MORAL HAZARD AND BERTRAND EQUILIBRIA UNDER REFERENCE DEPENDENCE AND LOSS AVERSION

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ABSTRACT

The paper characterizes the properties of the competitive equilibrium in the classic principal-agent model with moral hazard for the case where the agent has reference dependent preferences exhibiting loss aversion and diminishing sensitivity. The equilibrium contract consists of three regions. If output is below a critical value, the principal gives the smallest possible transfer—zero. When output is above this critical value but below an upper threshold, the agent receives a constant amount equal to expected output. And if output exceeds the upper threshold, there is a region with increasing additional rewards.

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INTRODUCTION

The study of incentives stands at the core of economic analysis. Yet how people respond to incentives crucially depends on the preferences under-laying their choices. This assertion is especially applicable to the case of contract theory. Fundamentally, the properties of a contract depend on the preferences of the parties involved, the production technology, and the set of feasible contracts that the agents can sign. Making valid inferences about contractual relationships thus requires, at a minimum, a thorough understanding of preferences.

The goal of this paper is to characterize the properties of the Bertrand-Rothschild-Stiglitz competitive equilibrium in the classic principal-agent model with limited liability and moral hazard for the case where the agent has reference dependent preferences exhibiting loss aversion and diminishing sensitivity. All other features of the standard model, including the expected utility form of the overall utility function, are preserved.

The conclusion is that in the presence of reference dependence and loss aversion, seemingly incompatible features of observed contracts, such as local full insurance (a guaranteed draw in labor contracts or a maximum out-of-pocket expense in insurance) combined with discrete harsh punishments (termination of employment in a labor contract or a limit on lifetime benefits in insurance) become integral parts of an optimal insurance-incentives mechanism. This is because, unlike the classic case of smooth risk aversion where both insurance and incentives are provided locally, in the presence of reference dependence and loss aversion, incentives are provided globally while insurance locally.

We first prove that a competitive equilibrium exists under quite general conditions. The equilibrium contract consists of three regions. If output is below a critical value, the principal gives the smallest possible transfer—zero. When output is above this critical value but below an upper threshold, the agent receives a constant amount equal to expected output. And if output exceeds the upper threshold, there is a region with increasing additional rewards. This is essentially a theory of full insurance locally in the middle part of the output distribution, rewards of increasing size for high output values and the biggest possible punishment for output below a critical level.

The flat region is due to reference dependence and loss aversion. Loss aversion makes the agent very risk averse around the reference point so full insurance is locally optimal. The agent is risk averse for gains relative to the reference transfer, hence incentives are partially provided by the region with increasing rewards. On the other hand, the agent is risk-loving for values below the reference point, making a corner solution, the zero transfer, optimal.

1.1 RELATED LITERATURE

The paper relates to several different strands of economic analysis. First, it contributes to the literature on the applications of prospect theory preferences. Camerer (2001) identifies ten market phenomena that are inconsistent with the classic formulation of expected utility theory but can be explained by some feature of prospect theory, typically loss aversion or diminishing sensitivity. The most prominent applications are in the area of finance–Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001)–

where it is argued that reference dependence and loss aversion can successfully account for the equity premium puzzle. But prospect theory preferences have also been used to explain behavior in a variety of other situations. Examples include inter-temporal choice–Bowman, Minehart, and Rabin (1999)–labor supply–Camerer et al. (1997), Goette, Fehr, and Huffman (2004), Rizzo and Zeckhauser (2003)–seller behavior in the residential real estate market–Genesove and Mayer (2001)–and the pricing strategy of a monopolist facing loss averse consumers–Heidhues and Koszegi (2008).

Initially, papers in this area aimed at explaining peculiar forms of behavior that violated predictions of the standard theory. Recently, however, works such as Barberis, Huang, and Santos (2001), Rizzo and Zeckhauser (2003), and Heidhues and Koszegi (2008) have been investigating more systematically the general implications of prospect theory preferences. Our paper contributes to this trend by incorporating prospect theory preferences in the classic principal-agent model with moral hazard. The conclusions of the model are then tested with available data and their merit evaluated on the basis of how well they explain that data.

Finally, our paper is part of the literature that tries to understand the effects of nonstandard preferences on competitive equilibrium in general and the case of moral hazard in particular. An example of the latter is Bennardo and Chiappori (2003). The literature on the former is extremely large. Classic references are Akerloff (1980) and Kahneman, Knetsch, and Thaler (1986), while Mas (2006) provides a recent example.

SETUP

Consider a static principal-agent relationship. Let Y denote the surplus from this relationship. Y is stochastic and can take on one of S possible values, where $Y_s \in [\underline{Y}, \overline{Y}]$ for $s = 1, 2, ..., S$ with $Y_1 < Y_2 < ... < Y_S$. The realized state of the world is observed by both parties and can be verified by an independent arbiter. The agent has no assets but can influence the probability, $p_{sn} > 0$, with which each state occurs by privately choosing one of N effort levels, $(1 - l_n) \in [0,1]$.

The timing of the interaction is summarized in figure 1. At stage 1, principals in the market compete by simultaneously offering contracts to the agents. The environment is hence one of Bertrand competition. Randomized contracts are enforceable so a contract may involve an ex ante lottery over effort levels as well an ex post lottery over nonnegative transfers in each state. The principals and the agents can commit to honoring the contracts. At stage 2, each agent exerts effort, output is realized and a transfer is made based on the observed state of the world.

2.1 PREFERENCES

The principals are risk neutral. The agents have preferences over the payoff and leisure sets $\Omega = R_+$ and $L = [0,1]$ that depend on a reference state $r = (r_w, r_l) \in \Omega \times L$,

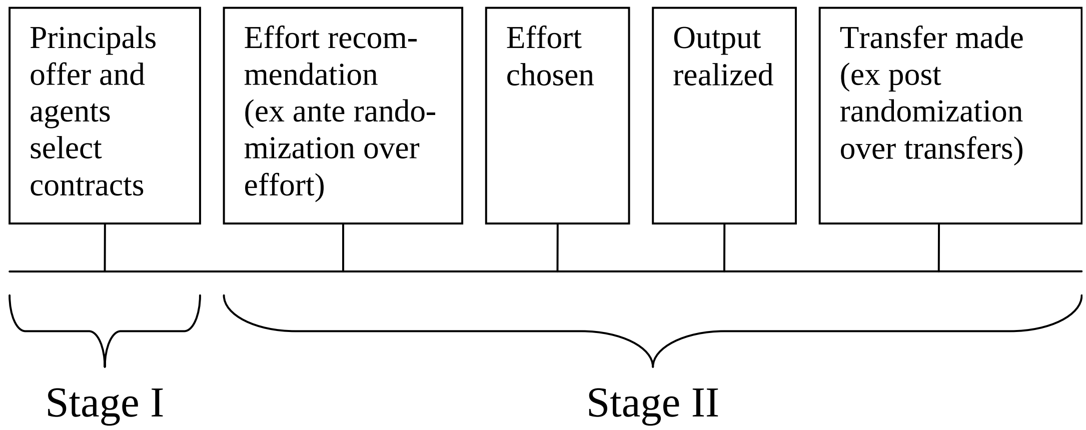

Figure 1: Timing of the principal-agent interaction.

which is context-dependent. Assume that for a given reference point, wants are independent so preferences over bundles $x = (w, l)$ can be represented by a strictly increasing continuous function U with

$$U(w, l; r_w, r_l) = u(w, r_w) + v(l, r_l). \quad (1)$$

Conditions for the existence of a reference dependent representation are given by Tversky and Kahneman (1991). The crucial point is that the attractiveness of a bundle here depends both on its intrinsic properties as well as on how it compares with the agent's reference point.

Assume that for a given reference point, the continuity and independence axioms are satisfied so that preferences over lotteries can be represented by a utility function with the expected utility form. Following prospect theory, assume that U exhibits loss aversion and diminishing sensitivity around the reference point. Formally, let U_x denote the partial derivative of U with respect to argument $x \in \{w, l\}$ and assume that for a given $r = (r_w, r_l)$:

(A1). $U(w, l; r_w, r_l)$ is continuous and strictly increasing in w and l and twice differentiable for $w \neq r_w$ and $l \neq r_l$;

(A2). For $a > b > 0$, $u(r_w + a, r_w) + u(r_w - a, r_w) < u(r_w + b, r_w) + u(r_w - b, r_w)$ and similarly for $v(.)$;

(A3). $U_x^-(r_x, r_x) > U_x^+(r_x, r_x) > 0$ where $U_x^-(r_x, r_x) = \lim_{x \uparrow r_x} U_x(x, r_x)$ and $U_x^+(r_x, r_x) = \lim_{x \downarrow r_x} U_x(x, r_x)$ are both finite;

(A4). $U_{xx}(x, r_x) < 0$ for $x \geq r_x$ while $U_{xx}(x, r_x) > 0$ for $0 \leq x \leq r_x$, with the derivatives at r defined as in $(A3)$;

(A5). $U_x(0, r_x) > U_x^+(r_x, r_x)$.

Assumption 1 is standard in economics. Assumptions 2 and 3 define loss aversion for large and small stakes respectively. Assumption 4 requires diminishing sensitivity globally for gains and losses relative to the reference point. Finally, $(A5)$ says that marginal utility is uniformly higher in the domain of losses than in the domain of gains. In the literature, this assumption is usually referred to as "strong loss aversion".

RESULTS

3.1 COMPETITIVE EQUILIBRIUM—EXISTENCE

As already mentioned, we assume that the unobservable effort is the only restriction imposed on trades. In particular, output is observable and randomized contracts are enforceable. Under these conditions, we show that a competitive equilibrium exists quite generally. We first describe the set of contracts that the parties can sign.

Definition 1: *For a given reference point r, a contract $C(r)$ consists of a menu of lotteries $\{W_n, \alpha_n\}_{n=1}^{N}$ over nonnegative transfers and effort levels, where for each recommended effort level n, $W_n = \{(w_{n,sq}, \gamma_{n,sq})_{q=1}^{Q(s)}\}_{s=1}^{S}$ is a sequence of S state-contingent lotteries. The contract $C^*(r)$ is **feasible** if the expected profit associated with $C^*(r)$ is nonnegative. That is, if*

$$\sum_n \alpha_n^* \sum_s p_{sn}[Y_s - \sum_q \gamma_{n,sq}^* w_{n,sq}^*] \geq 0.$$

*The contract $C^*(r)$ is **incentive compatible** if the agent always finds it optimal to choose the recommended effort level. That is, if the following incentive constraints (ICs) hold:*

$$\sum_s p_{sn} u^*_{sn}(r_w) + v(l_n, r_l) \geq \sum_s p_{sk} u^*_{sk}(r_w) + v(l_k, r_l) \quad \text{for all } n, k,$$

where $u^*_{sn}(r) = \sum_q \gamma^*_{n,sq} u(w^*_{n,sq}, r)$.

First, we provide a definition of competitive equilibrium.

Definition 2: *For r given, the contract $C^*(r)$ is a **competitive equilibrium** if it is a feasible and incentive compatible Nash equilibrium of the two-stage game.*

This definition is the moral hazard counterpart of the competitive equilibrium concept developed by Rothschild and Stiglitz (1976) for the case of adverse selection. In particular, as pointed out by Bennardo and Chiappori (2003), with moral hazard, the contract $C^*(r)$ is a competitive equilibrium if and only if it is robust to the introduction of additional profitable contracts. We now prove two lemmas that will be useful in proving that such equilibrium exists.

Lemma 1: *For r given, any competitive equilibrium $C^*(r)$ maximizes the agents' ex ante expected utility subject to the feasibility and incentive constraints.*

Proof: Suppose not. Take the incentive compatible contract $C^{\max}(r)$ that maximizes the agent's utility and consider $C^{\varepsilon}(r)$ such that $u_{sn}^{\varepsilon} = u_{sn}^{\max} - \varepsilon$. Clearly, this contract is feasible and incentive-compatible and features expected utility that is ε smaller than $C^{\max}(r)$. But then for any candidate other than $C^{\max}(r)$, there exists $\varepsilon > 0$ such that $C^{\varepsilon}(r)$ dominates this candidate. Hence only $C^{*}(r) = C^{\max}(r)$ can be a competitive equilibrium. Q.E.D.

Lemma 2: *For a given r, any equilibrium lotteries over transfers will feature lotteries only over the transfer levels 0 and r_w.*

Proof: To see that any non-degenerate lottery will involve only 0 and r_w, suppose not. Take a contract C^{ND} that includes such a lottery. Any such non-degenerate lottery (for example A or B in figure 2) can be dominated either by its certainty equivalent or by a lottery involving 0 and r_w. Hence for any such lottery there exists a degenerate lottery or a lottery between 0 and r_w that gives the same expected utility at a lower expected expenditures. Call this new contract C^{D}. Clearly then, we can always find $\varepsilon > 0$ such that $u_{sn}^{\varepsilon} = u_{sn}^{D} + \varepsilon$ and the expected profit associated with this new contract C^{ε} is greater

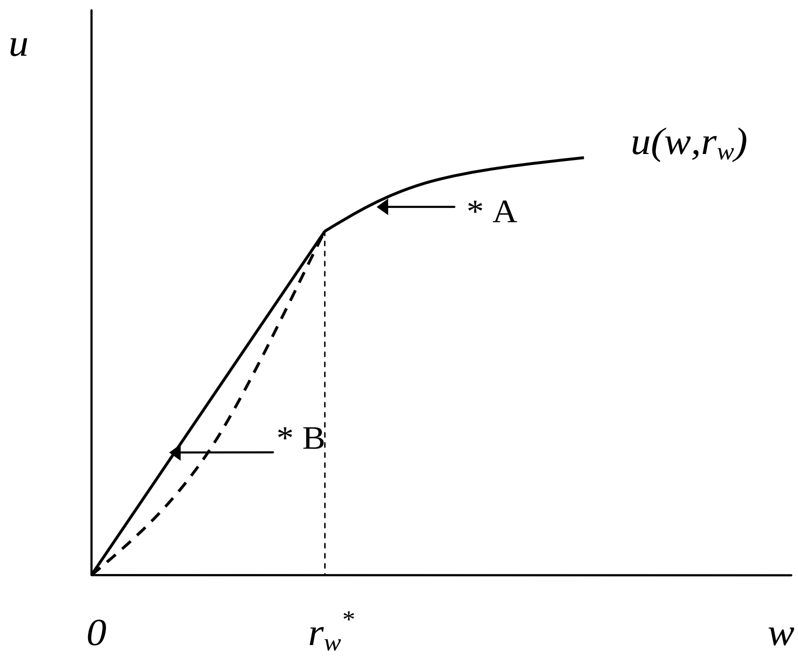

Figure 2: The utility possibilities set and two lotteries A and B that cannot be part of an equilibrium contract.

than the expected profit of C^{ND}. But then C^{ε} dominates C^{ND}. Hence C^{ND} cannot be a competitive equilibrium. Q.E.D.

The preference specification clearly suggests that the properties of a competitive equilibrium will depend on the value of the reference point. Without a theory of how this point is formed, we are left with the uneasy prospect that anything goes. This is where the theories developed by Rayo and Becker (2007) and Koszegi and Rabin (2006) come to help.

There are two important issues at hand. First, we have to postulate a law according to which each agent's reference point is formed, given the economic environment in question. Second, regardless of how the reference point is formed, we need to determine whether a principal believes that by offering a contract he can influence the agent's reference point. Since we have a competitive environment, it seems reasonable to assume that when each principal designs a contract, he takes the agents' reference points *as given*. A principal, in other words, does not believe that he can affect the reference point of agents by unilaterally deviating from the prevalent contract in the market.

$(A6)$. *Each principal is too small to influence the agents' reference point by unilaterally deviating from the prevalent contract in the market.*

We will be more specific about the exact form of the reference point function later. For now, it is sufficient to assume continuity.

$(A7)$. *The function* $r : R_+^{N \cdot S} \times [0,1]^{N^2 \cdot S} -> R_+ \times [0,1]$ *that maps contracts into reference points is continuous.*

Clearly then, it is possible to have a situation where even if $C^*(r^*)$ is a competitive equilibrium for the given value r^*, $r(C^*) \neq r^*$. The contract C^* is thus not consistent with the agent's preferences and will not be observed in equilibrium. Hence, to be equilibrium, a contract has to satisfy one last condition.

Definition 3: *Let* $r(C)$ *be the agents' reference point function and suppose that* $r(C)$ *satisfies* $(A7)$. *The contract* C^* *is a* ***consistent competitive equilibrium*** *if* C^* *is a competitive equilibrium for* $r(C^*)$.

We are now ready to prove that a competitive equilibrium exists.

Proposition 1: (Existence) *If* $(A1)-(A7)$ *are satisfied, a consistent competitive equilibrium* C^* *always exists.*

Proof: See Appendix A. The proof uses the Kakutani fixed point theorem. Q.E.D.

3.2 COMPETITIVE EQUILIBRIUM–CHARACTERIZATION

To characterize the properties of the competitive equilibrium, we impose several additional restrictions on the reference point function and the production technology.

$(A7')$. *If* C^* *is the prevalent contract in the market, then*

$r(C^*) = (E[\sum_n \alpha_n^* W_n^* \mid C^*], E[l \mid C^*])$.

Condition $(A7')$ presumes that the reference point is the agent's rational expectation conditional on the market environment. This specification is in line with the recent work of Koszegi and Rabin (2006). In addition, Rayo and Becker (2007) provide an evolutionary reasoning for why the reference point function should be equivalent to the expected value function. They argue that such preferences motivate agents to take the right course of action by conditioning the reward on ex post performance relative to what was expected given the environment in question. They then show that the expected value is the evolutionary optimal reference point.

We also impose two additional assumptions on the production process:

$(A8)$. $p_{sn}/p_{sm} \geq p_{tn}/p_{tm}$ *for all* $t < s$ *and all* $m < n$ *;*

$(A9)$. *For* $i < j < k$ *and* $\beta \in [0,1]$ *such that* $l_j = \beta l_i + (1-\beta) l_k$, *the condition* $P_{sj} \leq \beta P_{si} + (1-\beta) P_{sk}$ *holds for all* s, *where* $P_{sj} = \sum_{n=1}^{j} p_{sn}$.

Both of these assumptions are standard in principal-agent models with moral hazard as they allow us to use the first order approach. $(A8)$ is the standard monotone likelihood ratio condition (MLRC) which implies that exerting a higher effort level increases the likelihood with which a more favorable state of the world is realized. Furthermore, as in Grossman and Hart (1983), we also impose $(A9)$ –the cumulative distribution function of Y is convex in the effort level. This condition allows us to ignore all upward incentive constraints as they are slack if $(A9)$ is satisfied.

Proposition 2: (Properties of Competitive Equilibrium) *Suppose* $(A1)-(A6)$ *and* $(A7')-(A9)$ *hold. If* $C^* = \{(0,\gamma_{sn}^*; w_{sn}^*, 1-\gamma_{sn}^*)_{s=1}^S, \alpha_n^*\}_{n=1}^N$ *is a consistent competitive equilibrium, then for all* l_n *with* $\alpha_n^* > 0$*, the transfer to the agent is non-decreasing in* s *with:*

(I) $\gamma_{sn}^* = 1$ *for states where*

$$1+\sum_{k\neq n}\frac{\mu_k}{\alpha_n^*}\left(1-\frac{p_{sk}}{p_{sn}}\right) < \frac{\phi r_w^*}{u(r_w^*, r_w^*)-u(0,r_w^*)};$$

(II) $\gamma_{sn}^* \in (0,1)$ *and* $w_{sn}^* = r_w^*$ *with probability* $(1-\gamma_{sn}^*)$ *for*

$$1+\sum_{k\neq n}\frac{\mu_k}{\alpha_n^*}\left(1-\frac{p_{sk}}{p_{sn}}\right) = \frac{\phi r_w^*}{u(r_w^*, r_w^*)-u(0,r_w^*)};$$

(III) $\gamma_{sn}^* = 0$ *and* $w_{sn}^* = r_w^*$ *with probability* 1 *for*

$$\frac{\phi r_w^*}{u(r_w^*, r_w^*)-u(0,r_w^*)} < 1+\sum_{k\neq n}\frac{\mu_k}{\alpha_n^*}\left(1-\frac{p_{sk}}{p_{sn}}\right) \leq \frac{\phi}{U_w^+(r_w^*, r_w^*)};$$

(IV) for states where

$$1+\sum_{k\neq n}\frac{\mu_k}{\alpha_n^*}\left(1-\frac{p_{sk}}{p_{sn}}\right)>\frac{\phi}{U_w^+(r_w^*,r_w^*)},$$

$\gamma_{sn}^*=0$ *and* $w_{sn}^*>r_w^*$ *is given by the solution to*

$$1+\sum_{k\neq n}\frac{\mu_k}{\alpha_n^*}\left(1-\frac{p_{sk}}{p_{sn}}\right)=\frac{\phi}{U_w(w_{sn},r_w^*)},$$

where μ_k *and* ϕ *are the multipliers associated with the incentive and feasibility constraints and* $r^*=r(C^*)$ *is as defined in* $(A7')$.

Proof: See Appendix B. The proof consists of a straightforward manipulation of the first order conditions. Q.E.D.

DISCUSSION

Figure 3 provides a graphical representation of the equilibrium contract. It consists of three regions. If output is below a critical value, the principal gives a zero transfer. When output is above this critical value but below an upper threshold, the agent receives a constant amount equal to the reference transfer. And if output exceeds an upper threshold, there is a region with increasing additional rewards.

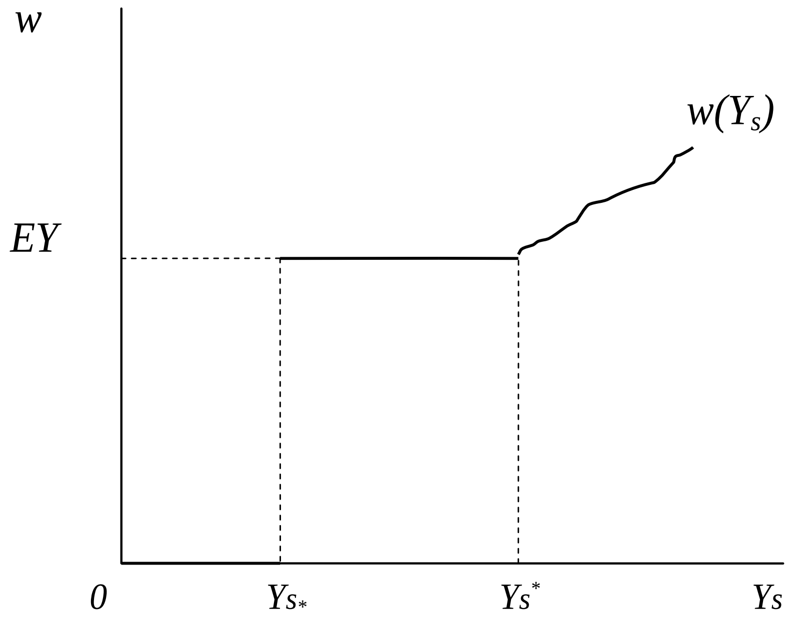

Figure 3: The competitive equilibrium contract.

What's the intuition behind this result? The flat region in the middle part of the distribution is due to reference dependence and loss aversion. Loss aversion makes the agent very risk averse around the reference point so full insurance is locally optimal. Mathematically, what's happening is that even though the sum of the likelihood ratios is increasing with output in that part of the distribution, the transfer is not, because of the discontinuity in marginal utility that's caused by the kink at the reference point. But we have moral hazard, so incentives have to be provided somehow. And since the agent is risk averse in the domain of gains, incentives are partially provided by the region with increasing rewards for high output values. On the other hand, the agent is risk-loving for values below the reference point, making a corner solution optimal. Thus whenever the sum of likelihood ratios in that region is above the critical value, we go to the reference transfer but when it falls below the critical value, implementing the biggest possible punishment becomes optimal.

Thus what we have is essentially a theory of full insurance locally in the middle part of the output distribution, rewards of increasing size for high output values and the biggest possible punishment for output below a critical level. Also, since the reference point forms before the lottery over effort is realized, the level of the flat region will not depend on the recommended effort level. The size of the region, however, will vary with the outcome of the effort lottery. The principal will adjust the rewards and punishments regions depending on the outcome of the effort lottery.

In this stylized, static model, the biggest punishment is a zero transfer. In reality, the biggest possible punishment that a principal can impose on an agent is to give a zero transfer for the rest of the agent's life, that is, to terminate the relationship. One can argue that some features of observed contracts, such as caps on lifetime insurance benefits and termination of employment following poor performance are consistent with such punishment.

APPENDIX A

PROOF OF PROPOSITION 1

The proof uses the Kakutani fixed point theorem. By Lemma 1, for a given r, $C^*(r)$ solves

$$(P1): \max_{\{\alpha_n, u_{sn}\}} \sum_n \alpha_n [\sum_s p_{sn} u_{sn} + v(l_n, r_l)]$$

s.t. $0 \le \alpha_n \le 1$,

$\sum_n \alpha_n = 1, u_{sn} \ge 0$

$$(ICs): \sum_s p_{sn} u_{sn} + v(l_n, r_l) \ge \sum_s p_{sk} u_{sk} + v(l_k, r_l) \quad \text{for all } k \ne n,$$

$$(FE): \sum_n \alpha_n \sum_s p_{sn} [Y_s - h_{sn}] \ge 0,$$

where $u_{sn} = \gamma_{sn} u(0, r_w) + (1 - \gamma_{sn}) u(w_{sn}, r_w)$ and $h_{sn} = (1 - \gamma_{sn}) w_{sn}$ with $w_{sn} \ge r_w$. Let $C^{\max}(r) = \text{argmax}\{P1\}$.

Step 1. $C^{\max}(r(C))$ *is upper hemi-continuous in* C *over* $R_+^{N \cdot S} \times [0,1]^{N^2 \cdot S}$.

First, note that for a given r, $(P1)$ has a unique global maximum and $C^{\max}(r)$ is upper hemi-continuous in r. In particular, the objective function in $(P1)$ is strictly quasi-concave and all constraints are convex. The only part which is not obvious is that (FE) is convex. Take $C' = \{W_n', \alpha_n'\}_{n=1}^N$ and $C'' = \{W_n'', \alpha_n''\}_{n=1}^N$ that satisfy (FE) and Let $\$'$ and $\$''$ denote their respective expected profits. Now clearly, for u_{sn} between 0 and $u(r_w, r_w)$, h_{sn} is linear in u_{sn} and γ_{sn}. However, for $u_{sn} > u(r_w, r_w)$, $h_{sn} = w_{sn}$ which is convex in u_{sn} since $u(w,r_w)$ is concave in w for $w > r_w$. But then for any β between 0 and 1, the contract $C^{\beta} = \beta C^{'} + (1-\beta)C^{''}$ will give an expected profit of $\$^{\beta} \geq \beta\$^{'} + (1-\beta)\$^{''} \geq 0$. Hence (FE) is convex. But we know that a strictly quasi-concave function over a convex constraint set has a unique global maximizer $C^{\max}(r)$ (Theorem M.K.4 in MasCollel et al. (1995)). Moreover, by the Theorem of the Maximum, $C^{\max}(r)$ is upper hemi-continuous in r. Finally, since $r(C)$ is continuous in C by assumption, $(C^{\max}r)(C)$ is upper hemi-continuous in C.

Step 2. The set of feasible contracts, $CONTR$, is compact.

Now in principle, the contract space $R_{+}^{N \cdot S} \times [0,1]^{N^2 \cdot S}$ is not compact. However, it can be shown that there exists a finite $w^{\max}$ such that if $w_{sn} > w^{\max}$, the feasibility constraint cannot be satisfied. To see this, note that for any $\alpha_n > 0$, (and there must be at least one such n), if $w_{s_* n} > E[Y \mid l_1]/p_{s_* n}$ with l_1 the lowest leisure (highest effort) level, then even if $h_{sm} = 0$ for all $(s,m) \neq (s_*, n)$, the feasibiblity constraint will be violated since $E[Y \mid l_1] > E[Y \mid l_m]$ for $m > 1$. Hence in reality, the contract space is restricted to $[0, w^{\max}] \times [0,1]^{N^2 \cdot S}$, which is a compact set.

Step 3. The Kakutani fixed point theorem applies.

Following the definition, C^* is a consistent competitive equilibrium if $(C^{\max}r)(C^*) = C^*$. That is, if C^* is a fixed point of $C^{\max}r : CONT -> CONT$. But since $C^{\max}r$ is an upper hemi-continuous correspondence over a compact set with $C^{\max}r(C)$ a singleton (hence nonempty and convex) for $C \in CONTR$, by Kakutani, $C^{\max} r$ has a fixed point over $CONTR$. Hence a consistent competitive equilibrium exists. Q.E.D.

APPENDIX B

PROOF OF PROPOSITION 2

Consider again $(P1)$ for $r^* = r(C^*)$. Following Lemma 2, without loss of generality we can restrict attention to the set of lotteries over transfers, Γ^*, consisting of

$$\Gamma^* = \{(0,\gamma; w, 1-\gamma) : w \geq r_w^*, 0 \leq \gamma \leq 1 \text{ and } \gamma = 0 \text{ for } w > r_w^*\}.$$

We thus have,

$$(P1'): \quad \max_{\{\alpha_n, w_{sn}, \gamma_{sn}\}} \sum_n \alpha_n \{\sum_s p_{sn}[\gamma_{sn} u(0, r_w^*) + (1-\gamma_{sn}) u(w_{sn}, r_w^*)] + v(l_n, r_l)\}$$

s.t. $0 \leq \gamma_{sn} \leq 1$,

$0 \leq \alpha_n \leq 1$

$\sum_n \alpha_n = 1,\ w_{sn} \geq r_w^*$

$$(ICs): \sum_s p_{sn}[\gamma_{sn} u(0, r_w^*) + (1-\gamma_{sn}) u(w_{sn}, r_w^*)] + v(l_n, r_l)$$

$$\geq \sum_s p_{sk}[\gamma_{sk} u(0, r_w^*) + (1-\gamma_{sk}) u(w_{sk}, r_w^*)] + v(l_k, r_l) \quad \text{for all } k \neq n,$$

$$(FE): \sum_n \alpha_n \sum_s p_{sn}[Y_s - (1-\gamma_{sn}) w_{sn}] \geq 0.$$

For any $\alpha_n^* > 0$, the equilibrium contract must satisfy the first-order conditions with respect to $\{w_{sn}\}$ and $\{\gamma_{sn}\}$, namely,

$$[\gamma_{sn}]: \quad [1 + \sum_{k \neq n} \frac{\mu_k}{\alpha_n^*}\left(1 - \frac{p_{sk}}{p_{sn}}\right)][u(w_{sn}, r_w^*) - u(0, r_w^*)] = \phi w_{sn} + \frac{\psi_{sn} - \xi_{sn}}{\alpha_n^* p_{sn}},$$

$$[w_{sn}]: \quad [1 + \sum_{k \neq n} \frac{\mu_k}{\alpha_n^*}\left(1 - \frac{p_{sk}}{p_{sn}}\right)](1-\gamma_{sn}) U_w(w_{sn}, r_w^*) = \phi(1-\gamma_{sn}) - \frac{\theta_{sn}}{\alpha_n^* p_{sn}},$$

where the multipliers are ϕ for (FE), $\{\mu_k\}$ for the incentive constraints, θ_{sn} for w_{sn}, and ψ_{sn} and ξ_{sn} for the downward and upward constraints on γ_{sn}.

By the construction of Γ^*, $\gamma_{sn} = 0$ for $w_{sn} > r_w^*$. Consider first the states where a (possibly degenerate) lottery between 0 and r_w^* is optimal. For interior values, the first order condition for γ_{sn} suggests that a non-degenerate lottery will be optimal in states which satisfy

$$1+\sum_{k\neq n}\frac{\mu_k}{\alpha_n^*}\left(1-\frac{p_{sk}}{p_{sn}}\right)=\frac{\phi r_w^*}{u(r_w^*,r_w^*)-u(0,r_w^*)}.$$

Note that the right-hand side of this condition does not depend on s or n. Hence a nondegenerate lottery will be optimal only for states where the left hand side is equal to this particular value, a rare occasion.

For $1+\sum_{k\neq n}\ (\mu_k/\alpha_n^*)(1-p_{sk}/p_{sn})\neq \phi r_w^*/[u(r_w^*,r_w^*)-u(0,r_w^*)]$, we then have $\gamma_{sn}=0$ or 1.

In particular, $1+\sum_{k\neq n}\ (\mu_k/\alpha_n^*)(1-p_{sk}/p_{sn})<\phi r_w^*/[u(r_w^*,r_w^*)-u(0,r_w^*)]$ requires that $\psi_{sn}<\xi_{sn}$, which implies $\gamma_{sn}=1$ since ψ_{sn} and ξ_{sn} cannot be both greater than 0.

Similarly, $1+\sum_{k\neq n}\ (\mu_k/\alpha_n^*)(1-p_{sk}/p_{sn})>\ \phi r_w^*/[u(r_w^*,r_w^*)-u(0,r_w^*)]$ implies that $\gamma_{sn}=0$.

From the $[w_{sn}]$ first order condition, $w_s>r_w^*$ is optimal if it satisfies

$$1+\sum_{k\neq n}\frac{\mu_k}{\alpha_n^*}\left(1-\frac{p_{sk}}{p_{sn}}\right)=\frac{\phi}{U_w(w_{sn},r_w^*)}.$$

Now $(A5)$ implies that $\phi/U_w^+(r_w^*,r_w^*)>\phi r_w^*/[u(r_w^*,r_w^*)-u(0,r_w^*)]$. Combining these requirements, we obtain the conditions (I)-(IV) described in Proposition 2. Finally, as shown by Grossman and Hart (1983), under $(A8)$ and $(A9)$, $1+\sum_{k\neq n}\ (\mu_k/\alpha_n^*)(1-p_{sk}/p_{sn})$ is non-decreasing in s. Hence the transfer to the agent must also be non-decreasing in s. Q.E.D.

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