McPeak
Lecture 6
PPA 723

Constrained consumer choice.

What is the optimal bundle?

The bundle of goods that makes the consumer as well off as possible given a particular budget constraint. The bundle consumed at the point where the indifference curve is tangent to the budget line.

Graph with points a, b, c, d.

B is the optimal bundle.
C is not, as it is not in the opportunity set.
A is in the opportunity set, but has less of good 1 and 2 than b, so must be worse.
D is in the opportunity set, and has more of good one, less of good 2. By looking at the indifference curves and knowing their properties, we can rule it out.

Technical note:

A solution is an interior solution if the optimal bundle involves the consumer consuming positive amounts of both goods.

A solution is a corner solution if the optimal bundle involves the consumer consuming zero of one of the goods.

We will focus on interior solutions.

For an interior solution, the marginal rate of substitution equals the marginal rate of transformation. We know these expressions have other forms, so we can write the expanded equality for the condition of an interior solution as follows:

$$MRS = -\frac{MU_{x1}}{MU_{x2}} = \frac{\Delta X_2}{\Delta X_1} = -\frac{P_1}{P_2} = MRT$$

A nice way to think about it is that the optimal bundle equates the marginal utility per dollar spent on good 1 with the marginal utility per dollar spent on good 2.

$$\frac{MU_{x1}}{p_1} = \frac{MU_{x2}}{p_2}$$

Example:
Price of apples is \$0.50, price of oranges is \$0.80.
Consumption bundle is 4 apples and 10 oranges.
Income is \$10.
MU_{apples}(apples=4, oranges=10)=3
$MU_{oranges}$(apples=4, oranges=10)=4

First, is the consumer on the budget line at the consumption bundle in question?

What does a budget line look like? $p_1 \cdot x_1 + p_2 \cdot x_2 = Y$ or $0.50 \cdot 4 + 0.80 \cdot 10 = 10$, OK, so on the budget line (not above, not below).

Draw with oranges on y axis, apples on x axis.

What is MRS at observed consumption bundle?

- MU_{apples}/ $MU_{oranges}$, or -3/4 =-0.75

What is MRT at observed consumption bundle? –price of apples/ price of oranges, or -0.5/0.8 or -0.625.

Market says I have to give up .625 oranges to get 1 more apple. My preferences say I am willing to give up .75 oranges to get 1 more apple to be equally happy. I can give up less than that and have some left over to make me move up to a higher indifference curve.

Or, MRS at that point says give up 3 oranges to get 4 apples, MRT says give up three oranges to get 4.8 apples.

Why give up oranges to get more apples and not the other way around? $\frac{MU_{x1}}{p_1} = \frac{MU_{x2}}{p_2}$

$$\frac{MU_{apples}}{p_{apples}} = \frac{MU_{oranges}}{p_{oranges}}$$

at optimal bundle, but

$$\frac{MU_{apples} = 3}{p_{apples} = 0.50} = 6 > \frac{MU_{oranges} = 4}{p_{oranges} = 0.80} = 5$$

at current bundle.

I need to bring down the marginal utility of apples, bring up the marginal utility of oranges.

By diminishing marginal utility, I get higher marginal utility by consuming less of a good, lower marginal utility by consuming more of a good.

So to bring MU_{apples} down, I need to consume more apples.

To bring $MU_{oranges}$ up, I need to consumer fewer oranges.

Thus more apples, fewer oranges (say 12 apples, 5 oranges which satisfies the budget constraint, and under the assumption that MU_{apples} (apples=12, oranges = 5) = 2.75, $MU_{oranges}$ (apples=12, oranges = 5) = 4.4).

Remember, MRS is the rate at which a consumer is willing to give up one good to get more of the other. It comes from the underlying preferences of the consumer.

MRT is the rate at which prices let a consumer give up one good to get more of the other. It comes from the underlying market conditions and has nothing to do with the consumer's preferences.

Show corner solution. In this case, the indifference curve is not tangent to the budget line, since reality intrudes.

The underlying preferences cause this, and since we take preferences as given, we can run across this. It happens when the consumer has a strong preference for one good as compared to the other.

Food stamp example.

Consider good 1 is food, good two is all other goods.

Food stamps are given to the consumer, and the value of the stamps can only be spent on food. $100 worth let us say.

Consider extreme examples, where all is spent on one good or the other again. However, note there is an issue about the constrained nature of the food stamp program.
Graph.

Shape of indifference curves matters here. Is the consumer better off getting $100 cash or $100 in food stamps (better off with food stamps than without in either case)?

Graph for equally well off whether the transfer is in cash or in food stamps

Graph for worse off when the transfer is in food stamps rather than cash.

Note in both cases, consumption of the other good has increased in spite of the fact that the program is targeted at food.

General result – a subsidy program that constrains consumer choice will do worse or no better than an unconstrained transfer.

Why do we constrain the transfer in the case of food stamps then?

What is the impact on the budget constraint of a black market in food stamps that lets you get 30 cents on the dollar face value?

Show graph.

(Chapter 5).

How do we derive a demand curve?

Remember that we began the course by taking a supply curve and a demand curve as given. Now we are going to find out where a demand curve comes from.

Ingredients: Budget line, Indifference curves, Variation

Go back to our basic budget line:

$p_{1*}\ x_1 + p_{2*}\ x_2 = Y$

first with:

$Y=100$, $p_1 = 2.5$, and $p_2 = 10$

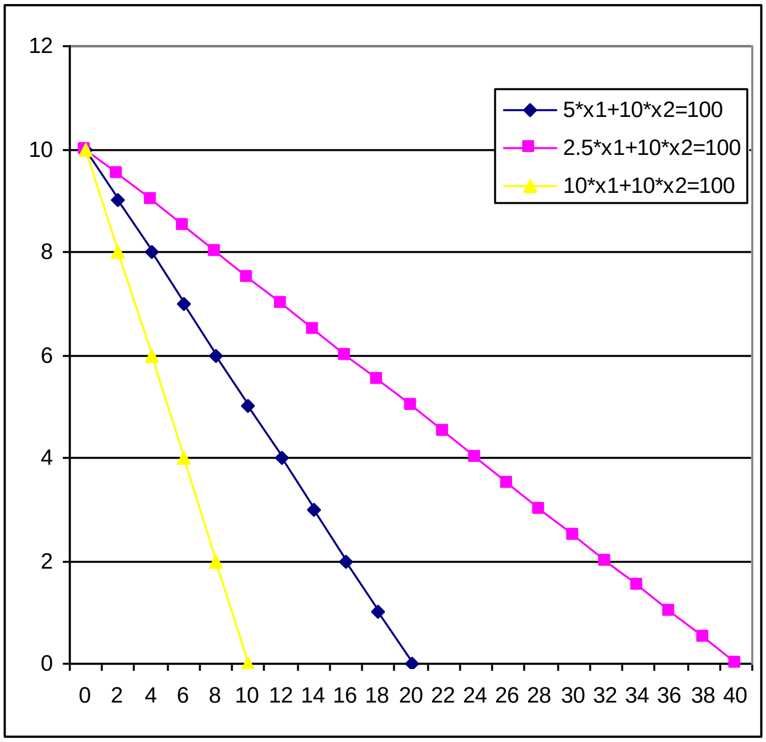
Then consider

$Y=100$, $p_1 = 5$, and $p_2 = 10$

$Y=100$, $p_1 = 10$, and $p_2 = 10$

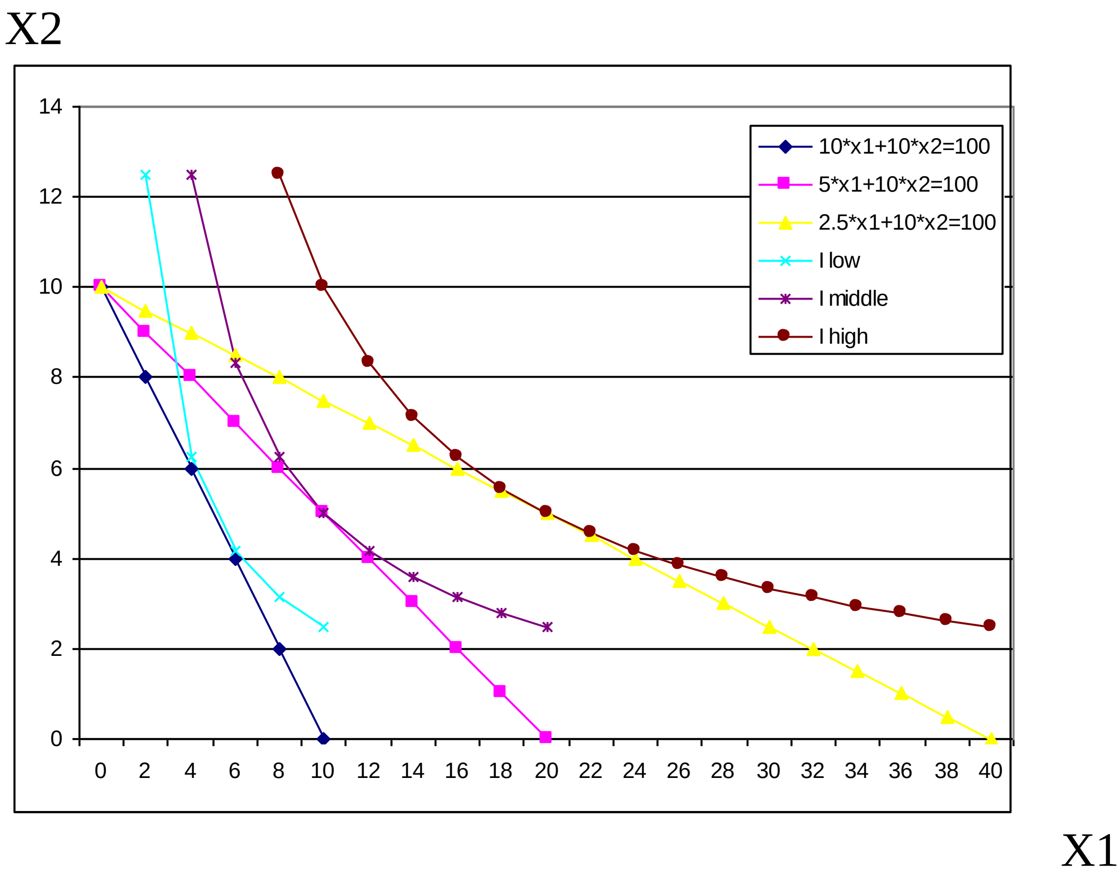
On the same graph.

X2

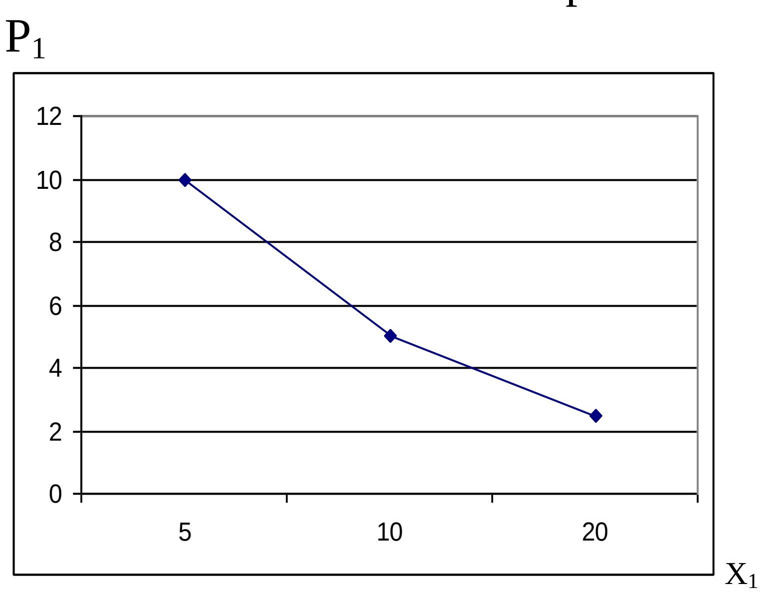
[graph] 

X1

Draw some indifference curves on this.



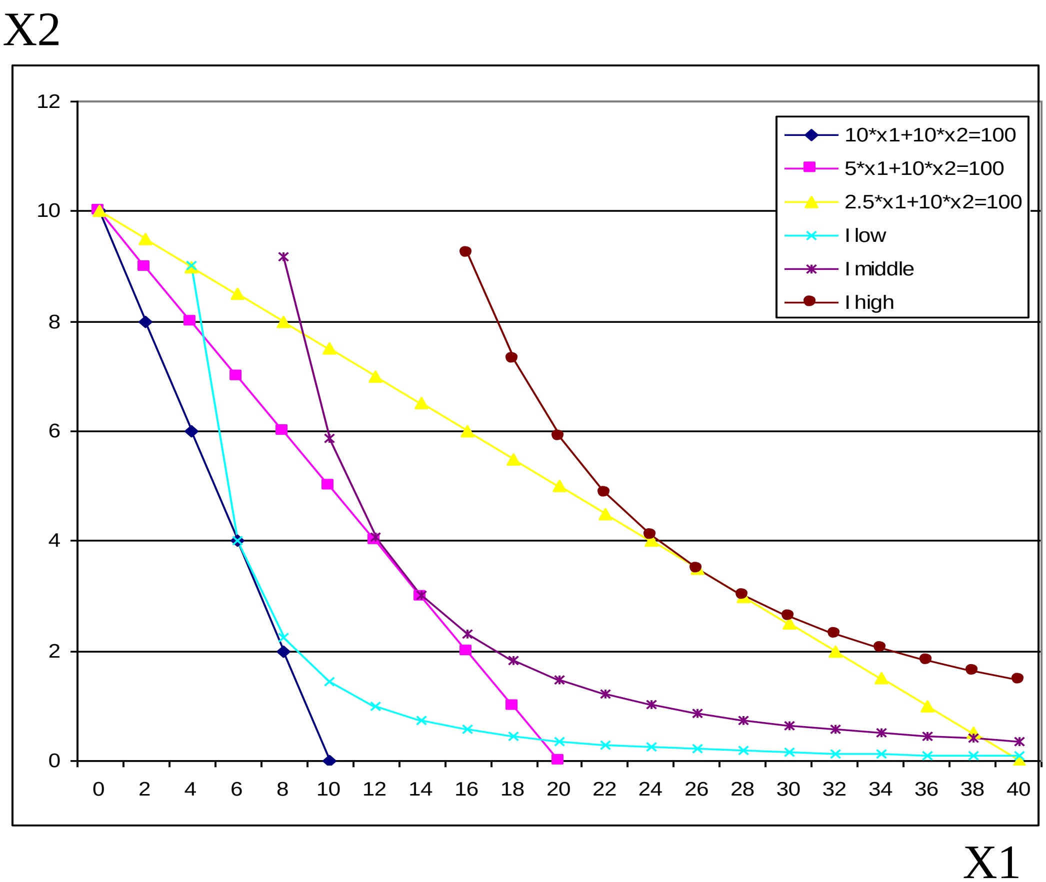
The line connecting these points traces out the price consumption curve (5,5) ; (10,5) ; (20, 5). In this case, it is a horizontal line due to the simple form I picked for utility.



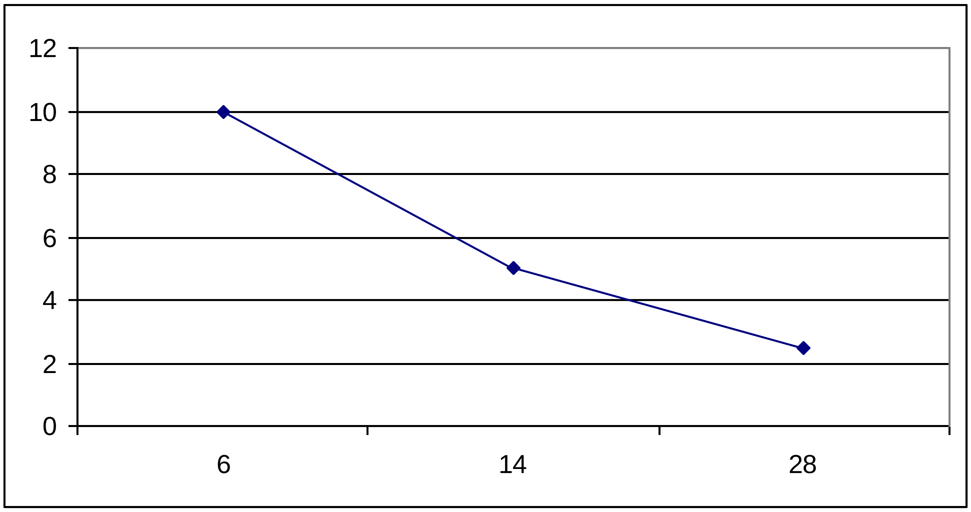
We can summarize the information in this form.

All else constant, how does a change in price impact the quantity demanded for a given good. Sound familiar? This is the derivation of the individual's demand curve for a particular good.

The line traced out by this, the price consumption curve, will reflect the individual's underlying preferences. For a different individual (one who has less of a preference for x2 than the individual just considered), we would get the following result.



Q of goods	Price
(6.4)	10
(14,3)	5
(28.3)	2.5

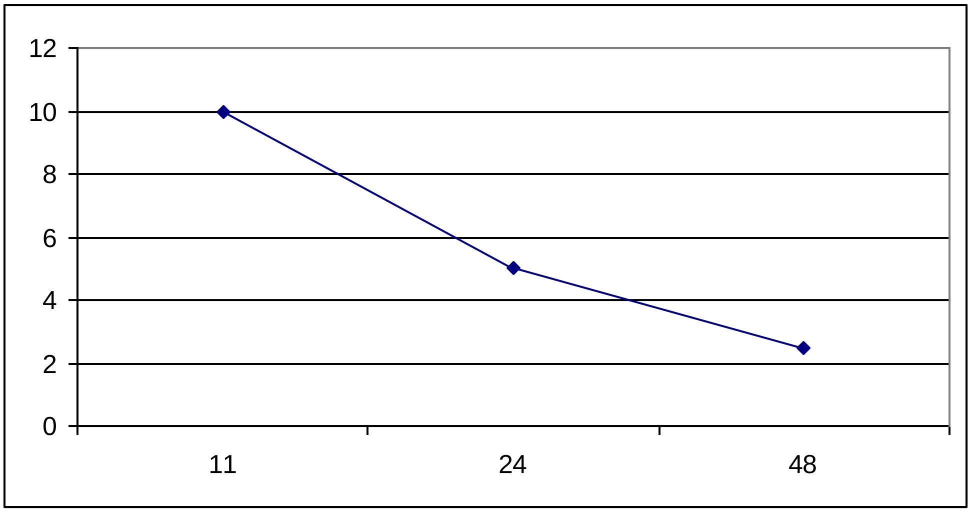


Now, if we take each individual's demand curve and sum them horizontally, we end up with the market demand.

At a price of 10, consumer 1 wants 5 and consumer 2 wants 6.
11

At a price of 5, consumer 1 wants 10 and consumer 2 wants 14.
24

At a price of 2.5, consumer 1 wants 20 and consumer 2 wants 28. 48

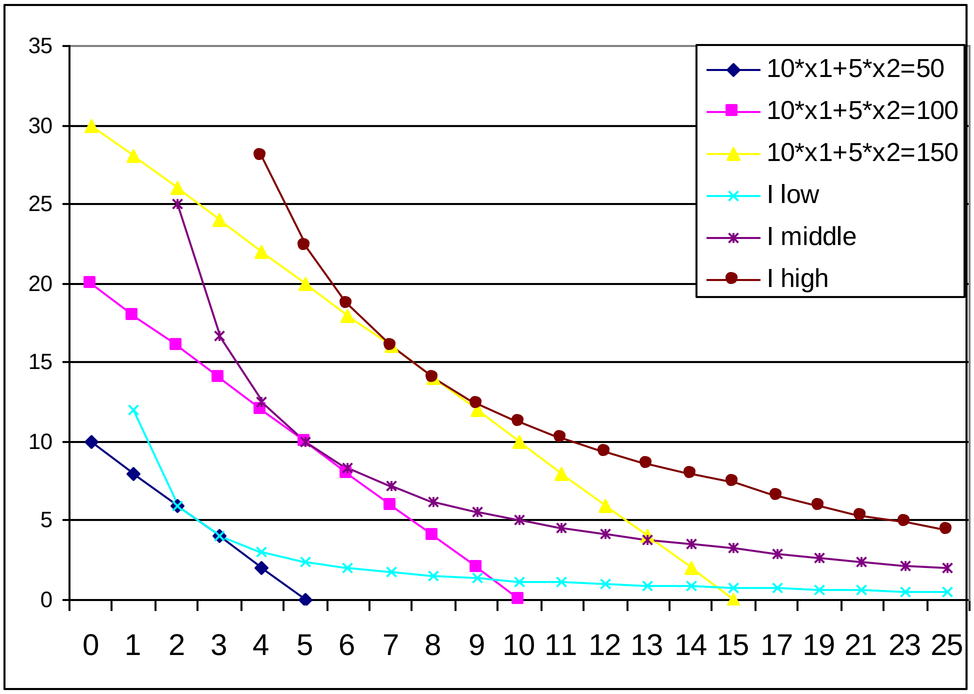


What about a change in income?

$p_{1*}\ x_1 + p_{2*}\ x_2 = Y$

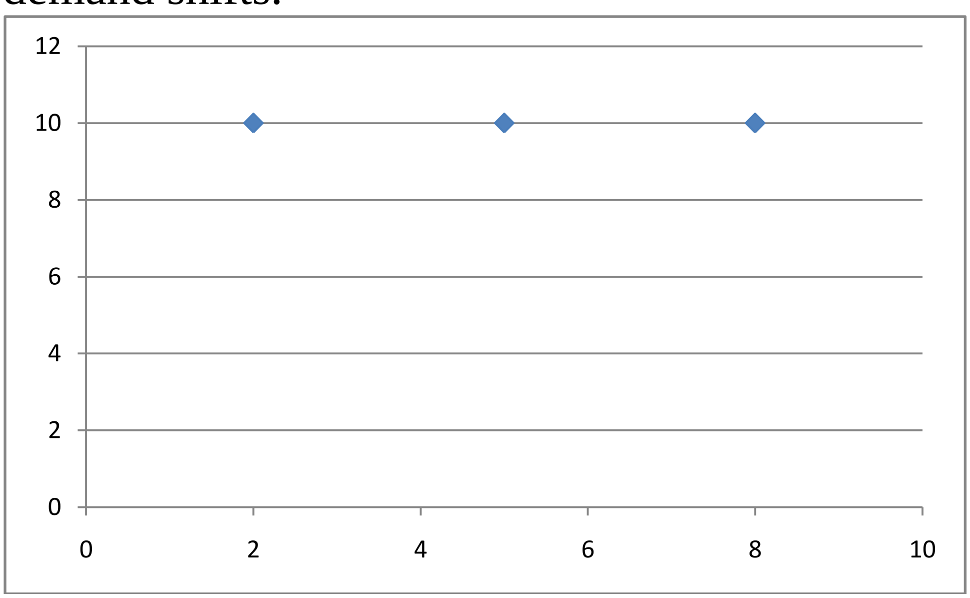
first with:

$Y=50$, $p_1 = 10$, and $p_2 = 5$
$Y=100$, $p_1 = 10$, and $p_2 = 5$
$Y=150$, $p_1 = 10$, and $p_2 = 5$

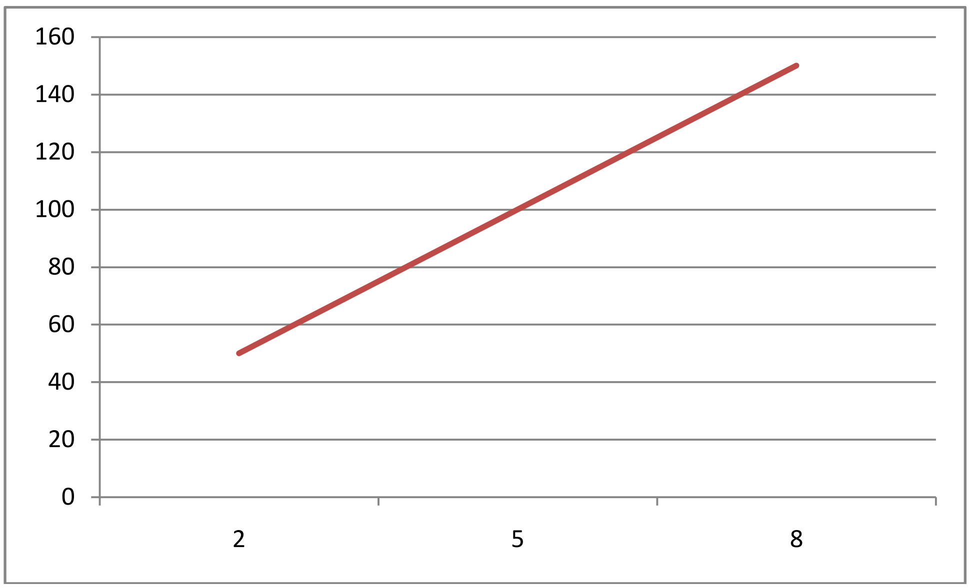


The line connecting the optimal bundles defined by increasing income is the Income – Consumption curve.

At a given price of 10 for good one, we can draw these quantities on a price quantity graph to illustrate the idea of demand shifts.



The graph traced out by placing income on the y axis and quantity on the x axis is called an Engel curve. It traces out the relationship between the quantity demanded of a single good and income.



Remember the idea of income elastiticites from before. Here is where they come from. A normal good has a positive income elasticity, an inferior good has a negative income elasticity.

What is the formula for an income elasticity?
(% change in q divided by % change in income)

A good can be normal for one person and inferior for another.

A good can be normal for one person at one point in their life and inferior at another.

A good can be normal at one level of income and inferior at another.

We can block out space from an optimal bundle to areas where both goods are normal, one good is normal and the other inferior.

Can both goods be inferior?

Show on graph.

What is the impact of a price change?

First, we identify the change in the quantity demanded as a result of a price change as the total effect.

We decompose this total effect into two distinct components.

1) The substitution effect. If utility and the price of the other good are held constant, as the price of one good rises, consumers substitute between the two goods to reflect the new price ratio.

Since MRS=MRT, price change leads to a change in MRT, the consumer will adjust the consumption bundle along the indifference curve to re-equate MRS and MRT.

2) The income effect. A change in price changes the consumer’s buying power. Recall the example of how a halving of prices is equivalent to a doubling of income. This is the core idea here. If the price of good one goes down all else held constant, that is going to increase the consumer’s buying power which is like increasing their income.

Total effect = substitution effect + income effect.

Since indifference curves are downward sloping, can we predict the sign of the substitution effect? The change in price and the substitution effect should have opposite signs. An increase in price leads to less of the good being consumed. A decrease in price leads to more of a good being consumed.

Show graph.

However, the income effect depends on whether the good is a normal or an inferior good.

A Giffen good is one for which a decrease in price causes quantity demanded to fall (the income effect outweighs the substitution effect, making the total effect negative). Not observed in reality, but an interesting example.

Show graphs for substitution effect for normal good, inferior good, and Giffen good.

For edification / inoculation:

This graphically represents what in economics is known as the Slutsky equation.

It describes the relationship between two different approaches to deriving the demand function: One, a money income held constant derivation (the Marshallian), and a second utility held constant derivation (the Hicksian).

Show three graphs for a price decrease: One, S+, I+,T+; Two, S+, I-, T+; Three S+, I-, T-.