

McPeak  
Lecture 7  
PPA 723  
Firms and Production

Firms:

Three main kinds.

- 1) Sole proprietorships
- 2) Partnerships
- 3) Corporations (Limited Liability)

Objective of the firm: make decisions so as to maximize profit.

Profit is defined as revenue, what it earns from selling the good, minus costs, what it costs to produce the good.

$$\pi = R - C$$

or, in slightly different form

$$\pi = p \cdot f(x) - w \cdot x$$

Necessary vs. sufficient conditions.

A necessary condition is in the nature of a prerequisite.

Statement A is true only if another statement B is true, then “A only if B” or “If A, then B”.

B is a necessary condition for establishing the truth of A.

If a person is a father (A), then they are a male (B).

Being male (B) is a necessary condition for being a father (A).

If Felix is a cat (A), then Felix hates baths (B).

Felix is a cat only if he hates baths.

If A (cat), then B (hates baths).

Can we turn it around: If B (hates baths), then A (cat)?

If Felix hates baths, can we assume Felix is a cat?

No, Felix might be a four year old boy for example.

Felix hating baths is a necessary condition for Felix to be a cat, but it is not sufficient. It is one characteristic of being a cat, but this characteristic is shared by non-cats as well.

Consider the situation where A is true if B is true, but A can be true when B is not true. B is a sufficient condition for A, but B is not a necessary condition for A.

A if B.

If B, then A.

A is “one can get to Chicago from Syracuse”,  
B is “There is a plane that flies to Chicago from Syracuse”,  
then the truth of B suffices for the establishment of the truth of A, but is not a necessary condition for A to be true. B is a sufficient condition for A but not a necessary one.

Consider where A and B imply each other.

A is “it is the month of February”. B is “there are less than 30 days in the month”.

A is a necessary and sufficient condition for B, and vice versa.

A if and only if B.

There is no way for A to be true without B being true. There is no way for B to be true without A also being true.

If A is false, there is no way B is true. If B is false there is no way A is true.

It is a definition.

Back to economics.

A: The point  $(x_1, x_2)$  is the optimal bundle.

B: The point  $(x_1, x_2)$  is on the budget line

Necessary: If A, then B.

Sufficient: If B, then A.

B is a necessary but not sufficient condition for A.

Yes it has to be on the budget line, but other conditions need to be met as well. A lot of non-optimal bundle points are on the budget line as well. We need to get a condition that takes care of these.

The reverse statement is a sufficient but not necessary condition: The point  $(x_1, x_2)$  is the optimal bundle, therefore it lies on the budget line (but there are other ways of being on the budget line besides being at the optimal bundle)

The point  $(x_1, x_2)$  is the point where  $MRS=MRT$  implies the point  $(x_1, x_2)$  the optimal bundle for an interior solution.

Optimal bundle for an interior solution iff  $MRS=MRT$   
 $MRS=MRT$  implies optimal bundle, and optimal bundle implies  $MRS=MRT$ .

Technologically efficient production is a necessary but not sufficient condition for profit maximization.

Profit maximization is a sufficient but not necessary condition to know we are using technologically efficient production.

Profit max implies we are technologically efficient, but being technologically efficient is not enough to know we are producing at a profit maximizing level.

Technologically efficient production: the firm can not produce more output given the amount of inputs it is using, and the firm cannot produce the amount of output it is producing by using fewer inputs.

Show sets.

Production function.

A firm gathers together inputs, or factors of production.

The firm then applies a technology, or production process to these inputs.

The result is an output – can be a good or a service.

No costs of input are involved yet.

No selling of the product is going on yet.

Just (boring old) production!!!!

Define a production function  $Q=f(K,L,E,M)$

Q is output of the good (in many studies, people use Y rather than Q – means the same thing)

K is capital

L is labor

E is energy

M is materials

f(.) defines the relationship between the quantities of inputs used and the maximum quantity of output that can be produced given current knowledge about technology and organization.

What is the nature of the f(.)? It can take on lots of different forms.

An important part of econometric work is to estimate the nature of the production function.

We can treat inputs in the production function as fixed or variable inputs.

In the short run, factors of production that can not easily be varied are viewed as fixed inputs.

A factor for which it is relatively easy to adjust the quantity quickly is a variable input.

The long run is the time span required to adjust all inputs. There is no precise definition of time period applied to these terms. It is a relative relationship.

All inputs are variable in the long run (there are no fixed inputs in the long run).

An example:

A commonly made assumption is that labor is the most variable of inputs, so we define it to be the variable input, hold the others constant in the analysis. (No law here, but convention)

$$Q = f(\bar{K}, \bar{L}, \bar{E}, \bar{M})$$

K “bar”, E “bar”, M “bar” mean fixed in short run. Holding fixed inputs constant, in the book Perloff defines a labor – output table.

[fill in units and total first, then revisit marginal and average]

Labor units	Total Output	Marginal product	Average product
0	0	NA	NA
1	5	5	5
2	18	13	9
3	36	18	12
4	56	20	14
5	75	19	15
6	90	15	15
7	98	8	14
8	104	6	13
9	108	4	12
...			

Marginal product of Labor: the change in total output resulting from the use of an additional unit of labor, all else constant.

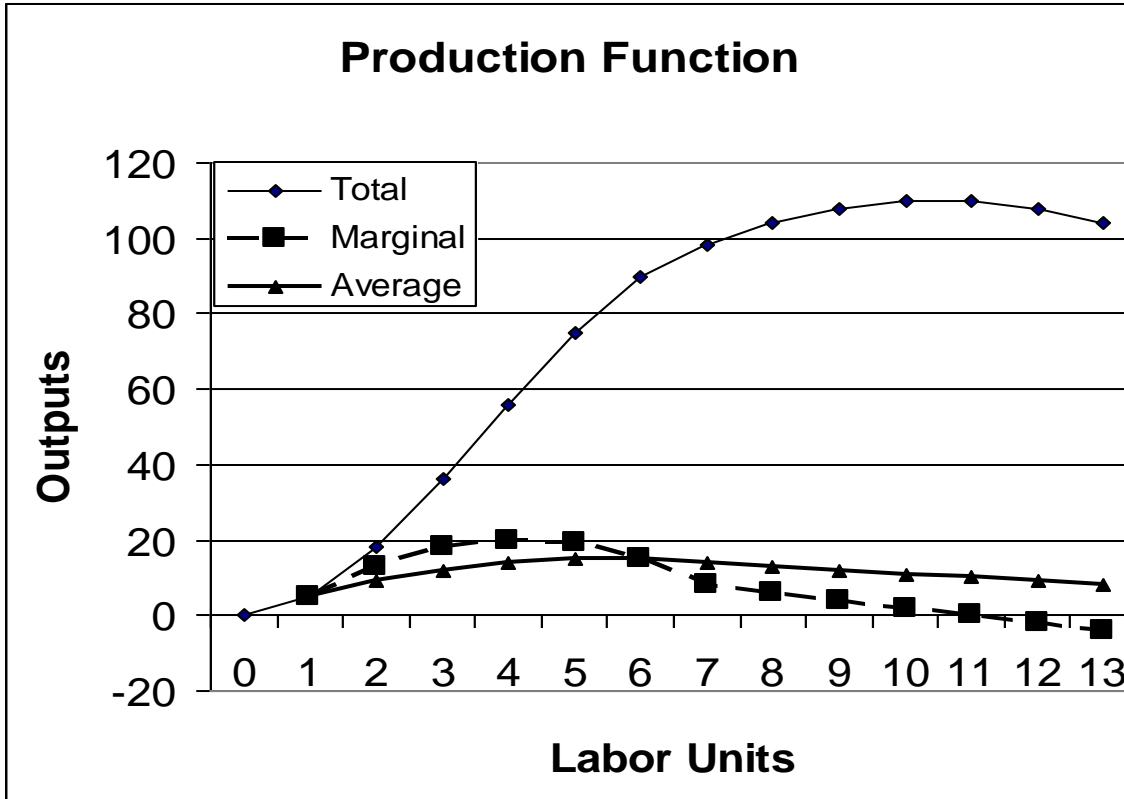
$$MP_L = \frac{\Delta Q}{\Delta L}$$

Note that this contrasts with the average product of labor: the ratio of the output to the number of workers used to produce this labor.

$$AP_L = \frac{Q}{L}$$

Note further that there is also a marginal product of capital, of materials, of energy... We are focusing on labor, but other inputs also have marginal and average measures as we have just defined for labor.

[fill these in on chart]



We can draw a graph of this information.

Why these shapes?

At low levels of labor, workers help each other do tasks that are hard for one person to do, or conversely, they specialize. This gives us initially a convex function.

Then, after we reach some critical level of labor,

They have to wait for each other to finish at a machine, or they get in each other's way... then we get a concave function.

If MP curve is above AP curve, then AP is upward sloping. If MP below AP, then AP downward sloping.

Think of heights for the intuition.

Geometrically, if you draw a ray from the origin to any point on the Total Product curve, you find the average product at that point. If you identify the slope of the total product curve at this point, you find the marginal product.

If AP steeper than MP, then  $AP > MP$ . If AP flatter than MP, then  $AP < MP$ . At some point,  $AP=MP$ .

The law of diminishing marginal returns. If a firm keeps increasing an input, holding all other inputs and technology constant, the corresponding increases in output will become smaller eventually.

Not diminishing returns, but diminishing marginal returns.

Long run production.

That was a discussion of variation in output due to different levels of labor holding other things (K, E, M) constant.

Now, we are in the long run so all inputs are variable.

Note that the short run implies that at least one input is being held fixed. Do not come away from this with the impression that the difference between the short run and the long run is one versus two inputs. That is not right. In the short run, at least one input and potentially more than one input is held constant while one or more other inputs are allowed to vary. In the long run all inputs are allowed to vary.

However, to keep things simple, we are going to assume there are only two inputs used in production of our good.

We will call them capital and labor. More than two are possible (likely) in reality.

We focus on two because it is easier to draw and the logic carries through to higher dimensions.

We can combine different quantities of these inputs in a variety of ways to produce a given level of output. Define a curve that traces out the minimum combinations of inputs required to produce a given level of output.

This is an isoquant. Again, if you want to think of this as a contour line, it is a contour line on the production function in 3-D space.

Properties:

- 1) The farther an isoquant is from the origin, the greater is the level of output. (remember more is better than less)
- 2) Isoquants do not cross, as that would imply inefficiency. (remember transitivity)
- 3) Isoquants slope downward, as they are efficient levels of production (remember there are tradeoffs).

[Draw an isoquant]

What will influence the shape of the isoquant?

How substitutable are inputs?

Production function of processed pork.

Processed Pork = pigs bought in New York + pigs bought in Pennsylvania

Straight line graph

Production function of peanut butter sandwiches.

Peanut butter sandwiches = minimum (dollops of peanut butter, (slices of bread / 2))

I have 10 dollops of peanut butter, and 4 slices of bread, I can only make 2 sandwiches.

Leontief graph

Most lie intermediary to these two extreme cases.

Show contrast on single graph, note nature of subs, connect at upper and lower extreme.

The slope of the isoquant is called the marginal rate of technical substitution.

This tells us the trade off between inputs in production. It is measured as the number of units of one input that have to be given up while increasing the other input to continue to produce a given level of output.

The MRTS, like the MRS, is a negative number since it is implicitly a trade-off. Here, we define it for the capital to labor MRTS.

$$\text{MRTS}_{\text{KL}} = \frac{\text{change capital}}{\text{change labor}} = \frac{\Delta K}{\Delta L}$$

Unless goods are perfect substitutes (MRTS=-1 for example) or perfect complements (MRTS=-∞/ undefined, or 0), the MRTS varies as different points are considered on the isoquant.

Remember we are on an isoquant. The quantity of output stays the same. Therefore, we know that if we change labor and change capital on a given isoquant, the total output should not change.

Recall that  $MP_K = \frac{\Delta TP}{\Delta K}$ , and a similar expression exists for the marginal product of labor.

So if we know the change in total product is zero by definition, and we know the definition of the marginal product is what we just saw, we can add zero plus zero to find the following.

$$\Delta K \cdot MP_K + \Delta L \cdot MP_L = \Delta TP = 0$$

Show math, note connection to calculus, and answer zero minus zero question if it comes up. Also note connection back to MRS and the Marginal Utility equations developed earlier.

While not overwhelming exciting, this allows us to gain the insight that the marginal rate of technical substitution is equal to the negative of the ratio of the marginal products (important: note the numerator – denominator relationship).

$$MRTS_{KL} = \frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K}$$

From an intuitive point of view, the movement along an isoquant is related to marginal changes.

I am getting this level of output using a specific mix of inputs, now I want to move over there to another mix of inputs holding output constant.

That is a marginal change.

Returns to scale.

Up until now, we have been considering adjustments to our input bundles, holding output constant. That is how we have defined an isoquant. Or we have been changing one input at a time, holding others constant. That is how we thought about a production function.

Now, however, we want to turn to the question of how changes in the total input bundle are related to changes in output.

What can we learn by comparing different isoquants rather than looking at movement along a given isoquant?

We are going to look at a specific type of change to the input bundle – blowups. Equal percentage change applied to all inputs.

I use labor and capital to produce my good. Let's say we can continue to ignore other inputs like materials and energy for production of our good.

What are the implications of different production functions for changing input levels?

Say I use 2 units of labor and 3 units of capital to produce 6 problem sets. In this case, assume the production function is defined by capital times labor. If I double both units (4 units of labor and 6 units of capital) I get 24 units of problem sets.

$$2*3=6$$

$$\text{new output} = (2*2)*(3*2) = 4*(2*3) = 4*6 = 24$$

Doubling inputs gives a four-fold increase in output.

$$(24/6)=4$$

Increasing Returns to scale. Doubling inputs leads to a more than double increase in outputs.

$$f(2*K, 2*L) > 2*f(K, L)$$

Say instead that the production function is capital plus labor (perfect substitutes in production).

$$2+3=5$$

$$\text{new output}=(2*2)+(2*3)=2*(2+3)=10$$

Doubling inputs gives a doubling of output.

$$(10/5)=2$$

Constant returns to scale. Case two (additive production function). Doubling inputs leads to a doubling of output.

$$f(2*K,2*L) = 2*f(K,L)$$

Finally, assume we have a production function that defines output as the natural log of capital times labor.

$$\ln(2*3)=1.79$$

$$\text{New output} = \ln((2*2)*(2*3))=\ln(4*6)=3.18$$

Doubling inputs increases output by 78%

$$(3.18/1.79)=1.78$$

Decreasing returns to scale. Doubling inputs leads to a less than double increase in output.

$$f(2*K,2*L) < 2*f(K,L)$$

Technical note:

Cobb-Douglas production function is useful, as it embeds these three cases.

$$q = \gamma \cdot L^{\alpha} \cdot K^{\beta}$$

If  $\alpha + \beta > 1$ , IRS.

If  $\alpha + \beta = 1$ , CRS.

If  $\alpha + \beta < 1$ , DRS.

While this defines the relationship between scale and the production function as either one or the other, it is important to note that there can be variation over the scale of production in the returns to scale.

In other words, returns to scale can depend on where you are in the production function.

A common pattern is IRS over low levels of input, CRS over moderate levels, and DRS with high levels of input.

If we think of isoquants as contour lines, they are close together near the origin, and spread further apart as we move away from the origin.

Show graph from book that illustrates IRS over low ranges, CRS mid range, and DRS higher range.

## Innovations.

Technological progress is one of the main driving factors of economic growth.

Different types:

Neutral technological progress. All inputs are equally affected. Allows same input bundle to be used, but generates more output.

Nonneutral technological progress. The innovation affects inputs unequally. This alters the proportions of the input bundles when generating more output.

Show progress on a single input production function,  
show on an isoquant neutral technological progress.