^a(\theta_s^a, \theta_l^a)$, must satisfy

$$\kappa^a(\theta_s^a, \theta_l^a) = 1 \quad (26)$$

At the autarky values of θ_s^a and θ_l^a , in order for a firm to be induced to offshore, it should make a positive profit from offshoring. Since an offshoring firm faces a world price of p_m for imported input, for it to have been induced to offshore, its unit cost evaluated at θ_s^a and p_m must have been less than unity:

$$\kappa^o(\theta_s^a, p_m) < 1 \quad (27)$$

where superscript o stands for offshoring. We show in the appendix that $\kappa^o(\theta_s, p_m)$ is increasing in θ_s and p_m . Therefore, starting from an autarky equilibrium θ_s^a and θ_l^a the lower the p_m the greater the likelihood of offshoring.

As firms offshore the input produced by unskilled labor, there are job losses for unskilled workers leading to a decrease in their market tightness and wage. The market tightness and wage of unskilled workers keep falling till firms become indifferent between offshoring and not offshoring and also firms make zero profits. In this case we will assume that some firms are offshoring and others are not offshoring. Denoting the post-offshoring equilibrium values of θ_s and θ_l by θ_s^o and θ_l^o , zero profit condition of an offshoring firm is given by

$$\kappa^o(\theta_s^o, p_m) = 1 \quad (28)$$

while the zero profit condition of non-offshoring firms is given by¹⁶

$$\kappa^{no}(\theta_s^o, \theta_l^o) = 1 \quad (29)$$

¹⁶We can rule out the case of the complete offshoring of all unskilled jobs as follows. If unskilled unemployment is at 100 percent, then any vacancy posted will be filled up with certainty (with probability one). The cost of hiring (wage plus recruitment cost) a domestic unskilled worker in that case is $(1 - \beta)^{-1}c_l$ from lemma 2. We assume that $(1 - \beta)^{-1}c_l$ is low enough that for any θ_s^o satisfying (28) non-offshoring firms would make positive profits.

It is easily seen from (27) and (28) that $\theta_s^o > \theta_s^a$. That is, the offshoring of unskilled labor jobs reduces the unemployment of skilled workers. Intuitively, starting at an autarky equilibrium, firms make profits by offshoring and the resulting expansion of output increases the demand for skilled labor and consequently the skilled unemployment decreases and the skilled wage increases.

Along the same lines as proving $\kappa^o(\theta_s, p_m)$ is increasing in θ_s and p_m , it can be shown that $\kappa^{no}(\theta_s, \theta_l)$ is increasing in θ_s and θ_l , which again is very intuitive. Also, note, as is trivial to see, that the $\kappa^{no}(\cdot)$ function takes exactly the same form as $\kappa^a(\cdot)$. Given all this, (26) and (29) imply $\theta_l^o < \theta_l^a$. That is, the unemployment of unskilled workers is higher and the unskilled wage is lower in an offshoring equilibrium compared to autarky. This result arises from the competitive pressure on unskilled labor exerted by its substitute, namely the cheap imported, offshored input.

In addition to comparing autarky equilibrium with offshoring equilibrium, one can also do comparative statics with respect to the cost of the offshored input, p_m . This would capture the impact of a reduction in the trading cost of inputs or a productivity improvement in the country providing the offshored input. It is easy to verify from the zero profit condition (28) of offshoring firms that a decrease in p_m must lead to an increase in θ_s^o , and consequently an increase in skilled wage and a decrease in skilled unemployment. Since θ_s^o increases, the zero profit condition of non-offshoring firms (29) requires a decrease in θ_l^o , and consequently a decrease in unskilled wage and an increase in unskilled unemployment. Therefore, the impact of a decrease in p_m on the two labor markets is exactly in the same direction as a movement from autarky to offshoring. The results on the impact of offshoring are summarized below.

Proposition 3: *In the presence of search frictions in the markets for both skilled and unskilled labor (and no additional distortions), offshoring of jobs performed by unskilled labor leads to an increase in skilled wage and a decrease in skilled unemployment. As well, there is an increase in unskilled unemployment and a decrease in unskilled wage. While wage inequality increases, the impact on aggregate unemployment is ambiguous. Starting from an offshoring equilibrium, a decrease in the cost of imported input has the same effect on the two labor markets as a move from autarky equilibrium to offshoring equilibrium.*

While proposition 1 showed that moving from an unconstrained equilibrium to a constrained one

had an ambiguous effect on the skilled labor market due to the appearance of the “strategic effect” in the latter case in net terms, proposition 2 showed an unambiguous increase in skilled unemployment and a decrease in skilled wage consequent upon an increase in the fairness parameter τ . In contrast, proposition 3 above says that moving from autarky to an offshoring equilibrium leads to similar changes in the skilled labor market as in the case of a decrease in the cost of offshored input p_m . Earlier, while the “strategic effect” tended to increase the demand for skilled labor and hence had a positive effect on the skilled labor market, the direct effect of fairness constraint, resulting in a higher cost of unskilled labor, had a negative effect on the skilled labor due to complementarity between the two inputs. Now, both the net “strategic effect” and the direct effect of offshoring resulting in cheaper imported input have a positive spillover for the skilled labor market.

5.2 The interaction between offshoring and the fair-wage constraint in our search model

5.2.1 Impact of offshoring in the presence of a fair-wage constraint

We have seen that in the absence of a fair-wage constraint, offshoring increases the skilled wage but reduces the unskilled wage. It follows from this result that in the presence of fairness considerations, offshoring increases (cannot reduce) the likelihood of the fair-wage constraint to bind. Irrespective of whether the fair-wage constraint is binding or not, (27) and (28), which must always hold in the case of offshoring, imply $\theta_s^o > \theta_s^a$, i.e., skilled wage increases and skilled unemployment decreases post-offshoring¹⁷. If the fair-wage constraint does not bind in autarky, then $w_l^a > \tau w_s^a$. However, if offshoring now makes this constraint binding, we have $w_l^o = \tau w_s^o$.¹⁸ Given that $w_s^o > w_s^a$, it is possible for unskilled wage to increase upon offshoring. In fact, because $w_s^o > w_s^a$ is always true, when the fair-wage constraint binds both before and after offshoring, the proportionality between skilled and unskilled wage will ensure that unskilled wage also increases upon offshoring. Unskilled unemployment always has to rise due to the competitive pressure exerted by the offshored input on

¹⁷Again we rule out offshoring by all firms by assuming that for any θ_s^o satisfying (28) fairness constrained non-offshoring firms would make positive profits if the unskilled unemployment is 100%. A sufficient condition for this to be true is $p_m > \frac{\tau \beta c_s}{(1-\beta)q(\theta_s^o)} + c_l$.

¹⁸Note that in the post-offshoring equilibrium the issue of fairness constraint binding or not is applicable only to firms that continue to remain completely domestic (do not offshore).

domestic unskilled labor. Moreover, this effect on unskilled unemployment gets further accentuated if offshoring converts a nonbinding fair-wage constraint into a binding one. As well, it is easy to verify that starting from an offshoring equilibrium, the impact of a decrease in p_m on the two labor markets is exactly in the same direction as a movement from autarky to offshoring. The results are summarized below.

Proposition 4: *As in the unconstrained case (with only search frictions), even in the presence of an additional distortion in the form of a fairness constraint, offshoring leads to an increase in skilled wage and a decrease in skilled unemployment. Even if the constraint is not binding in autarky, it may become binding upon offshoring. In this case, the impact on unskilled wage is ambiguous. If the constraint binds both before and after offshoring, then the unskilled wage increases unambiguously.*

5.2.2 Impact of an increase in the fairness parameter, τ , on offshoring and unemployment

Recall from lemma 7 that in autarky the cost of hiring unskilled labor is increasing in τ : $\omega'(\tau) > 0$. Next, compare the first order conditions of a fair wage constrained firm in autarky given in (18) and (19) with that of an offshoring firm given in (23) and (24). It is obvious that starting from a constrained autarky equilibrium, in order for a firm to offshore it must be the case that $\omega(\tau) > p_m$. The implication is that for a given p_m the greater the τ the more likely it is that firms offshore.

Next, we compare the impact of an increase in τ on unemployment and wages starting from an offshoring equilibrium where the fairness constraint is binding. Since the zero profit condition for offshoring firms given in (28) is unaffected by an increase in τ , for any p_m the θ_s^o satisfying (28) remains unchanged as well. Therefore, an increase in τ leaves the skilled wage and skilled unemployment unaffected. However, unskilled wage increases because $w_l^o = \tau w_s^o$. Since θ_s^o is unchanged, while the unskilled wage has increased, in order for the zero profit condition of the non-offshoring firms given in (29) to be satisfied, θ_l^o must decrease to leave the total cost of hiring unskilled labor unchanged and equal to p_m . That is, in an offshoring equilibrium, $\omega'(\tau) = 0$, since $\omega(\tau)$ must equal p_m .

Note also from the first order conditions of offshoring and non-offshoring firms that the relative demand s/l for non-offshoring firms and the relative demand s/m for offshoring firms are equal to

each other and remain unchanged with changes in τ , as long as the fair-wage constraint remains binding. Since the effective relative supply of domestic skilled labor rises due to increased unemployment of unskilled workers, a greater amount of offshoring must be taking place now. Therefore, an increase in τ leads to an increase in offshoring. We summarize the impact of an increase in τ in an offshoring equilibrium in the proposition below.

Proposition 5: *Starting from an offshoring equilibrium where the fairness constraint is binding, an increase in τ leads to an increase in offshoring, an increase in unskilled wage, and an increase in unskilled unemployment, while the unemployment rate and wage of skilled workers remain unchanged.*

It is worth comparing the result in proposition 2 with that in proposition 5. In autarky, an increase in τ has a negative spillover effect on the skilled labor market. However, when the input produced by unskilled labor can be offshored, any impact of a rise in the cost of domestic unskilled labor induced by increased fairness considerations is absorbed by increased offshoring leaving the skilled labor market unaffected.

6 Discussion of possible extensions

Here we discuss some extensions where we relax some of the assumptions of the model. First, we allow the fair wage to be determined within the firm. Next we relax the assumption that τ is a scalar (constant). We allow it to depend on the unemployment rate of unskilled workers. Finally, look at the implications of allowing for free adjustment of the imported input under offshoring.

6.1 Endogenous firm-specific fair wage

In our model the fair-wage constraint operated at the level of the industry. That is, the firm took the fair-wage as given while deciding on its employment decision in the first period. If the fair wage is determined at the firm level, then a firm takes this into account while choosing its employment in the first stage. To be more precise, the objective function of the firm given in (4) now becomes

$$Max_{s,l} \left\{ F(s,l) - w_s(s,l)s - Max\{w_l(s,l), \tau w_s(s,l)\}l - \frac{c_s}{q(\theta_s)}s - \frac{c_l}{q(\theta_l)}l \right\} \quad (30)$$

where w_l^* has now been replaced by $\tau w_s(s, l)$ since it is determined within the firm. As a result, the first order condition for the optimal choice of s in the case where the constraint is expected to bind is given by

$$f'(t) - w_s - s \frac{\partial w_s}{\partial s} - l\tau \frac{\partial w_s}{\partial s} = \frac{c_s}{q(\theta_s)} \quad (31)$$

Note the extra term $l\tau \frac{\partial w_s}{\partial s}$ which captures the fact that any change in skilled wage affects the fair wage as well, which is taken into account by the firm. Using the Nash bargaining as before, the wage of the skilled workers is given by the following differential equation

$$w_s(t) = \beta[f'(t) - (t + \tau)w'_s(t)] \quad (32)$$

It can be verified that the solution to the above differential equation is given by

$$w_s(t) = (t + \tau)^{-\frac{1}{\beta}} \int_0^t (x + \tau)^{\frac{1-\beta}{\beta}} f'(x) dx \quad (33)$$

Intuitively, since hiring an extra skilled worker reduces the fair wage by lowering the skilled wage, the value of a skilled worker for a firm is higher than in the case when the fair wage is exogenous from the firm's point of view. Therefore, the "strategic effect" in the hiring of skilled workers becomes stronger than in the case discussed earlier. Note that when the firm takes the fair wage as given, then both in autarky and in the case of offshoring, the only "strategic effect" is that of skilled employment affecting skilled wage. When the fair wage is determined inside the firm, there is an additional "strategic effect" in autarky because skilled employment affects fair wage, however, it vanishes in the case of an offshoring firm because the firm takes the price of the offshored input as given. In autarky, because of the additional strategic effect, we would expect the relative demand for skilled labor to be higher than when the fair wage was external to the firm. Therefore, the possibility of skilled unemployment decreasing (positive spillover for the skilled labor market) when the fair-wage constraint binds becomes stronger when the fair wage is determined at the firm level. The impact of offshoring on the two labor markets should be qualitatively similar to what we found in the case of the external fair wage.

6.2 Endogenous τ

Let us relax the assumption that τ is a scalar and assume that τ is a decreasing function of unskilled unemployment, which means it is an increasing function of the market tightness for

unskilled workers. In other words, given a skilled wage, what unskilled wage is considered fair is lower, the higher is the unskilled unemployment rate. When there is a vast number of jobless unskilled individuals, unskilled workers will be willing to accept a lower wage as a fair wage. In proposition 2 we derived the equilibrium θ_l as a decreasing function of τ . This is represented in Figure 2 by the downward sloping curve, $[\theta_l(\tau)]^a$, where a stands for autarky. The level of τ the society is willing to accept as a norm is given by the upward sloping curve, $\tau^n(\theta_l)$. Let us assume that it is $[\tau^n(\theta_l)]_1$ in the beginning. The intersection of $[\tau^n(\theta_l)]_1$ with $[\theta_l(\tau)]^a$ gives us our initial autarky equilibrium. We know from here the equilibrium θ_l and τ . We can now draw our *RLS-RLD* diagram for this value of τ to get $\frac{\theta_s}{\theta_l}$, and since we know θ_l , we get θ_s . Thus we are able to obtain wages and unemployment rates. Our main model with a scalar τ is a special case of this analysis where $\tau^n(\theta_l)$ is vertical. So starting from an initial point of equilibrium, an increase in the preference for fairness will be represented by a rightward shift in the $\tau^n(\theta_l)$ curve. We see that with an upward sloping $\tau^n(\theta_l)$ curve, we get a smaller reduction in θ_l and therefore, a smaller increase in unskilled unemployment as well as a smaller increase in skilled unemployment than would be the case with the same rightward horizontal shift of a vertical τ^n curve (starting from the same initial point). The reason is that the increase in the intrinsic preference for fairness is offset to a certain extent by the increase in unemployment that by itself has a negative effect on τ . Similarly the autarky curve, $[\theta_l(\tau)]^a$ shifting down to $[\theta_l(\tau)]^o$ under offshoring will result in a smaller increase in unskilled unemployment than would have prevailed under a scalar τ passing through the initial equilibrium. Thus the effects of shocks and additional distortions are moderated if τ is decreasing in unemployment. Any shock that increases unskilled unemployment also results in the acceptability of a higher wage inequality, which brings unskilled unemployment down. However, overall the results are qualitatively (in terms of direction) the same as in the case of a scalar τ .

6.3 Allowing the free adjustment of the imported input under offshoring

We end this section on extension by looking at, under offshoring, the implications of allowing the adjustment of the imported input at any stage, in particular after the skilled employment has been chosen and at the same time as (or even after) they bargain over wages with skilled workers. As argued earlier, this would better capture the case of imported input, as opposed to the service offshoring case discussed earlier. In this case the optimal choice of imported input must always

satisfy $F_2(s, m) = p_m$, and the equilibrium t will be pinned down by this condition. Therefore, the “strategic effect” will disappear ($\frac{\partial w_s}{\partial s} = 0$), and the first-order condition for the choice of skilled labor will become $f'(t) - w_s(t) = \frac{c_s}{q(\theta_s)}$, and the Nash bargained wage will simply be $w_s(t) = \beta f'(t)$. The results regarding the increase in unskilled unemployment and a decline in skilled unemployment obtain even in this case. Recall the discussion following proposition 3 that the direct effect of offshoring and the “strategic effect” go in the same direction as far as the impact on the skilled labor market is concerned. Therefore, the disappearance of the “strategic effect” in the case of freely adjustable imported input leaves the qualitative result unchanged.

7 Concluding remarks

In this paper, we have studied the effects of additional distortions or shocks, such as a fair-wage constraint and/or the possibility of offshoring unskilled jobs, in a two-factor general equilibrium model of unemployment with search frictions. While the direct point of impact of such shocks is on unskilled workers, we also find interesting indirect spillover effects on skilled workers.

We can summarize our key findings of this paper as follows. Firstly, we find that search frictions and the fair-wage constraint can interact in interesting ways to determine wages, wage inequality and unemployment rates. These interaction effects spillover from one labor market to another. For example, a binding fair-wage constraint in a search model increases the unskilled unemployment rate and can also lead to an increase in the skilled unemployment rate as compared to an equilibrium where fairness considerations are absent or non-binding. This is consistent with the differences we observe between Europe and the US.

Secondly, offshoring always leads to an increase in skilled wage, a decrease in skilled unemployment and an increase in unskilled unemployment. The presence of fairness considerations increases the adverse impact of offshoring on unskilled unemployment. At the same time, allowing for offshoring makes it more likely that the fair-wage constraint binds. In other words, a dormant fair-wage constraint can be made operational through offshoring, which thereby facilitates the interaction between the fair-wage distortion and the search friction.

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8 Appendix

8.1 Proof of Lemma 1

Re-write (8) and (9) as

$$w_s(t) = \beta[f'(t) - tw'_s(t) - w'_l(t)] \quad (34)$$

$$w_l(t) = \beta[f(t) - tf'(t) + t^2w'_s(t) + tw'_l(t)] \quad (35)$$

The above is a system of differential equations in the argument t . We guess the following solution:

$w_s(t) = \beta f'(t); w_l(t) = \beta(f(t) - tf'(t))$, and verify that it satisfies (34)-(35).

The guessed solution implies

$$w'_s(t) = \beta f''(t); w'_l(t) = \beta[-tf''(t)] \quad (36)$$

Therefore, $tw'_s(t) + w'_l(t) = 0$, and hence (34)-(35) imply $w_s(t) = \beta f'(t); w_l(t) = \beta(f(t) - tf'(t))$. *QED*

8.2 Proof of Lemma 2

The first-order conditions in (5) and (6) can be written as

$$f'(t) - w_s(t) - tw'_s(t) - w'_l(t) = \frac{c_s}{q(\theta_s)} \quad (37)$$

$$f(t) - tf'(t) - w_l(t) + t^2w'_s(t) + tw'_l(t) = \frac{c_l}{q(\theta_l)} \quad (38)$$

Using $tw'_s(t) + w'_l(t) = 0$, and $w_s(t) = \beta f'(t)$ from lemma 1 in (37) we get

$$w_s(t) = \frac{\beta}{1 - \beta} \frac{c_s}{q(\theta_s)}$$

Similarly, $w_l(t) = \frac{\beta}{1 - \beta} \frac{c_l}{q(\theta_l)}$. *QED.*

8.3 Proof of lemma 4

(a) Recall that $w_s(t)$ is given by

$$w_s(t) = t^{-\frac{1}{\beta}} \int_0^t x^{\frac{1-\beta}{\beta}} f'(x) dx$$

Therefore,

$$w'_s(t) = -\frac{1}{\beta} t^{-\frac{1}{\beta}-1} \int_0^t x^{\frac{1-\beta}{\beta}} f'(x) dx + \frac{f'(t)}{t}$$

or,

$$w'_s(t) = -\frac{1}{\beta} t^{-\frac{1}{\beta}-1} (\beta f'(t) t^{\frac{1}{\beta}} - \beta \int_0^t x^{\frac{1}{\beta}} f''(x) dx) + \frac{f'(t)}{t}$$

or

$$w'_s(t) = t^{-\frac{1}{\beta}-1} \left(\int_0^t x^{\frac{1}{\beta}} f''(x) dx \right) < 0 \text{ since } f''(x) < 0$$

(b) Use the differential equation for the skilled wage, $w_s(t) = \beta[f'(t) - tw'_s(t)]$, to write $f(t) - tf'(t) + t^2w'_s(t)$ as

$$f(t) - \frac{tw_s(t)}{\beta}$$

Now, take the derivative with respect to t to get

$$f'(t) - \frac{w_s(t)}{\beta} - \frac{tw'_s(t)}{\beta}$$

Next, note again from the differential equation for the skilled wage that $\frac{w_s(t)}{\beta} = f'(t) - tw'_s(t)$.

Therefore, the above becomes

$$tw'_s(t) - \frac{tw'_s(t)}{\beta} > 0$$

The last equality follows from $\beta < 1$.

8.4 Constrained RD lies to the right of the unconstrained one in $(\frac{s}{l}, \frac{\theta_s}{\theta_l})$ space

Start with any $\frac{\theta_s}{\theta_l}$. If $\tau = \frac{c_l}{c_s} \left(\frac{\theta_s}{\theta_l}\right)^{\delta-1}$, then the constraint is just binding. Below we show that at this τ the constrained relative demand is greater than the unconstrained relative demand. The relative demands in the unconstrained and constrained cases can be written as follows.

$$\frac{f'(t)}{f(t) - tf'(t)} = \frac{c_s}{c_l} \left(\frac{\theta_s}{\theta_l}\right)^{1-\delta} \quad (39)$$

$$\frac{\frac{1-\beta}{\beta} w_s(t)}{f(t) - tf'(t) + t^2 w'_s(t) - \tau w_s(t)} = \frac{c_s}{c_l} \left(\frac{\theta_s}{\theta_l}\right)^{1-\delta} \quad (40)$$

We show that at each t the l.h.s of (40) is higher than the l.h.s of (39) when $\tau = \frac{w_l}{w_s} = \frac{f(t) - tf'(t)}{f'(t)}$.

That is, we are comparing relative demands in the unconstrained and constrained cases when the constraint is just binding.

$$\frac{\frac{1-\beta}{\beta} w_s(t)}{f(t) - tf'(t) + t^2 w'_s(t) - \frac{f(t) - tf'(t)}{f'(t)} w_s(t)} > \frac{f'(t)}{f(t) - tf'(t)}$$

or

$$\frac{1-\beta}{\beta} w_s(t) > f'(t) + t^2 w'_s(t) \frac{f(t) - tf'(t)}{f'(t)} - w_s(t)$$

or

$$\frac{w_s(t)}{\beta} > f'(t) + t^2 w'_s(t) \frac{f(t) - tf'(t)}{f'(t)}$$

substituting out $w'_s(t)$ using (16) we get

$$\frac{w_s(t)}{\beta} > f'(t) + t(f'(t) - \frac{w_s(t)}{\beta}) \frac{f(t) - tf'(t)}{f'(t)}$$

or

$$\frac{w_s(t)}{\beta} \left(\frac{f'(t) + t(f(t) - tf'(t))}{f'(t)} \right) > f'(t) + t(f(t) - tf'(t))$$

or

$$w_s(t) > \beta f'(t)$$

which is true from (16) since $w'_s(t) < 0$ as shown in lemma 4(a). What we have shown is that for any $\frac{\theta_s}{\theta_l}$ and $\tau = \frac{c_l}{c_s} \left(\frac{\theta_s}{\theta_l}\right)^{\delta-1}$ and the corresponding unconstrained relative demand denoted by t^u , the following is true

$$\frac{\frac{1-\beta}{\beta} w_s(t^u)}{f(t^u) - t^u f'(t^u) + t^{u2} w'_s(t^u) - \tau w_s(t^u)} > \frac{c_s}{c_l} \left(\frac{\theta_s}{\theta_l}\right)^{1-\delta}$$

Since the relative demand curve is downward sloping, it implies that the relative demand t^c satisfying

$$\frac{\frac{1-\beta}{\beta} w_s(t^c)}{f(t^c) - t^c f'(t^c) + t^{c2} w'_s(t^c) - \tau w_s(t^c)} = \frac{c_s}{c_l} \left(\frac{\theta_s}{\theta_l}\right)^{1-\delta}$$

exceeds t^u . It is also easy to see from the above that any further increase in τ implies an increase in the relative demand for skilled labor, t^c at that given $\frac{\theta_s}{\theta_l}$, which means the constrained relative demand exceeds the unconstrained one for any $\frac{\theta_s}{\theta_l}$ for which the fair-wage constraint is binding.

8.5 Proof of lemma 5

Suppose not. Suppose $\theta_l^c > \theta_l^u$. Since $\frac{\theta_s^c}{\theta_l^c} > \frac{\theta_s^u}{\theta_l^u}$, it implies $\theta_s^c > \theta_s^u$. Let the input combination per unit of output be (s^u, l^u) and (s^c, l^c) in the unconstrained and constrained equilibrium respectively. Then we have $\frac{1}{1-\beta} \frac{c_s}{q(\theta_s^u)} s^u + \frac{1}{1-\beta} \frac{c_l}{q(\theta_l^u)} l^u = 1$ and $\frac{1}{1-\beta} \frac{c_s}{q(\theta_s^c)} s^c + \left[\frac{\tau\beta}{1-\beta} \frac{c_s}{q(\theta_s^c)} + \frac{c_l}{q(\theta_l^c)} \right] l^c = 1$. From the optimality conditions of the profit maximization problem of an unconstrained firm in equilibrium, it must be true that $1 - \left[\frac{1}{1-\beta} \frac{c_s}{q(\theta_s^u)} s^c + \frac{1}{1-\beta} \frac{c_l}{q(\theta_l^u)} l^c \right] < 1 - \left[\frac{1}{1-\beta} \frac{c_s}{q(\theta_s^u)} s^u + \frac{1}{1-\beta} \frac{c_l}{q(\theta_l^u)} l^u \right] = 0$, which implies that $\frac{1}{1-\beta} \frac{c_s}{q(\theta_s^c)} s^c + \frac{1}{1-\beta} \frac{c_l}{q(\theta_l^c)} l^c > 1$ here. Now since $\theta_l^c > \theta_l^u$ and $\theta_s^c > \theta_s^u$, and since a binding (or a just binding) fair-wage constraint (at the constrained equilibrium) implies $\tau \geq \frac{c_l q(\theta_s^c)}{c_s q(\theta_l^c)}$, we have $\frac{1}{1-\beta} \frac{c_s}{q(\theta_s^c)} s^c + \left[\frac{\tau\beta}{1-\beta} \frac{c_s}{q(\theta_s^c)} + \frac{c_l}{q(\theta_l^c)} \right] l^c \geq \frac{1}{1-\beta} \frac{c_s}{q(\theta_s^c)} s^c + \frac{1}{1-\beta} \frac{c_l}{q(\theta_l^c)} l^c > 1$. Thus, the zero-profit condition does not hold in the constrained case and we have a contradiction. Therefore, it has to be true that $\theta_l^c < \theta_l^u$.

8.6 Proof of lemma 6

Proof: Note that the absence of “strategic effect” in wage setting in the constrained case implies $\frac{\partial w_s}{\partial s} = 0$. Therefore, the solution to (16) is simply $w_s(t) = \beta f'(t)$. The two first-order condition

become

$$\begin{aligned}(1 - \beta)f'(t) &= \frac{c_s}{q(\theta_s)} \\ f(t) - tf'(t) - w_l^* &= \frac{c_l}{q(\theta_l)}\end{aligned}$$

Therefore, the relative demand for skilled labor is given by

$$\frac{(1 - \beta)f'(t)}{f(t) - tf'(t) - w_l^*} = \frac{c_s}{c_l} \left(\frac{\theta_s}{\theta_l} \right)^{1-\delta}$$

Using $w_l^* = \tau w_s(t) = \tau \beta f'(t)$, the above can be re-written as

$$\frac{(1 - \beta)f'(t)}{f(t) - tf'(t) - \tau \beta f'(t)} = \frac{c_s}{c_l} \left(\frac{\theta_s}{\theta_l} \right)^{1-\delta} \quad (41)$$

Next, comparing the above with the relative demand in the unconstrained case given in (39), verify that at $\tau = \frac{w_l}{w_s} = \frac{f(t) - tf'(t)}{f'(t)}$, the l.h.s of (39) is identical to the l.h.s. of (41), while at higher τ the constrained demand (41) is higher. Therefore, unlike in the case with “strategic effect”, there is no jump-shift in the relative demand in the absence of the “strategic effect”. Therefore, there is no change in unemployment when the constraint is just binding.

However, starting from an unconstrained equilibrium with θ_l^u and θ_s^u as the equilibrium values, if $\tau > \frac{c_s}{c_l} \left(\frac{\theta_s^u}{\theta_l^u} \right)^{1-\delta}$, then it is obvious that the constrained equilibrium $\frac{\theta_s}{\theta_l}$ and t are higher. Since the skilled wage is still $\beta f'(t)$, an increase in t implies a lower skilled wage, which in turn implies a lower θ_s as well. Since $\frac{\theta_s}{\theta_l}$ has increased, θ_l must be lower as well. Also, since $f(t) - tf'(t) = w_l^* + \frac{c_l}{q(\theta_l)}$ in the constrained case and $f(t) - tf'(t) = w_l + \frac{c_l}{q(\theta_l)}$ in the unconstrained case, and given that we have shown that θ_l is lower in the constrained case, with a higher t in the constrained case (that leads to a higher $f(t) - tf'(t)$), unskilled wage is higher in the constrained case. *Q.E.D*

8.7 Proof of $\kappa^o(\theta_s, p_m)$ increasing in θ_s and p_m

The following are the first order conditions of the profit maximization problem of an offshoring firm:

$$f'(t) = w_s(t) + tw'_s(t) + \frac{c_s}{q(\theta_s)} \quad (42)$$

$$f(t) - tf'(t) = p_m - t^2 w'_s(t) \quad (43)$$

Dividing one by the other gives us

$$\frac{f'(t)}{f(t) - tf'(t)} = \frac{w_s(t) + tw'_s(t) + \frac{c_s}{q(\theta_s)}}{p_m - t^2w'_s(t)} \quad (44)$$

which tells us that the ratio of marginal products of the two factors should equal the ratio of their marginal factor costs (the ratio of the marginal expenditures on each of them). We define unit cost of a firm at given θ_s and p_m as the cost of the input bundle required to produce a unit of output that satisfies the above ratio condition. Let us denote this input bundle by (s^*, m^*) . Let $t^* = s^*/m^*$. Thus the unit cost at this point is $\left[w_s(t^*) + \frac{c_s}{q(\theta_s)} \right] s^* + p_m m^*$. The above ratio condition implies that $\left[w_s(t^*) + \frac{c_s}{q(\theta_s)} \right] s^* + p_m m^* \leq \left[w_s(t) + \frac{c_s}{q(\theta_s)} \right] s + p_m m$ for all (s, m) such that $F(s, m) = 1$, where $t = s/m$. Let (s_1^*, m_1^*) , where $t_1^* = s_1^*/m_1^*$, be the input bundle corresponding to the unit cost when $p_m = p_m^1$ at a given θ_s . Also, let (s_2^*, m_2^*) , where $t_2^* = s_2^*/m_2^*$, be the input bundle corresponding to the unit cost when $p_m = p_m^2 > p_m^1$ at the same θ_s . It is easy to see that at given θ_s , it should be the case that $\left[w_s(t_2^*) + \frac{c_s}{q(\theta_s)} \right] s_2^* + p_m^1 m_2^* < \left[w_s(t_2^*) + \frac{c_s}{q(\theta_s)} \right] s_2^* + p_m^2 m_2^*$. By the ratio condition, as explained above, $\left[w_s(t_2^*) + \frac{c_s}{q(\theta_s)} \right] s_2^* + p_m^1 m_2^* \geq \left[w_s(t_1^*) + \frac{c_s}{q(\theta_s)} \right] s_1^* + p_m^1 m_1^*$. Thus, we have $\left[w_s(t_1^*) + \frac{c_s}{q(\theta_s)} \right] s_1^* + p_m^1 m_1^* < \left[w_s(t_2^*) + \frac{c_s}{q(\theta_s)} \right] s_2^* + p_m^2 m_2^*$, which means unit cost under offshoring is increasing in p_m . Since $q(\theta_s)$ is decreasing in θ_s , along the same lines as above, we can show that this unit cost is also increasing in θ_s .

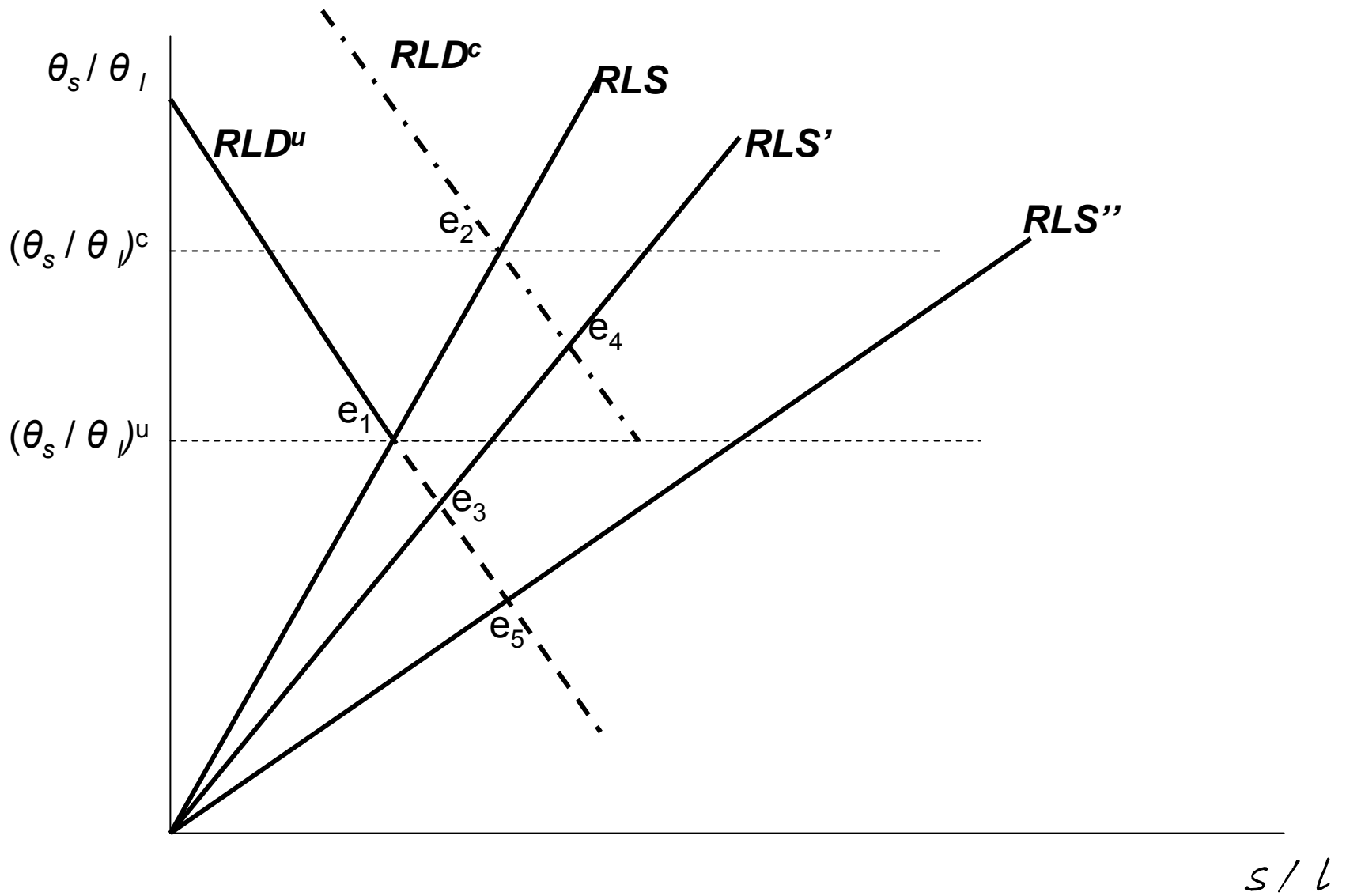


Figure 1: Determination of equilibrium

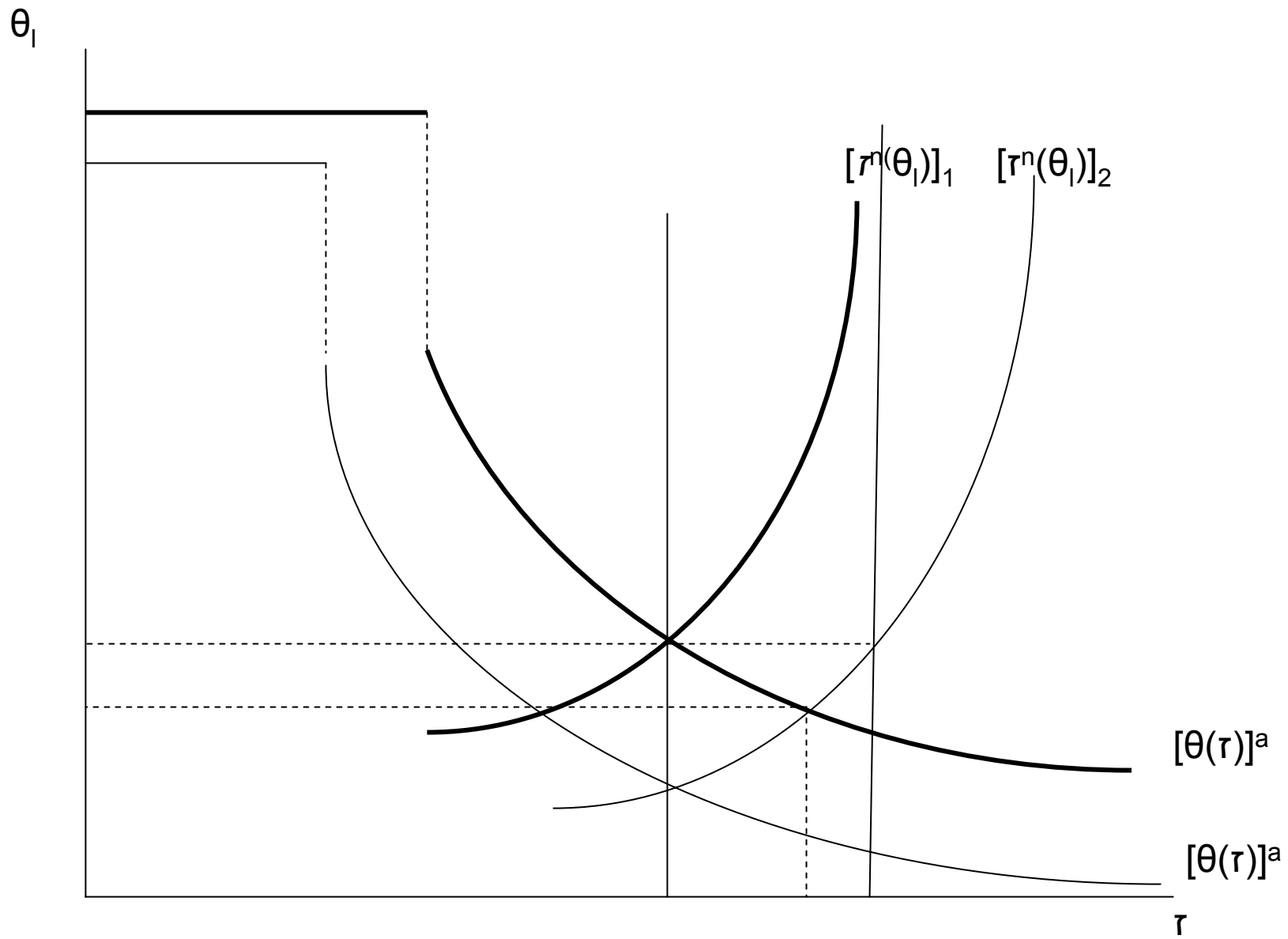


Figure 2: Equilibrium With Endogenous τ