7 Appendix to "Offshoring and Unemployment: The Role of Search Frictions and Labor Mobility" by Devashish Mitra and Priya Ranjan

7.1 Maximization problem of the firm in the autarky case

The firm maximizes (10) subject to (9), and (7). Denoting the Lagrangian multiplier associated with (9) by $\lambda$, and with (7) by $\phi$, the current value Hamiltonian for each firm can be written as

$$H = P_z Z - w_z N - c_z V + \lambda[q(\theta_z)V - \delta N] + \phi[N - m_h - m_p]$$

where $Z$ is given in (6). The first order conditions for the above maximization are follows.

$$m_h : \quad P_z \tau m_h^{\rho-1} (\tau m_h^\rho + (1 - \tau)m_p^\rho)^{\frac{1}{\rho}} = \phi \quad (26)$$

$$m_p : \quad P_z (1 - \tau)m_p^{\rho-1} (\tau m_h^\rho + (1 - \tau)m_p^\rho)^{\frac{1}{\rho}} = \phi \quad (27)$$

$$V : \quad c_z = \lambda q(\theta_z) \quad (28)$$

$$N : \quad w_z + \lambda \delta - \phi = \dot{\lambda} - r \lambda \quad (29)$$

Now, (26) and (27) imply

$$\frac{m_h}{m_p} = \left(\frac{\tau}{1 - \tau}\right)^{\frac{1}{1-\rho}} \quad (30)$$

using the above in (26) gives

$$\tau' P_z = \phi \quad (31)$$

Since the value of marginal product of labor, given by $\tau' P_z$, is constant, using the result from Cahuc and Wasmer (2001) mentioned in footnote 11, we have treated wage to be exogenous in deriving (29) above.

Next, note from (28) that for a given $\theta_z$, $\lambda$ is constant. Using $\dot{\lambda} = 0$, (28), and (31) in (29) we get

$$\tau' P_z - w_z = (r + \delta) \lambda = \frac{(r + \delta)c_z}{q(\theta_z)} \quad (32)$$

$\lambda$ is the shadow value of an extra job.

7.2 Wage Determination

Let $U_j^z$ denote the income of the unemployed worker-$j$ searching for a job in the $Z$ sector. The asset value equation for the unemployed in this sector is given by

$$r U_j^z = \xi_j^z + b + \theta_z q(\theta_z)[E_j^2 - U_j^z] \quad (33)$$
where $E^j_z$ is the expected income from becoming employed in the $Z$ sector, which is the sum of the idiosyncratic benefit, $\varepsilon^j_z$, the unemployment benefit $b$, and the expected capital gain from the possible change in state from unemployed to employed.

The asset value equation for employed worker-$j$ in sector $Z$ is given by

$$rE^j_z = \varepsilon^j_z + w^j_z + \delta(U^j_z - E^j_z) \Rightarrow E^j_z = \frac{\varepsilon^j_z}{r + \delta} + \frac{w^j_z}{r + \delta} + \frac{\delta U^j_z}{r + \delta}$$ \hfill (34)

Again the return on being employed is the sum of the idiosyncratic benefit, $\varepsilon^j_z$, the wage, and the expected change in the asset value from a change in state from employed to unemployed. Next, (34) implies that

$$E^j_z - U^j_z = \frac{\varepsilon^j_z}{r + \delta} + \frac{w^j_z}{r + \delta} - \frac{r U^j_z}{r + \delta}$$ \hfill (35)

Assume the rent from a vacant job to be zero which is ensured by no barriers to the posting of vacancy. Now, denote the surplus for a firm from a job occupied by worker-$j$ by $J^j_z$. From (32) above,

$$J^j_z = \frac{\tau P_z - w^j_z}{r + \delta}$$ \hfill (36)

The Nash-bargained wage is obtained by

$$\arg \max_{w^j_z} (E^j_z - U^j_z)^\beta (J^j_z)^{1-\beta}$$

where $E^j_z - U^j_z$ is given in (35) and $J^j_z$ is given by (36). The first-order condition for the bargained wage is

$$E^j_z - U^j_z = \frac{\beta}{1 - \beta} J^j_z = \frac{\beta}{1 - \beta} \frac{c_z}{q(\theta_z)}$$ \hfill (37)

where the last equality follows from the fact that the value of an occupied job, $J^j_z$, equals $\frac{c_z}{q(\theta_z)}$ as discussed in (13) in the text. Plugging the value of $E^j_z - U^j_z$ from above into the asset value equation for the unemployed in (33) we have a simplified version of this asset value equation

$$rU^j_z = \varepsilon^j_z + b + \frac{\beta}{1 - \beta} c_z \theta_z$$ \hfill (38)

Use (37) to substitute out $E^j_z - U^j_z$ and (38) to substitute out $rU^j_z$ in (35) to get the following simplified wage equation:

$$w^j_z = b + \frac{\beta c_z}{1 - \beta} \left[ \theta_z + \frac{r + \delta}{q(\theta_z)} \right]$$

Note that $w^j_z$ is the same for all $j$. Similarly, in the case of the $X$ sector, we obtain $w_x = b + \frac{\beta c_x}{1 - \beta} [\theta_x + \frac{r + \delta}{q(\theta_x)}]$.

### 7.3 Proofs of Lemmas 2 and 3

At $p = \frac{1}{\mu}(w_x)$ the relative supply of $Z$ is given by

$$\frac{(\tau^\sigma + (1 - \tau)^\sigma) \tilde{w} [L_z(1 - u_z) - N^\sigma] + \tau^{-\sigma} (\tau^\sigma + (1 - \tau)^\sigma) \tilde{w} N^\sigma}{L_x(1 - u_x)}$$ \hfill (39)
where the total domestic employment of the offshoring firms is denoted by \( N^o \) and \( N^o \in [0, L_z(1 - u_z)] \), and \( L_x \) and \( L_z \) are given in (18). Since \( \tilde{\varphi} \) is a function of \( p, L_z \) and \( L_x \) are functions of \( p \) as well. Therefore, the denominator remains constant while the numerator increases with \( N^o \).

To find the offshoring relative supply when \( p > \underline{p}(w_s) \), we need to obtain the amount of labor affiliated with each sector, which in turn depends on \( \tilde{\varphi} \) given in (24). Denote the \( \tilde{\varphi} \) with the possibility of offshoring by \( \hat{\varphi}^o(p) \), where the superscript \( o \) stands for offshoring, and as in autarky, \( \hat{\varphi} \) is a function of \( p \). For \( p < \underline{p}(w_s) \), allowing for offshoring leaves \( \hat{\varphi}(p) \) unchanged. For \( p > \underline{p}(w_s) \), \( P_z \) and \( P_x \) are still given by (3). Therefore, \( \theta_x \) and \( w_x \) remain unchanged from autarky for each \( p \). However, \( \theta_z \) and \( w_z \) are now determined by (25). We know from lemma 1 that \( \theta_z \) and \( w_z \) are higher than in autarky. Since \( \theta_z \) is higher while \( \theta_x \) is unchanged for each \( p > \underline{p}(w_s) \), (24) implies that the \( \hat{\varphi}^o(p) \) curve lies to the left of the \( \hat{\varphi}^A(p) \) curve as shown in Figure 2a.

Note that the expressions for the amount of labor going to each sector in the case of offshoring are still given by (18) with \( \hat{\varphi}^A \) being replaced by \( \hat{\varphi}^o \). The relative supply for each \( p > \underline{p}(w_s) \) is given by

\[
\left( \frac{Z}{X} \right)^s = \frac{\tau^{-\sigma} (\tau^\sigma + (1 - \tau)^\sigma \omega \sigma^{-1}) \frac{\hat{\varphi}(p)}{(1 - u_z)} (1 - u_z)}{(1 - u_x) \exp(\hat{\varphi}(p)/\alpha)}
\]  

(40)

where \( u_i, \omega, \) and \( \hat{\varphi}^o \) are functions of \( p \). For each \( p > \underline{p}(w_s) \), \( \omega > 1 \), \( \hat{\varphi}^o(p) < \hat{\varphi}^A(p) \), \( u_x \) is unchanged from autarky, while \( u_z \) is lower than in autarky, therefore, the expression on the r.h.s above exceeds the expression on the r.h.s of (20). This proves lemma 2.

In the limit, when \( \alpha \to 0, \varphi^j \to 0 \) \( \forall j \). In this case the relative supply is zero for any \( p < p^0 \) because \( c_x \theta_x > c_z \theta_z \), and hence no one wants to work in the Z sector, and it becomes horizontal at \( p = p^0 \) since all workers are indifferent between working in the two sectors. Also, \( \hat{\varphi}^o(p) < \hat{\varphi}^A(p) \) implies \( p^o < p^A \). Next we prove that \( p^o > \underline{p}(w_s) \). Note that, there is no offshoring for \( p < \underline{p}(w_s) \). Therefore, \( p^o \geq \underline{p}(w_s) \). Suppose \( p^o = \underline{p}(w_s) \).

Now, by assumption \( w_s < \tilde{w}_z \), which leads to offshoring (Recall that the superscripts “\( o \)” and “\( A \)” denote the equilibrium values of variables under offshoring and autarky, respectively). At \( p = \underline{p}(w_s) \) we have \( \tilde{w}_z^o = w_s < \tilde{w}_z^A \). This implies, from the wage curve equation for \( Z \), that \( \theta_z^o < \theta_x^A \). Additionally, \( \tilde{w}_z^o < \tilde{w}_z^A \) in conjunction with the numeraire condition (or the zero-profit condition for the numeraire good \( C \)) implies that \( \tilde{w}_x^o > \tilde{w}_x^A \), which in turn from the wage curve equation for \( X \), gives us \( \theta_x^o > \theta_x^A \). By the no arbitrage condition, we start in autarky from a situation where \( \theta_x^A = \theta_x^A \). Given that \( \theta_z^o < \theta_x^A \) and \( \theta_x^o > \theta_x^A \), this implies \( \theta_z^o < \theta_x^A \). Thus the no arbitrage condition is not satisfied under offshoring. This is a contradiction because at \( p^o \) the no arbitrage condition must be satisfied by definition. Therefore, \( p^o > \underline{p}(w_s) \). This proves lemma 3.

The expression in (40) makes it clear that the relative supply is independent of \( \alpha \) for \( \hat{\varphi}^o(p) = 0 \). Therefore, the offshoring relative supply curves with different values of \( \alpha \) all pass through the same point at \( p = p^o \). Using the same argument as in the case of autarky, we can verify that a decrease in \( \alpha \) leads to a clockwise rotation of the relative supply curve at \( p = p^o \). This pins down the relative positions of the offshoring relative supply curves corresponding to \( \alpha_1 \) and \( \alpha_2 \), respectively, in Figure 3.
7.4 Proof of Lemma 4

Assume \( c_x = c_z \) and the following production function for \( C \)

\[
C = \left( \gamma z^{\frac{\phi - 1}{\phi}} + (1 - \gamma) x^{\frac{\phi - 1}{\phi}} \right)^{\frac{\phi}{\phi - 1}}
\]

where \( \phi \) is the elasticity of substitution between \( X \) and \( Z \). The production function for \( C \) implies the following cost function.

\[
\left( \gamma (P_z)^{1 - \phi} + (1 - \gamma) (P_x)^{1 - \phi} \right)^{\frac{1}{1 - \phi}}
\]

Since \( C \) is the numeraire, the unit cost of \( C \) must equal 1. Note that the relative demand \((4)\) for \( Z \) when the production function for \( C \) is of the CES type is given by

\[
\left( \frac{Z}{X} \right)^d = \left( \frac{\gamma P_x}{(1 - \gamma) P_z} \right)^{\phi}
\]

The relative demand for \( Z \) equal to relative supply in autarky equilibrium can be written as

\[
\frac{\tau' L_z}{L - L_A} = \left( \frac{\gamma P_z}{(1 - \gamma) P_z} \right)^{\phi} \frac{1 - u_x}{1 - u_z}
\]

Next, \( c_x = c_z \) implies \( \theta_x = \theta_z \), which in turn implies \( w_x = w_z \), and hence \( P_x = \tau' P_z \) where \( \tau' \equiv [\tau^\sigma + (1 - \tau)^\sigma]^{\frac{1}{1 - \sigma}}. \) Also, \( \theta_x = \theta_z \) implies \( u_x = u_z. \) Therefore, from (43)

\[
\frac{L_A}{L - L_A} = \frac{1}{\tau'} \left( \frac{\gamma \tau'}{(1 - \gamma)} \right)^{\phi}
\]

where \( L_A \) is the amount of labor in the \( Z \) sector in autarky equilibrium. Note that if there was no labor market friction in the model, the expression for \( L_z \) in autarky would be exactly the same as in (44).

Similarly, the relative demand equals relative supply in the offshoring equilibrium can be written as

\[
\frac{\tau^{-\sigma} (\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1})^{\frac{1}{\sigma - 1}} L_z}{L - L_z} = \left( \frac{\gamma P_x}{(1 - \gamma) P_z} \right)^{\phi} \frac{1 - u_x}{1 - u_z}
\]

Again, \( c_x = c_z \) implies \( \theta_x = \theta_z \) and hence \( u_x = u_z. \) Also, \( \theta_x = \theta_z \) and \( w_x = w_z \) imply \( P_x = (\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1})^{\frac{1}{\sigma - 1}} P_z. \)

Therefore, (45) can be written as

\[
\frac{L_A^o}{L - L_A^o} = \frac{\tau^\sigma}{(\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1})^{\frac{1}{\sigma - 1}}} \left( \frac{\gamma (\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1})^{\frac{1}{\sigma - 1}}}{(1 - \gamma)} \right)^{\phi}
\]

where \( L_A^o \) is the amount of labor in the \( Z \) sector in the offshoring equilibrium. Again, if there is no labor market friction then the expression for \( L_z \) in an offshoring equilibrium would be the same as in (46). The only difference would be that \( \omega \) would be the ratio of domestic wage to foreign wage rather than being the ratio of domestic labor cost to foreign wage.
Comparing (44) and (46) note that $L^A_z > (<) L^o_z$ if the following inequality holds.

\[
\left( \frac{\tau^\sigma + (1 - \tau)^\sigma}{\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1}} \right)^{\frac{\phi - 1}{\sigma - 1}} > (<) \frac{\tau^\sigma}{\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1}}
\]

(47)

We get the following possibilities:

Case I: $\phi = 1$. In this case the l.h.s of (47) exceeds the r.h.s if $\tau^\sigma < 1$, if which is true for any $\sigma$. Therefore, if the production function for $C$ is Cobb-Douglas, then irrespective of the elasticity of substitution in $Z$ production, labor always moves from $Z$ to $X$ upon offshoring.

Case II: $\phi = \sigma$. In this case the l.h.s of (47) exceeds the r.h.s if $1 + \left( \frac{1 - \tau}{\tau} \right)^\sigma > 1$, which is always true implying $L^A_z > L^o_z$.

Case III: $\phi < 1$, $\sigma > 1$. In this case the l.h.s of (47) exceeds 1 since $\frac{\tau^\sigma + (1 - \tau)^\sigma}{\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1}} < 1$ and $\frac{\phi - 1}{\sigma - 1} < 0$, while the r.h.s is less than 1. Therefore, again $L^A_z > L^o_z$.

Case IV: $\phi < 1$, $\sigma < 1$. Again, the l.h.s of (47) exceeds 1 because $\frac{\tau^\sigma + (1 - \tau)^\sigma}{\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1}} > 1$ and $\frac{\phi - 1}{\sigma - 1} > 0$. Therefore, again $L^A_z > L^o_z$.

Case V: $\phi > 1$, $\sigma > 1$. Again, the l.h.s of (47) exceeds 1 because $\frac{\tau^\sigma + (1 - \tau)^\sigma}{\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1}} > 1$ and $\frac{\phi - 1}{\sigma - 1} > 0$. Therefore, again $L^A_z > L^o_z$.

Case VI: $\phi > 1$, $\sigma < 1$. In this case $\sigma < 1$ implies $\frac{\tau^\sigma + (1 - \tau)^\sigma}{\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma - 1}} > 1$, but $\phi > 1$, $\sigma < 1$ implies $\frac{\phi - 1}{\sigma - 1} < 0$. Therefore, the l.h.s of (47) is less than 1. Since both the l.h.s and the r.h.s are less than 1, it is possible for the r.h.s to exceed l.h.s in which case $L^A_z < L^o_z$.