Hedonic Markets and Explicit Demands:
Bid-Function Envelopes for Public Services and Neighborhood Amenities

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Abstract

Hedonic regressions with house value as the dependent variable are widely used to study public services and neighborhood amenities. This paper builds on the theory of household bidding and sorting to derive a bid-function envelope, which provides a form for these regressions. This approach allows for household heterogeneity, yields estimates of the price elasticities of demand directly from the hedonic without needing a Rosen two-step procedure, and provides tests of key hypotheses about sorting. An application to data from the Cleveland area in 2000 yields price elasticities for school quality and neighborhood ethnic composition and supports the sorting hypotheses.

Introduction

House-value regressions, also called hedonic regressions, are a central empirical tool of urban economics and local public finance. This tool has been used to study many topics, including the demand for public services and environmental quality, the impact of property taxes on housing markets, the trade-off between housing and commuting costs, and racial prejudice and discrimination. With a few important exceptions, however, the empirical specifications have not been based on theory. This paper connects the theoretical and empirical literatures by introducing the logic of bid functions into the specification of a house-value regression. This approach makes it possible to test some basic tenets of bid-function theory, facilitates consideration of household heterogeneity, leads to direct estimation of service and amenity demand elasticities, and exposes inconsistencies in some previous studies.

The foundation of this paper is the theory of household bids for housing in locations with different traits. The approach developed here brings together bids for housing based on public services, neighborhood amenities, commuting costs, and local taxes. Two key steps define this approach. First, I introduce constant-elasticity demand functions for public services, neighborhood amenities, and housing. Second, I introduce household heterogeneity into the public service, amenity, and housing demand functions. Third, based on the main theorem from the literature on household sorting (namely, that households sort according to the slopes of their bid functions), I derive the envelope of the household bid functions across household types and show how this envelope can be incorporated into a house-value regression. This derivation emphasizes the distinction in Rosen (1974) between the amount a given type of household is willing to pay for an increment in an amenity and the movement along the bid-function envelope, which involves a change in household type. I show how to separate these two effects and to test the sorting theorem. This approach also can accommodate cases in which an "amenity" has positive value for some households and negative value for others, a feature that proves to be important empirically. In addition, I show that many studies of service/amenity demand make inconsistent assumptions about the forms of the envelope and of the underlying bid functions.

The second part of the paper estimates this new approach to a house value regression using all the house sales in the Cleveland area in 2000. The methods developed here, combined with extensive controls for neighborhood traits, yield precise estimates of the price elasticity of demand and of the sorting parameters, plus support for the sorting theorem, for a school district's high school performance and for the percentage of a neighborhood's population that is black. Bidding and sorting are more difficult to separate (with my approach or any other) for several other school performance measures and for a neighborhood's percent Hispanic.

Preview of the Literature

This section introduces the literature; comparisons with my approach appear in a later section. The core studies on which I draw model how households bid for housing across communities with various levels of public services and property taxes and then sort into communities (Ellickson 1971; Epple, Filimon, and Romer 1984; Epple and Platt 1998; Henderson 1977; and Wheaton 1993). This literature, reviewed in Ross and Yinger (1999), predicts that property taxes and public service levels are capitalized into the price of housing. A large empirical literature on capitalization, much of it inspired by Oates (1969), has also appeared. Recent contributions on public service capitalization include Bayer et al. (2007), Black (1999), Brasington (2002, 2007); Brasington and Haurin (2006); Clapp, Nanda, and Ross (2008); and Kane, Riegg, and Staiger (2006).[1] Studies on tax capitalization are reviewed in Yinger et al. (1988) and Ross and Yinger (1999).

A related literature concerns hedonic regressions, which estimate the impact of product attributes on product prices. In the case of housing, the attributes include features of the house's location, such as school quality. This literature, reviewed in Taylor (2008), goes beyond estimating the impact of public services on house values to explore the underlying demand for public services. Most studies follow the two-step procedure in Rosen (1974): (1) estimate a hedonic regression and. (2) find implicit prices of product attributes by differentiating the hedonic and estimate the demand for each attribute as a function of its implicit price.

Brown and Rosen (1982) and Epple (1987) explain that with a nonlinear hedonic equation the implicit price depends on the quantity selected by the household and therefore is endogenous. Epple (1987), Ekeland, Heckman, and Nesheim (2004), and Bajari and Benkard (2005) provide alternative solution to this problem. Ekeland et al. also identify a "problem with the current literature," namely, that the "full economic content of the hedonic model is not being exploited" (p. S74). This paper exploits the economic content in bidding and sorting models to derive a hedonic price function with which the service/amenity price elasticities can be directly estimated, without a second step. Sorting is brought into a hedonic regression by Epple and Sieg (1999) and Epple, Romer, and Seig (2001). These studies estimate a general equilibrium model in which households sort into communities and then select the level of local public services

This literature extends beyond public services. The standard Alonso/Mills/Muth model of urban residential structure (reviewed in Brueckner 1987), predicts that commuting costs will be reflected in housing prices. Polinsky and Shavell (1976) introduce an exogenous amenity such as air pollution into an urban model, and many studies, such as Chay and Greenstone (2005), estimate the impact of pollution on property values. Yinger (1976) extends an urban model to include an endogenous amenity, neighborhood ethnic composition. Empirical studies on this topic include Bayer et al. (2007) and Zabel (2008).

The Theory of Bidding and Sorting

This section builds a bidding framework based on the standard model with constant elasticity demand functions, develops a new method to account for household heterogeneity and sorting, and incorporates the results into a house-value regression. The standard model assumes that households maximize utility over public service quality, *S*, housing services, *H*, and a composite good, *Z*. Households make bids on housing based on *S* and the effective property tax rate, τ, and households with different incomes and preferences sort into different jurisdictions. Households are assumed to be mobile, so a key equilibrium condition is that all households in an income-taste class achieve the same utility. Households live in an urban area with many local governments financed by a property tax. Everyone who lives in a given jurisdiction receives the same level of *S*, and the only way to gain access to the *S* in a jurisdiction is to live there. All households are owners, but, depending on assumptions about property tax incidence, these models could be applied to renters, as well. The household budget constraint sets household income, *Y*, equal to *Z* plus housing consumption, *PH*, where *P* is the price per unit of *H*, plus property taxes. A household's property tax payment is τ multiplied by its house value, $V = PH/r$, where *r* is a discount rate (and $\tau^* = \tau/r$).

$$Y = Z + PH + \tau V = Z + PH\left(1 + \frac{\tau}{r}\right) = Z + PH(1+\tau^*)\ . \tag{1}$$

A straightforward way to derive housing bids is to determine the maximum amount a household would pay per unit of *H* in different locations, holding utility constant (Wheaton 1993). Solving (1) for *P*, this approach leads to the following maximization problem:

$$\text{Maximize} \quad P = \frac{Y - Z}{H(1+\tau^*)}, \quad \text{subject to } U\{Z, H, S\} = U^0\ , \tag{2}$$

where U^0 is the utility level obtained by households in this income-taste class, *Z* and *H* are choice variables, and *S* and τ are parameters. Applying the envelope theorem yields

$$P_S = \frac{U_S / U_Z}{H(1+\tau^*)} = \frac{MB_S}{H(1+\tau^*)} \tag{3}$$

$$P_\tau = -\frac{P}{(r+\tau)} = -\frac{rP}{(1+\tau^*)} , \tag{4}$$

where subscripts indicate partial derivatives. The numerator of (3) is the marginal rate of substitution between *S* and *Z*, also called the marginal benefit from *S* or MB_S.

The differential equation (4) can be solved using the initial condition that the before-tax price, $\hat{P}$, which depends on *S*, equals the after-tax price, *P*, when τ equals zero. The solution

$$P\{S,\tau\} = \frac{\hat{P}\{S\}}{(1+\tau^*)} . \tag{5}$$

Differentiating (5) with respect to *S* yields another helpful result:

$$P_S = \frac{\hat{P}_S}{(1+\tau^*)} . \tag{6}$$

Now let us assume that the demands for *S* and *H* take a constant-elasticity form. First,

$$S = K_S N^\delta Y^\theta W^\mu , \tag{7}$$

where *W* equals tax price, K_S is a constant, and *N* is a vector of preferences and other variables that influence the demand for *S*. Second,

$$H = K_H M^\rho Y^\gamma \left(P(1+\tau^*)\right)^\nu = K_H M^\rho Y^\gamma \hat{P}^\nu , \tag{8}$$

where K_H is a constant and *M* is a vector of variables that influence housing demand. LaFrance (1986) shows that these forms can be derived from a model of "incomplete" demand, in which one set of commodities (*Z*) is not observed but affects the observed commodities (*S* and *H*) through a price index, which appears in *N* and *M*. More specifically, LaFrance shows that these forms are consistent with the integrability requirements of a demand system if observed commodities all have the same income elasticity of demand[2] and cross-price elasticities equal zero. In the case of (7) and (8), these conditions require that $\theta = \gamma$, that *W* not be an element of *M*, and that $\hat{P}$ not be an element of *N*. Under these conditions, "there is sufficient information embodied in the structure of the quasi-expenditure, indirect utility, and direct utility functions to permit exact welfare measures for changes in income and prices of the commodities of interest" (p. 551). Equations (7) and (8) also have proven useful in empirical applications. Several studies, including Duncombe and Yinger (2011), use (7) to estimate community-level service demand equations, and (8) has been widely used to study housing demand (Zabel 2004).

The inverse demand function associated with (7) is:

$$W = \left(\frac{S}{K_S N^{\delta} Y^{\theta}} \right)^{1/\mu} \equiv MB_S \ . \tag{9}$$

Combining (3), (6), (8), and (9) yields

$$\hat{P}_S \hat{P}^{\nu} = \frac{S^{1/\mu}}{\left(K_S N^{\delta}\right)^{1/\mu} K_H M^{\rho} Y^{(\theta/\mu)+\gamma}} = \psi S^{1/\mu} \ , \tag{10}$$

where

$$\psi = \left(\left(K_S N^{\delta}\right)^{1/\mu} K_H M^{\rho} Y^{(\theta/\mu)+\gamma} \right)^{-1} . \tag{11}$$

With C as a constant of integration, Equation (10) can be solved for the bid function:

$$\hat{P}\{S\} = \left((1+\nu)\left(C + \left(\frac{\psi\mu}{1+\mu} \right) S^{(1+\mu)/\mu} \right) \right)^{1/(1+\nu)} . \tag{12}$$

According to the standard model, different household types sort into different jurisdictions because their bid functions have different slopes with respect to *S*. As shown by Ellickson (1971), Henderson (1977), Yinger (1982, 1995), Epple et al. (1984), and Wheaton (1993), the household type with the steepest bid function wins the competition for housing in the jurisdiction with the highest *S*. The same result arises in models in which household types are determined by a distribution of income (Epple et al. 1984) or of income and preferences (Epple and Platt, 1998). These models all rely on the single-crossing assumption, which says that if a household type has a steeper bid function than another at one value of *S*, it also has a steeper bid function at other values of *S*. My approach to sorting builds on the single-crossing assumption and the theorem that people sort according to the relative slopes of their bid functions.

The slope of household's bid function is the derivative of $\hat{P}$ with respect to *S*, which is given by (10). A household's relative slope is determined by all the variables in (10) that are unique to that household, that is, everything except for *S* and $\hat{P}$. These terms are collected in ψ as defined by (11). In other words, ψ can be interpreted as in index of the steepness of a household's bid function—and hence an index of the value of *S* into which the household sorts.

To translate this observation into an estimating equation, we can derive the envelope of bid functions with various values of ψ. This envelope defines the set of winning bids. Consider two household types whose bid functions cross at a given value of S, say S^*. These households have different values of ψ and hence different bid-function slopes, but, by the definition of "cross," they also have the same bid, $\hat{P}$, at S^*. As shown in Figure 1, the bid function with the steeper slope must have a smaller intercept, C. The derivation of an envelope therefore involves finding a formula for C such that $d\hat{P}/d\psi = 0$ when S is held constant at $S\{\psi\}$, that is, at the value of S where the bid functions cross. Applying this condition to (12) yields

$$\left.\frac{dC}{d\psi}\right|_{S=S\{\psi\}} = \frac{-\mu}{1+\mu} S\{\psi\}^{\frac{1+\mu}{\mu}} \quad . \tag{13}$$

To solve this differential equation, that is, to find the envelope, we need to specify $S\{\psi\}$.[3] This function depends on the distribution of factors that influence ψ, and the distribution of S by jurisdiction size. In other words, it depends on the demand and supply of S. No existing model of the housing and public service markets accounts for household and supplier heterogeneity in a general way or yields a tractable form for $S\{\psi\}$. Instead of deriving $S\{\psi\}$ theoretically, therefore, I approximate it with a general form, the parameters of which can be estimated:

$$S\{\psi\} = (\sigma_1 + \sigma_2\psi)^{\sigma_3} \quad . \tag{14}$$

Substituting (14) into (13) and integrating yields:

$$C = C_0 - \left(\frac{\mu}{1+\mu}\right)\left(\frac{\mu}{\sigma_3(1+\mu)+\mu}\right)\left(\frac{(\sigma_1+\sigma_2\psi)^{\frac{\sigma_3(1+\mu)+\mu}{\mu}}}{\sigma_2}\right), \tag{15}$$

where C_0 is a constant of integration. Solving (14) for ψ and substituting the result and (15) into (12) leads to the bid envelope, identified with superscript E:

$$\hat{P}^E\{S\} = \left((\nu+1)\left(C_0 - \left(\frac{\sigma_1}{\sigma_2}\right)\left(\frac{\mu}{1+\mu}\right)S^{\left(\frac{1+\mu}{\mu}\right)} + \left(\frac{1}{\sigma_2}\right)\left(\frac{1}{\frac{1+\mu}{\mu}+\frac{1}{\sigma_3}}\right)S^{\left(\frac{1+\mu}{\mu}+\frac{1}{\sigma_3}\right)}\right)\right)^{1/(\nu+1)} \quad . \tag{16}$$

Equation (16) makes it clear that the impact of S on bids depends not only on a household's willingness to pay for S, as indicated by μ, but also on sorting, as measured by the σ parameters. Thus, any estimated impact of S on housing prices describes movement along the bid-function envelope, not the willingness to pay of a particular type of household.

Further explanation is provided by Figure 2, which builds on figures in Rosen (1974) and Follain and Jimenez (1985). The bid-function envelope, illustrative bid functions, and illustrative offer functions appear in the top panel. The slopes of these functions are plotted in the bottom panel. Even though people with steeper bid functions sort into higher-S locations, the slope of the envelope declines as S increases because all the underlying individual bids are affected by movement down the individual demand curves for S. This decline in the slope of the envelope is moderated by the increase in the slope of the bid functions as groups with steeper slopes win the competition at higher levels of S. An illustrative $S\{\psi\}$ function, which captures the second effect, appears in the bottom panel. For ease of interpretation, this function is anchored by bid function i; it indicates how much the slope must increase to go from the slope of bid function i (which includes just the first effect) to the slope of the envelope (which includes both effects). The positive slope of the $S\{\psi\}$ function corresponds to the theorem that household types with steeper bid functions win the competition for housing at higher levels of S. The estimated slope of this function is determined by the sign of σ_2 in (16); so long as σ_3 is positive, which is assumed in all my estimations, a positive σ_2 supports the sorting theorem.

With a suitable adjustment to the constant, (16) can be re-written using Box-Cox forms:

$$\left(\hat{P}^E\right)^{(\lambda_1)} = C_0' - \frac{\sigma_1}{\sigma_2} S^{(\lambda_2)} + \frac{1}{\sigma_2} S^{(\lambda_3)} \tag{17}$$

where $\lambda_1 = 1 + \nu$; $\lambda_2 = (1+\mu)/\mu$; $\lambda_3 = \lambda_2 + 1/\sigma_3$; $X^{(\lambda)} = (X^\lambda - 1)/\lambda$ if $\lambda \neq 0$; and $X^{(\lambda)} = \ln\{X\}$ if $\lambda = 0$. Most functional forms that have been used for hedonic estimation are special cases of (17). When $\sigma_3 = \infty$, for example, the two terms on the right side have the same exponent and (17) reduces to a standard Box-Cox form. This is an extreme case; according to (14), $\sigma_3 = \infty$ implies that sorting does not take place; that is, S is not a function of ψ. Another common form regresses the log of housing price on S. This form arises when $\nu = -1$ and $\sigma_3 = -\mu = \infty$, which indicates no sorting and a horizontal demand curve for S. Assuming $\nu = \mu = -1$ and $\sigma_3 = \infty$ leads to a double log form. Forms consistent with sorting always have two terms on the right side. A quadratic form arises, for example, with a horizontal demand curve ($\mu = -\infty$,) and a linear approximation to the sorting equilibrium ($\sigma_3 = 1$). With $\mu = -1$ and $\sigma_3 \neq \infty$, the first term is a log form but the second is not. With μ = -0.5 and σ_3 = 1, the first term is an inverse and the second is a log.

This approach can be generalized to many public services and neighborhood amenities (henceforth "amenities"). The LaFrance (1986) derivation of constant elasticity demands applies to any number of amenities and requires the cross-price elasticities of amenity demand to be zero. With this assumption the bid function for S_i does not depend on any S_j, $j \neq i$, or on its price.[4] Thus, the envelope for S_j can be incorporated into (16) through the constant of integration, C_0, and the multi-S envelope is (16) or (17) with a subscript on S, say i, and a summation over i in front of both right-side terms. One complication is that a household does not select S_i and S_j, but instead decides how much to bid at given values of S_i and S_j, and the amount a household bids to live in a community with a certain S_i might depend on how much S_j the community provides. However, the assumption that the cross-price elasticities equal zero is mathematically indistinguishable from the assumption that S_j does not appear in the demand function for S_i. As a result, the same generalization to a multi-S case can be used with this formulation. Moreover, this formulation makes it possible to introduce an indirect form of interaction across Ss through the community-level determinants of S. Households may demand more S_i in a community that has relatively low costs for producing S_j, and is therefore more likely to select a high value of S_j. The systematic determinants of S_j can be treated as components of N in (7) for S_i, and therefore cause no complications for the derivation or for the interpretation of estimated parameters.

An extension to many amenities must also assume that the table of values is full enough that each amenity can be selected independently. A high correlation between S_i and S_j, for example, might force households to select from a limited number of (S_i, S_j) pairs, so that the marginal conditions derived here do not hold. In this case, S_i and S_j can be combined into a single index, a procedure I follow below in averaging passing rates across various state tests.

A neighborhood's ethnic composition might influence house values for at least three reasons (Zabel 2008). First, ethnic prejudice may lead some households to prefer neighborhoods in which their own ethnic group is concentrated. According to surveys, some people in every ethnic group have this type of prejudice (Charles 2000), so prejudice alone is unlikely to generate a monotonic relationship between neighborhood ethnic composition and housing price. Second, historically disadvantaged minority groups tend to cluster in neighborhoods with poor amenities, even controlling for income (Deng, Ross, and Wachter 2003). This clustering reflects past discrimination and the attendant disparities in wealth and other factors that influence housing choices.[5] In addition, some people might bid more to live where their ethnic group is concentrated to gain access to ethnic restaurants or social organizations. Third, households in certain ethnic groups might face discrimination in obtaining housing or mortgages. Although discrimination has declined over time, recent audit studies find evidence of continuing discrimination against blacks and Hispanics (Ross and Turner 2005).

Equation (17) permits an "amenity" to be good for some households and bad for others. Specifically, (14) implies that a household's marginal willingness to pay for an "amenity," ψ, is negative if the sorting theorem holds and $(\sigma_1)^{\sigma_3} > S$. This type of household heterogeneity is therefore revealed by the estimation.[6] Nevertheless, a negatively sloped envelope is not entirely consistent with the theory that leads to (17); (7) indicates that a negative ψ implies a negative S, which is not how the model is specified. One way to resolve this inconsistency is to define S^* as the minimum point on the envelope and then to define the amenity as (S^* - S) for values of S below S^* and as ($S - S^*$) for values above S^*. With this approach, implemented below, S^* is a parameter to be estimated. In models without S^*, this inconsistency can be minimized by defining the amenity to be positive for most households. Because most households in the Cleveland area are white, non-Hispanic households who may be prejudiced against blacks and/or Hispanics, I specify the two key amenity variables as share non-black and share non-Hispanic.[7]

The Cleveland metropolitan area has long exhibited a high degree of racial segregation, and in 2000 about 60 percent of the majority-black CBGs were located in a black ghetto, defined as a set of contiguous neighborhoods at least 80 percent black. The regressions also identify neighborhoods in or near a black ghetto. These variables control for unmeasured neighborhood quality variables in the ghetto, discriminatory restrictions on the ability of blacks to move outside the ghetto, and expectations about potential racial transition near the ghetto.

A standard urban model specifies housing bids as a function of time or distance to a central worksite (Brueckner 1987), and households sort according to the slopes of their bid functions. One key extension of the standard model is to multiple worksites (White 1999). Thus, the regressions include average employment-weighted distance to major worksites, u.

Finally, following (1) and (5) we can combine bids with τ and H, which is a function of structural housing characteristics, X. If β is the degree of property tax capitalization, then

$$V = \frac{P^E H\{X\}}{r} = \frac{\hat{P}^E\{S_1, S_2, ..., S_n, u\} H\{X\}}{(r + \beta\tau)} , \qquad (18)$$

where $\hat{P}^E$ reflects (17) and the comparable terms for u. I specify $H\{X\}$ as multiplicative; Sieg et al. (2002) show that this form is consistent with the assumption used here that V is the product of P (a function of locational characteristics) and H (a function of the Xs).[8]

Equations (18) and (11) define a two-step procedure. Estimates of μ and the σs for each amenity can be obtained by estimating (18) (after inserting (17)). These estimates make it possible to solve (14) for ψ and to estimate (11). This two-step procedure differs from Rosen's (1974) because it obtains μ from the first step and because the coefficient of Y in the second step is $-(\theta/\mu + \gamma)$, not θ. "Normal" sorting is defined as an equilibrium in which higher-income households live in locations with higher S. Henderson (1977) shows that normal sorting requires $(\theta/\mu + \gamma) < 0$, so the coefficient of Y in (11) provides a direct test of normal sorting.[9] Many variables in (11) may be correlated with Y, so S may be positively correlated with Y even if this elasticity condition is not met. Tests for direct and indirect normal sorting are presented below.

Comparison with Previous Approaches

Rosen's (1974) hedonic price function is "a joint envelope of a family of value functions and another family of offer functions" (p. 44). Some scholars derive joint envelopes that are consistent with equilibrium when the value (or bid) and offer functions take certain forms (Epple 1987; Ekeland et al. 2004). As Ekeland et al. point out it, however, this "approach is computa-tionally complicated and does not transparently deliver identification of the deep structural parameters" (p. S71). A key problem, therefore, is how to specify the hedonic regression.

Most studies use Rosen's two-step method, which calls for estimating the first-step hedonic with a flexible functional form. The introduction of the Box-Cox hedonic by Goodman (1978) and Halvorsen and Pollakowksi (1981) was an important innovation for this approach. Equation (17) shows, however, that a standard Box-Cox, which has a common exponent and one term for each amenity, assumes that all amenities have the same price elasticity and rules out sorting.[10] This discussion reveals three potential problems in many previous studies. First, the estimated envelopes are based on implicit assumptions about amenity price elasticities or the nature of the sorting equilibrium; if these assumptions are incorrect, estimated results—i.e. the implicit prices—may be biased or inconsistent. Second, some forms, including linear, log, and standard Box-Cox forms on the right side, implicitly rule out sorting, which is central to the theory on which hedonic functions are based. Third, the implicit assumptions about amenity demands in the Rosen first stage may be logically inconsistent with the demand functions estimated in the second stage. A standard semi-log specification, for example, assumes that $\mu = -\infty$, so it is inconsistent to estimate μ in the second stage based on implicit prices derived from this specification. These conclusions are based, of course, on the constant-elasticity specifications of (7) and (8), but similar inconsistencies could arise with other specifications.

Some studies, including Bayer et al. (2007) estimate linear hedonic equations. This specification appears to contradict Bayer et al.'s household choice model, which asumes a linear utility function for household *i* in house *h* of the form $V_{ih} = \alpha_{0i} + \sum_j \alpha_{ji} X_{hj} - \alpha_{hi} P_h$, where *V* stands for utility, the *X*'s denote housing and neighborhood traits, and *P* is the price of housing.[11] A bid function for a given household is the price, say P^*, that holds utility constant (at V^*) over various *X*'s or $P^*_{hi} = \left(\alpha_{0i} - V^*_i\right) / \alpha_{hi} + \sum_j \left(\alpha_{ji} / \alpha_{hi}\right) X_{hj}$. Households sort according to the steepness of this bid function, so households with higher $\left(\alpha_{ji} / \alpha_{hi}\right)$ live in locations with higher values of X_j. In Rosen (1974) the hedonic equation is the envelope of these bid-functions.[12] Consider a single amenity, re-write the bid function as $P^*_i = a_i + b_i X$, and assume that the sorting equilibrium can be approximated by $X_i = \sigma_1 + \sigma_2 b_i$. Then the procedure presented earlier implies that the envelope is $P^E = C - (\sigma_1 / \sigma_2) X + X^2 / (2\sigma_2)$, where *C* is a constant. Thus, with my procedure the Bayer et al. assumptions imply a quadratic not a linear hedonic.[13]

Epple and Sieg (1999) and Epple et al. (2001) avoid inconsistency (and the standard endogeneity problem) by estimating general equilibrium models in which people care about one amenity index and sort according to income and a utility parameter. Their form for the indirect utility function is consistent with constant elasticity demand for *H* but not for *S*. These models are estimated by solving for the parameter values that produce the best correspondence between the distributions of outcomes generated by the model and the distributions of outcomes in a sample of communities. The strengths of these studies are that they (1) incorporate both household sorting and the determination of *S* through voting and (2) explicitly solve for an equilibrium and incorporate the solution into the estimating method. My approach also draws on the theory of bidding and sorting, but it cannot do either of these things. Instead, my approach overcomes some of the empirical limitations of these studies, namely, that they are based on aggregate (community level) data, are difficult to extend beyond a single amenity, consider only two dimensions of household heterogeneity, and require an unusual statistical technique.

Hedonic regressions may also be subject to omitted-variable bias. An approach to this problem that shifts the focus to school quality *within* a school district appears in Black (1999), Kane et al. (2006), and Bayer et al. (2007). This approach identifies segments of school attendance-zone boundaries and defines fixed effects for the houses within a certain distance of each segment to control for unobserved neighborhood traits. Observations far from these boundaries are dropped. Kane et al. (2006) and Bayer et al. (2007) point out that this approach needs to consider sorting. As Bayer et al. put it, "even if a school boundary was initially drawn such that the houses immediately on either side were identical, households with higher incomes and education levels might be expected to sort onto the side with the better school" (p. 590). This sorting leads to differences in the demographics of the neighborhoods on either side of a segment; if households care about their neighborhood's demographics, these differences may affect housing prices and hence be a source of bias. To address this problem, Bayer et al. include neighborhood demographics in their hedonic regression.

According to the analysis in this paper, the key problem is that sorting results in different bids on either side of the boundary, as indicated by different values of ψ in (12). Consider (12) with v = -1, one school variable (S), and one shared neighborhood variable (N):

$$\ln\{\hat{P}\} = C + \psi_S\left(\frac{\mu_S}{1+\mu_S}\right)S^{(1+\mu_s)/\mu_s} + \bar{\psi}_N\left(\frac{\mu_N}{1+\mu_N}\right)N^{(1+\mu_N)/\mu_N} + \left(\psi_N - \bar{\psi}_N\right)\left(\frac{\mu_N}{1+\mu_N}\right)N^{(1+\mu_N)/\mu_N} \quad . (19)$$

Thanks to sorting, the households on one side of the boundary differ from those on the other side and have different ψ_N. Neighborhood fixed effects remove the average price impact in each neighborhood (the first term containing $\bar{\psi}_N$) but ignore deviations around this impact associated with sorting (the last term). Because the average effect is shared by households on both sides of the boundary, it is not highly correlated with S and is therefore not an important source of bias. In contrast, the last term is likely to be correlated with S, and a source of bias, because it reflects household traits associated with sorting. In short, sorting leads to differences in neighborhood demographics that households may care about and to differences in bids for the same (unobserved) neighborhood traits on either side of a zone border. To the extent that income and education pick up variation in ψ, the Bayer et al. approach, including income and education in the regression, is analogous to (12) not to (16), which makes these variables endogenous.

I recognize that some neighborhood demographic traits may serve as amenities that directly affect housing bids. To minimize problems of interpretation, I distinguish ethnicity, which can be readily perceived by households, from other demographic traits, such as average income or education, that are determinants of amenity demand and difficult for households to perceive. Then I assume that demographic traits in the first category are amenities that affect housing bids whereas those in the second category serve only as demand variables. Although this assumption cannot be directly tested, indirect evidence comes from the estimated impact of variables in the second category on amenity demand. In addition, I minimize bias from omitted amenity variables by controlling for many non-demographic neighborhood amenities.[14]

Estimation Procedures

Although (18) could be estimated directly, I estimate it in two stages: The first stage is a regression of the log of sales price on housing characteristics, within neighborhood differences, and neighborhood fixed effects. The second stage uses the coefficients of the fixed effects as the dependent variable to estimate (17), controlling for other an extensive set of geographic variables.[15] This two-stage approach has two advantages. First, it prevents omitted variable bias in the coefficients of housing characteristics due to the correlation between these characteristics and unobserved neighborhood traits. This approach is analogous to the one used by Goldhaber and Brewer (1997), who use school fixed effects to eliminate bias in the estimated impact of student traits on student performance caused by the correlation between student traits and unobserved school traits. Second, as explained below, it facilitates consideration of many functional forms for the bid-function envelope.

This study builds on the data set described in Brasington (2007) and Brasington and Haurin (2006), which consists of all the house sales in Ohio in 2000, and, in particular, on the sub-sample for the Cleveland MSA. This data set indicates sales price, housing characteristics, housing location, census block group characteristics, school performance, crime rate, and air quality. These data were supplemented with various school and neighborhood characteristics.[16]

The first-stage hedonic faces two main challenges. The first is to define the neighborhood fixed effects. I use census block groups (CBGs) as neighborhood units, although I break up the few CBG's that fall into more than one school district. My results are based on CBG/school units (still called CBGs for conciseness) with at least two observed house sales.

The second challenge is to account for locational traits that vary within neighborhoods. Nine distance-based variables vary across houses within a CBG: distance to worksites, to the nearest public elementary school, to the nearest private school, to the nearest environmental hazard, to Lake Erie, to the nearest black ghetto, to the Cleveland airport, to the center of the CBG, and to a high-crime location. Houses close to an airport might be more valuable due to the convenient access or less valuable due to the noise. Houses located on the outskirts of an established neighborhood might be less valuable, but it is not clear whether CBGs accurately identify established neighborhoods. Because the hedonic equation includes CBG fixed effects, these variables are specified to measure only within-CBG variation. These variables are specified as a difference between the distance to the house and the distance to the center of the CBG. The first-stage hedonic also controls for differences between a house and the center of the CBG in the test scores of the nearest elementary school and in location within a historic district or near a public housing project. These differences equal zero for most, but not all, observations.

The commuting variable is average employment-weighted distance to the Cleveland area's five major worksites. A worksite was defined as a set of zip codes with at least 5,000 jobs that are within 6 miles of a central point, that contain at least one zip code with 25,000 jobs, and contain at least 45,000 total jobs. Because many jobs are clustered near a beltway around Cleveland, one worksite was defined as a ring 7 to 11 miles from the center of the downtown site. The five resulting centers are downtown Cleveland, the beltway, Mentor/Painesville, Bedford/Solon, and Lorain/Elyria. Except for the beltway, the worksite center was set near concentrations of business buildings (identified using satellite images on the U.S. Geological Survey web site) near the job-weighted centroid of the zip-code cluster. The radius of the beltway, 8.59 miles, was selected to minimize the squared employment-weighted distance to the associated zip codes. These five worksites accounted for 75.8 percent of the jobs in the Cleveland area in 2000, with job shares of 35.3, 40.7, 9.3, 7.7, and 7.0 percent, respectively. Each CBD was assigned to the closest worksite consistent with the requirement that the share of workers assigned to each site (based on CBG data) equals the share of employment located at that site (based on zip code data). The regression includes five variables measuring the difference between house and CBG weighted distance to worksites—one variable for each worksite.

Variable definitions and results for the basic hedonic are presented in Table 1. The hedonic is estimated with the "areg" option in Stata using 22,880 observations. The R-squared is 0.7893. The housing characteristics are, except for "One Story," significant with the expected sign. The set of geographic fixed effects is also highly significant. The commuting variables are all significant with similar coefficients and the expected negative sign. Additional small but significant impacts on housing prices come from the distance from a private school (negative), from an environmental hazard (positive), from Lake Erie (negative), from a black ghetto (negative), from the center of the CBG (negative), and from a high-crime location (positive). In addition, prices are higher near small family public housing projects but lower near large ones.

The second equation is the bid function envelope. The dependent variable is the set of coefficients for the CBG fixed effects and the number of observations, 1,665, is the number of CBGs with at least two house sales. In unlogged form with the first-stage constant term included, this variable has a mean of 84,836 and a standard deviation of 23,331 (Table 2).

As indicated in Table 2, the key amenity variables are the test scores of the neighborhood elementary school relative to the district average (Elementary), the district's passing rate on twelfth grade tests (High School), a measure of a district's elementary value added (Value Added), the share of minority teachers in the district (Minority Teachers), and two measures of neighborhood ethnic composition (Share Non-Black and Share Non-Hispanic). Test scores are measured by the passing rate on all five examinations (mathematics, reading, writing, science, and citizenship) used in the state's accountability system. Elementary, which refers to fourth grade scores, is the difference between the passing rate in the nearest elementary school and the average score in the district. High School is the share of students who enter the twelfth grade who pass all the state's twelfth grade tests. This variable equals the passing rate on the tests, which reflects only those students who do not drop out, multiplied by the graduation rate, which indicates the share of students who stay in school. Value Added is a school district's sixth grade passing rate in 2000-2001 minus its fourth grade passing rate in 1998-99. Thus, it is a test score change for a given cohort, but movement in or out of that cohort is not considered. Minority Teachers is the share of a district's teachers who belong to a minority group. The average value for Elementary is 0.3148 and the average value for High School is 0.320. The minimum value for this variable is astonishingly low; just under 5 percent of students starting twelfth grade in Cleveland pass all five state exams. In the best districts, over three-quarters of entering students reach this standard. Value Added, to which a constant is added so that it is always positive, ranges from 1 to 50 percent, and Minority Teachers ranges from 0 to 61 percent. In addition, the average CBG was 80 percent non-black and 96 percent non-Hispanic.

Cleveland and its neighbor, East Cleveland, are unlikely to exhibit the same relationship between school quality and housing prices as other school districts. Starting in 1996-97, children in the Cleveland district, but not in any other district in the area, became eligible for vouchers to help pay private tuition (GAO 2001). Vouchers were granted by lottery, with priority given to families with incomes below 200 percent of the poverty line. For these families, the voucher equaled the lesser of 90 percent of tuition and $2,250. First-time recipients had to be in a grade from kindergarten to third, and recipients were guaranteed vouchers through eighth grade. At the end of 1999-2000, 3,400 students were receiving vouchers, which they used in 52 private schools. In addition, legislation allowing charter schools (called community schools) was passed in Ohio in 1997. By 2001 Cleveland had 7 elementary charter schools, all of which were within two miles (and three were within 0.62 miles) of a CBG with elementary passing rates more than 10 points below the district average.[17] Only one charter school had opened outside Cleveland. The East Cleveland school district received a huge state grant for school construction in the 1998-2000 period (Ohio Schools Facility Commission 2000). The total grant was almost $96 million, compared to annual own-source revenue of about $36 million. East Cleveland was the only district in the area to receive this type of funding. In short, housing prices in these two districts may be higher than expected at a given measured school quality. Moreover, the link between vouchers and income, the placement of charter schools near low-performing schools, and the likely focus of construction on the most distressed schools may alter the relationship between housing prices and Elementary in these two districts. Thus, I include a fixed effect for the Cleveland and East Cleveland school and estimate a separate Elementary envelope there.[18]

Tax variables are defined in Table 3. A few school districts levy an income tax with a rate of 0.75 or 1.0 percent. Ohio requires assessment at market value, but some slippage exists and nominal property rates are translated into effective rates using assessment-sales data.[19] The average effective rate is 3.1 percent. See Table 2. Most school districts coincide with cities. The regressions also include city property tax rates (net of school taxes) and the reduction in the city tax rate due to exemptions. Indicator variables account for school districts without assessment-sales data, 13 percent of the sample, and for non-city districts, 14 percent.

As discussed earlier, the commuting variable is average job-weighted distance to five major worksites. This variable is entered in quadratic form to approximate the impact of sorting, which arises in urban models as well as in models of local public services. The regression also control for neighborhood traits other than school quality and ethnicity that influence property values. Household demands for these traits are undoubtedly heterogeneous, but many of these traits do not fit neatly into the framework developed in this paper. The impact of a neighborhood trait on property values may depend, for example, not only on the value of that trait within a CBG, but also on its value in nearby CBGs. In addition, property values may also depend on distance from certain neighborhood amenities, and this impact may disappear once a certain distance has been reached. These variables were entered with simple specifications that are consistent with sorting but do not yield explicit sorting parameters. See Tables 3 and 4.

One key neighborhood trait is crime. Households care about crime in their own neighborhood but may also be concerned about living near a high-crime location. Thus, I define variables for within-neighborhood crime and distance from crime "hot spots" (Table 3). Because property and violent crime are highly correlated (ρ = 0.8), I use four indicators of within-neighborhood crime; below-average property and violent crime (the omitted category), below-average property and above-average violent crime, above-average property and below-average violent crime, and above-average property and violent crime. Following Bowes and Ihlanfeldt (2001), crime rates are defined per unit area. Crime hot spots were identified by plotting locations above the 95th percentile of the sample on both violent and property crime. These locations were all census tracts in three nearby locations in Cleveland. I found the population-weighted centroid of each hot spot and defined rings around each centroid.

Households may also care about pollution (Table 3). I identified three clusters of CBGs where a relatively high volume of pollutants was released into the air and found the pollution-weighted centroid of each cluster. The final variables measure whether a CBG was within 20 miles of the nearest cluster and the distance from the cluster if it was less than 20 miles. One of the clusters is located less than 20 miles to the northwest of another, and the other two clusters are located on Lake Erie, with only water to the northwest; thus, the pollution distance variable recognizes that distance away from a cluster to the northwest is not expected to lower prices. Another pair of pollution variables indicates distance from a hazardous waste site.

The Cleveland area's "ghettos" were defined by plotting all CBGs with a population at least 80 percent black and then identifying large clusters of these CBGs. Two such clusters were found and Ghetto and Near Ghetto (see Table 7) were defined based on the population-weighted centroids of these clusters. The eastern suburbs of Cleveland receive unusual amounts of snow (Table 7). The "snowbelt" variables make it possible to determine whether housing prices are different in locations with particularly heavy snowfall. Snowfall maps from the years just preceding 2000 indicate that the heaviest snows are east of the town of Pepper Pike and about 10 miles from Lake Erie. The quadratic specification of these variables estimates the distance from the lake at which the snowfall effect in the eastern suburbs appears is maximized.

These steps make it possible to estimate the bid function envelope, (18) without $H\{X\}$. My strategy is to estimate several different models, corresponding to different assumptions about the parameters of (17), and to test whether the more general models are supported by the data. The first two models, linear and quadratic, can be estimated with OLS. As indicated by (17), the linear model corresponds to the assumption that $-\mu = \sigma_3 = \infty$ for each amenity, and the quadratic model corresponds to the assumptions that $\mu = -\infty$ and $\sigma_3 = 1$. The third model retains the assumption that $\sigma_3 = 1$, which indicates a linear approximation to the sorting equilibrium, and uses nonlinear least squares (NLLS) to estimate a value of μ for each amenity. The final model explores alternative assumptions for σ_3. To facilitate interpretation and keep the estimation manageable, this model selects, for each amenity, the value of σ_3 that minimizes the sum of squared errors from the set (1/4, 1/3, 1/2, 1, 2, 3, 4) and then uses NLLS to estimate the μs and other parameters based on these value for the σ_3s. These estimates all use $\ln\{P^E\}$ as the dependent variable; for every model I test the associated assumption that $v = -1$.

For the first three models, standard errors were estimated using the "hc3" option in Stata, which is recommended by Davidson and MacKinnon (2004) for models with heteroskedasticity. Because the fourth model selects a value for σ_3 before the NLLS regression is estimated, standard errors obtained through NLLS may be biased. Again following Davidson and MacKinnon (2004), the standard errors for this model are obtained from a jackknife procedure.[20]

Empirical Results

Table 5 presents alternative envelope specifications for Elementary, High School, Share Non-Black, and Share Non-Hispanic. (Results for Value Added and Minority Teachers raise additional issues and are discussed below.) The four columns correspond to the four models described above. With the linear model (column 1) Elementary, High School, and the two ethnicity variables are all significant with the expected signs, and the hypothesis that $v = -1$ cannot be rejected. In column 2 (quadratic), the squared terms for High School and Share Non-Black are significant, but the other terms are not. Nevertheless, the last row shows that first model can be rejected in favor of this one, which implies that hedonic models need to account for sorting. Again, we cannot reject $v = -1$. The NLLS estimates in column 3 include significant values for σ_1 and σ_2 for all four amenities; three of the estimated μs fall between zero and one and are significant, whereas the estimated μ for Share Non-Black is -56 and is not significant. This column also indicates that the quadratic specification can be rejected in favor of this one and that we still cannot reject $v = -1$.

The model in the last column allows μ and σ_3 to vary across amenities. Although this model has a slightly higher R-squared than the model in column 3, we cannot reject that model in favor of this one. We also cannot reject the hypothesis that $v = -1$. Compared to the third model, the value of μ for High School increases in absolute value to -1.05 and is still significant, whereas the value of μ for Share Non-Black changes to a more reasonable -0.81 and becomes significant. The SSE-minimizing values for σ_3 are 4.0 for High School and 1/3 for Share Non-Black. The insignificant results in this column for the other amenities indicate that the functional form restrictions in (17) are not sufficient to identify μ and σ_3

in some cases. Elementary's variation is limited because it omits Cleveland and East Cleveland and Table 2 indicates that Share Non-Hispanic does not exhibit nearly as much variation as Share Non-Black. This limited variation may explain why these parameters are so difficult to estimate. This problem cannot be solved with the Rosen two-step procedure, which does not bring in any additional information to inform this separation and may use contradictory assumptions in estimating the first-stage envelope and the second-stage demand function.

For all three models in which it is estimated (columns 2 through 4 of Table 5), the value of σ_2 is positive and significant for High School and Share Non-Black. These results provide strong support for the sorting theorem. The results are not so clear for Elementary and Share Non-Hispanic. For these two amenities, σ_2 is statistically significant only in column 3.

Envelopes for the school variables are plotted in Figure 3. The full nonlinear envelopes are based on column 4 of Table 5.[21] Envelopes based on a quadratic specification (column 2) are presented for comparison. In the case of Elementary, the quadratic and full nonlinear specifications yield similar envelopes except at the bottom of the distribution. Recall that Elementary is adjusted to have a minimum of 0.001, which corresponds to a test score 30.8 percentage points below the district average. Outside of Cleveland, housing prices are 22.9 percent lower for a house with an elementary school scoring 30.75 percentage points below the district average than for a house with a school scoring 24.6 above the district average. However, only 4 observations have values of Elementary of 0.1 or below. Housing prices are only 5.8 percent higher for the maximum value of Elementary than for a value of 0.1, which is about the same as with a quadratic envelope (5.0 percent). The second panel of Figure 3 plots the envelopes for High School, excluding Cleveland and East Cleveland. This envelope has an anomaly at low values for this variable, namely a downward slope. However, this portion of the envelope applies to only four school districts. Three of those school districts have housing prices that are less than 1 percent above the minimum price and the fourth has housing prices only 3.5 percent above the minimum. Moreover, the slope of the envelope at this minimum point is not significantly different from zero. In contrast, school districts with the highest values for High School have housing prices that are 30.0 percent above the minimum price.[22] The quadratic envelope indicates an even higher difference between these districts, namely 32.5 percent.

Envelopes for the ethnicity variables are plotted in Figure 4. The first panel shows that both full nonlinear and quadratic envelopes for Share Non-Black have a U-shape. The minimum points are at 16.6 percent non-black for the full non-linear envelope and at 25.1 percent non-black for the quadratic envelope. The full non-linear envelope implies that housing prices are 4.3 percent higher at zero percent non-black and 25.7 percent higher at 100 percent non-black than at this minimum point. Moreover, the slope of the full nonlinear bid function at zero percent non-black is significantly different from zero. These results support the view that households have heterogeneous preferences for neighborhood ethnic composition, with some households preferring an increase and other a decrease in the percent non-black.

As discussed earlier, this finding is not fully consistent with the assumptions used to derive (17). An alternative model without this inconsistency introduces S^*, defined as the value of the amenity at which housing bids are at a minimum, and re-defines the amenity to be (S^* - S) below S^* and as ($S - S^*$) above S^*. An envelope based on this alternative model (with $\sigma_3 = 1$) is drawn in Figure 4. The estimated S^* is 0.249 and is statistically significant. The estimates of μ are -0.79 below S^* and -1.55 above S^* but are not significant. Figure 4 indicates that this envelope looks similar to the full nonlinear envelope. This approach has a disadvantage, however, namely that it predicts unreasonably high housing prices close to S^*. To preserve a reasonable scale, Figure 4 does not plot about 50 observations near this minimum point. Overall, the estimates in column 4 of Table 5 appear to provide reasonable estimates of the envelope in this case, but further research is needed to find a form that fully resolves these issues.

The second panel of Figure 4 plots envelopes for Share Non-Hispanic. The full nonlinear envelope reaches a minimum at a share of 0.493, but the quadratic envelope slopes upward for all observed values of this variable. On the full nonlinear envelope, housing prices decrease by 12.6 percent from the minimum observed share, 0.367, to a share of 0.493 and then increase by 30.7 percent from a share of 0.493 up to a share of 1.0. The slope of the envelope at a share of 0.367 is significantly different from zero. These results point to extensive heterogeneity in preferences for Hispanic neighbors. The alternative model discussed above can also be estimated for this amenity; the resulting split envelope is plotted in Figure 4. This model yields a highly significant value of S^* equal to 0.492 and highly significant values of μ equal to -0.635 below S^* and -0.644 above S^*. As in the case of Share Non-Black, the split envelope is similar to the full nonlinear envelope if one excludes values of the amenity near S^*.

Results for other variables are presented in Tables 3 and 4. The two commuting variables are significant with the expected signs. The coefficients of the property tax variables reflect both property tax capitalization and the impact of public services that are correlated with the property tax rate but not accounted for by the public service variables.[23] The positive estimated coefficient for the school tax rate indicates that the second of these effects is larger for school services. The signs for the two city tax coefficients point to the first effect but these coefficients are not significant. The income tax coefficient is also not significant.

The crime variables have large, significant impacts on property values. Compared to houses in CBGs with low property and violent crime, houses in CBGs with high property and low violent crime sell for 3.4 percent less, houses in CBGs with low property and high violent crime sell for 8.8 less, and houses in CBGs with high property and violent crime sell for 7.8 less. Moreover, houses located within one-half mile of a crime hotspot sell for a huge discount: 20.4 percent. This discount declines at greater distances from the hotspot, but still equals 4.1 percent for houses two to five miles away. Property values are significantly lower in villages and townships and places that receive their police services from the county. Outside of Cleveland, property values decline with city population. Compared to Glenwillow (population = 449), the city with the second largest population, Parma (population = 85,655), has prices that are 23.5 percent lower. These effects account for unidentified variation in public services or neighborhood quality across cities of different sizes. Table 3 also indicates that home buyers care about pollution. Houses located at the center of an air pollution cluster sell for 23.8 percent less than houses with clean air; this effect declines 0.8 percentage points per mile and has essentially disappeared at a distance of 20 miles. Location next to an environmental hazard lowers prices by 6.2 percent, an effect that fades out within a mile.

Table 3 also presents result for additional school variables. The relationship between housing prices and both Value Added and Minority Teachers is hill-shaped, which is not expected for an envelope. In the case of Value-Added, housing prices are at a maximum when this amenity equals 32.8, a value somewhat above its average value of 24.0. Thus, as expected, housing prices increase (by 20.3 percent) from the lowest Value-Added to 32.8, but then, contrary to expectations, they decline (by 5.0 percent) as Value-Added goes from 32.8 to its maximum. Because the regressions control for High School, a relatively high Value Added indicates a school district with a low fourth-grade test score compared to other districts with the same twelfth grade score. Home buyers apparently appreciate relatively successful elementary schools but are also concerned about those that start too far behind.[24] Housing prices also reach a maximum when Minority Teachers equals 0.23. Although the underlying coefficients are not significant, this result suggests that households may prefer an integrated teaching force.[25] Because of these results, no attempt is made to separate the bidding and sorting parameters for these two variables. Table 3 also reveals that the coefficients Elementary are significantly different in

Cleveland and Cleveland Heights than elsewhere in the area, and indeed decline with Elementary for much of its range. As discussed earlier, these results probably indicate that house buyers associated poor elementary schools with unique voucher and charter school opportunities in Cleveland and with major facility improvements in East Cleveland.

Table 4 presents results for other geographic variables. Housing prices are 7.1 percent higher within two miles of Lake Erie. This effect declines with distance from the lake, although the phase-out is not significant. The coefficients of the two snowbelt variables imply a maximum positive impact on housing prices at 11 miles from Lake Erie in the eastern suburbs, which is where snowfall is the greatest. Households apparently value snowfall or some topographical feature, such as ridge lines, with which it is correlated. Housing prices are boosted a small amount by neighborhood parks but lowered by location near a railroad. Finally, housing prices are about 9 percent lower for houses located near a large family public housing project.

The estimates in Table 5 can be used to calculate ψ using (14), to estimate (11), and to test the "normal sorting" hypothesis. The first test is based on the exponent of Y in (11) controlling for other demand determinants. As explained earlier, normal sorting requires this coefficient to be positive. The second test recognizes that the correlation between Y and S depends on both the direct impact of Y on S and the impact on S of other demand determinants correlated with Y. According to the theorem for omitted-variable bias, regressing $\ln\{\psi\}$ on $\ln\{Y\}$ yields a coefficient with an expected value equal to the true coefficient of $\ln\{Y\}$ plus the sum across other demand variables of their true coefficients multiplied by their correlations with $\ln\{Y\}$. The coefficient of $\ln\{Y\}$ therefore provides an indirect test for normal sorting.

The first panel of Table 6 presents the indirect tests. The coefficient of $\ln\{Y\}$ is positive and significant in three of the four cases, providing support for the indirect normal sorting hypothesis. Direct tests are presented in the second panel. These tests, which control for a range of family and educational characteristics, support normal sorting for High School and Share Non-Black.[26] Indeed, the coefficient of $\ln\{Y\}$ is large in magnitude and highly significant in both cases. These estimates imply that a one standard deviation in $\ln\{Y\}$ leads to a 16.4 percent increase in High School but only a 2.1 percent increase in Non-Black. In contrast, the income coefficient is positive but insignificant and close to zero for the other two amenities. Because they show that income and education are often key determinants of amenity demand, these regressions also highlight the potential for endogeneity bias when income and education are treated as neighborhood amenities, as in Bayer et al. (2007).

Conclusions

In contrast to Taylor (2008), who writes "Because the hedonic price function is an envelope function, there is no theoretical guidance for its specification" (p. 20), this paper takes the view that this functional form can be derived precisely because it is an envelope. Using constant elasticity demand functions for amenities and housing and the theory of household sorting, I derive the bid function envelope with heterogeneous households. This envelope, equation (17), consists of three Box-Cox forms. I also show that specifications with a single Box-Cox form (or one of its special cases) instead of two such forms on the right side implicitly rule out sorting. My approach has three main advantages. First, it accounts for both observable and unobservable factors in the demands for amenities and housing. Second, it yields estimates of the price elasticities of demand for amenities directly from the hedonic estimation and thereby avoids the potential inconsistency and endogeneity in the Rosen two-step procedure. Third, it provides tests of the hypotheses that households sort according to the slopes of their bid functions and that higher-income households live in locations with more desirable amenities.

This approach was implemented using data from the Cleveland area in 2000. The regressions, which control for a wide range of housing and neighborhood characteristics, focus on four measures of school quality and two measures of neighborhood ethnicity. For two of these measures (the passing rate on high school exams and neighborhood percent non-black), the specification derived in this paper leads to precise estimates of the price elasticities of demand (-1.05 and -0.81, respectively) and of the sorting parameters. Moreover, the ethnicity results confirm what surveys have long indicated, namely, that some households prefer largely white neighborhoods, whereas others prefer neighborhoods in which blacks are concentrated.

For two other measures, relative elementary test scores and a neighborhood's share non-Hispanic, the results are mixed. With linear approximations to the sorting equilibriums, I obtain precise results and evidence that preferences for Hispanic neighbors are also heterogeneous. The price elasticity and sorting parameters cannot be precisely estimated, however, with more general approximations. Although a regression based on linear approximations cannot be rejected in favor of a more general case, these results show that it can be difficult to separate the bidding and sorting parameters. Another school quality measure, a district's elementary school value added, raises issues of interpretation that preclude separate estimation of the bidding and sorting parameters. Increases in elementary value added above a certain point lower housing prices, presumably because parents prefer not to start their children in a school with particularly low initial performance even if, on average, that disadvantage eventually disappears.

Finally, the results in this paper support the standard model of household bidding and sorting. Results for the high school and percent non-black variables (and results for the relative elementary score and percent non-Hispanic variables with a linear approximation to the sorting equilibrium) support the theorem that households sort into locations according to the slopes of their bid functions. Moreover, the results confirm that sorting is usually "normal," with higher-income households sorting into locations where the services and amenities are more desirable.

Challenges for future research include considering other demand or sorting specifications and accounting for changes in sorting over time. Because the second-stage regressions identify the distribution of amenity demands, the approach in this paper could also be used to simulate the impact of policy changes, such as school finance reform, on household sorting.

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Endnotes

[1] Recent school capitalization studies are reviewed in Nguyen-Hoang and Yinger (2011).

[2] Or the income elasticity can equal zero for some for observed commodities and one for others.

[3] Rouwendal (1992) derives a quadratic envelope based on a linear utility function and the assumption that C is a quadratic function of observable variables. Rouwendal claims, as I do, that "The second stage of Rosen's procedure is circumvented in this way and so are the problems associated with it" (p. 59). However, his equation does not identify demand parameters.

[4] Epple (1987) derives a quadratic envelope with across-commodity interactions, but it requires strong assumptions about the distributions of household tastes and product supply.

[5] Ondrich, Ross, and Yinger (2003) find that real estate brokers steer blacks toward neighborhoods with low average house values or high average house ages.

[6] A discrete-choice approach to estimating heterogeneity in household ethnic preferences is developed by Bayer et al. (2007).

[7] Estimated envelopes are similar using share black and share Hispanic.

[8] The model and estimation in this paper are based on (17) with a multiplicative $H\{X\}$. Unless $\mu = \nu = -1$ and $\sigma_3 = \infty$, therefore, the *S*s and the *X*s are treated differently. An alternative model would treat the *X*s the same way as the *S*s, so (17) would apply to the entire estimating equation.

[9] For more on normal sorting, see Wheaton (1993) and Yinger (1995).

[10] The quadratic Box-Cox in Halvorsen and Pollakowski has two terms for each amenity with exponents λ and 2λ. Consistency with (17)—and sorting—requires λ to be interpreted as $(1+\mu)/\mu$ and σ_3 t to equal $\mu/(1+\mu)$, which is a nonsensical condition. This form also has interactions between services, which are not consistent with the integrability of the demand functions.

[11] This formulation leaves off several terms in the Bayer et al. formulation, their equation (2); this simplification facilitates the exposition but do not alter the conclusions in the text.

[12] Bayer et al. also estimate a multinomial discrete choice model of sorting. Each household is allocated to the house that gives it the highest utility (net of price), so an equilibrium allocation is built into their estimation procedure. In this context, they argue (p. 620) that "there is likely to be only a slight difference between the mean preferences estimated in the heterogeneous sorting model and the coefficients of the hedonic price regression." In the Rosen framework, however, heterogeneous preferences result in an envelope that can diverge from mean preferences.

[13] Bajari and Benkard (2005) and Bajari and Kahn (2005) avoid endogeneity by assuming that utility is linear in Z and $\ln\{X\}$. This implies that μ = -1 so (3) and (9) yield a second-step with $P_X X$ on the left and nothing endogenous on the right. In this case, (17) implies that, with $\sigma_3 = 1$, the envelope is quadratic in $\ln\{X\}$. The squared term is not included in their hedonic regression.

[14] Bajari and Benkard (2005) develop a method to account for unobserved product characteristics and Bajari and Kahn (2005) apply it to housing. However, this method complicates estimation of the envelope (see note 12) and is not attempted here.

[15] As is well known (see, for example, Goldhaber and Brewer 1997), the second-stage standard errors must be corrected for heteroskedasticity.

[16] Appendixes explaining data collection procedures, estimating techniques, and detailed results are available on the author's web site [to be added].

[17] See http://www.charterschoolsearch.com/.

[18] Identification is aided by the fact that the Cleveland School District (CSD) is not coterminous with the city of Cleveland. Several CBGs in the City of Cleveland are in the Shaker Heights School District, and a few of the CBGs in the CSD are in small cities on Cleveland's border.

[19] When assessment-sales ratios were not available, the nominal rates were corrected by the average observed assessment-sales ratio and the No A-to-S dummy was set to 1.0.

[20] A bootstrap (or wild bootstrap) procedure is not feasible for these regressions because many of the selected samples must drop variables due to collinearity.

[21] Envelopes based on column 3 are similar, although bid functions may not be. The relative envelope heights are selected for presentation purposes.

[22] Starting at the mean, a one standard deviation increase in High School raises housing prices by 7.6 percent, which is similar to previous studies (Brasington and Haurin 2006; Nguyen-Hoang and Yinger 2011). In contrast to previous studies, however, this effect is not constant.

[23] The property tax variables approximate (5) (and 18) because $\log\{1-a\} \approx a$ when a is small. Nonlinear estimation of the tax terms yields results similar to those in in Table 3.

[24] Other explanations are possible, of course, such as omitted variable bias.

[25] The Minority Teachers coefficients are significant in a quadratic estimation (Table 5, col. 2).

[26] The discrete choice estimates in Bayer et al. (2007, Table 8) also indicate that willingness to pay for a whiter neighborhood increases with income.

Figure 1: Deriving the Bid-Function Envelope

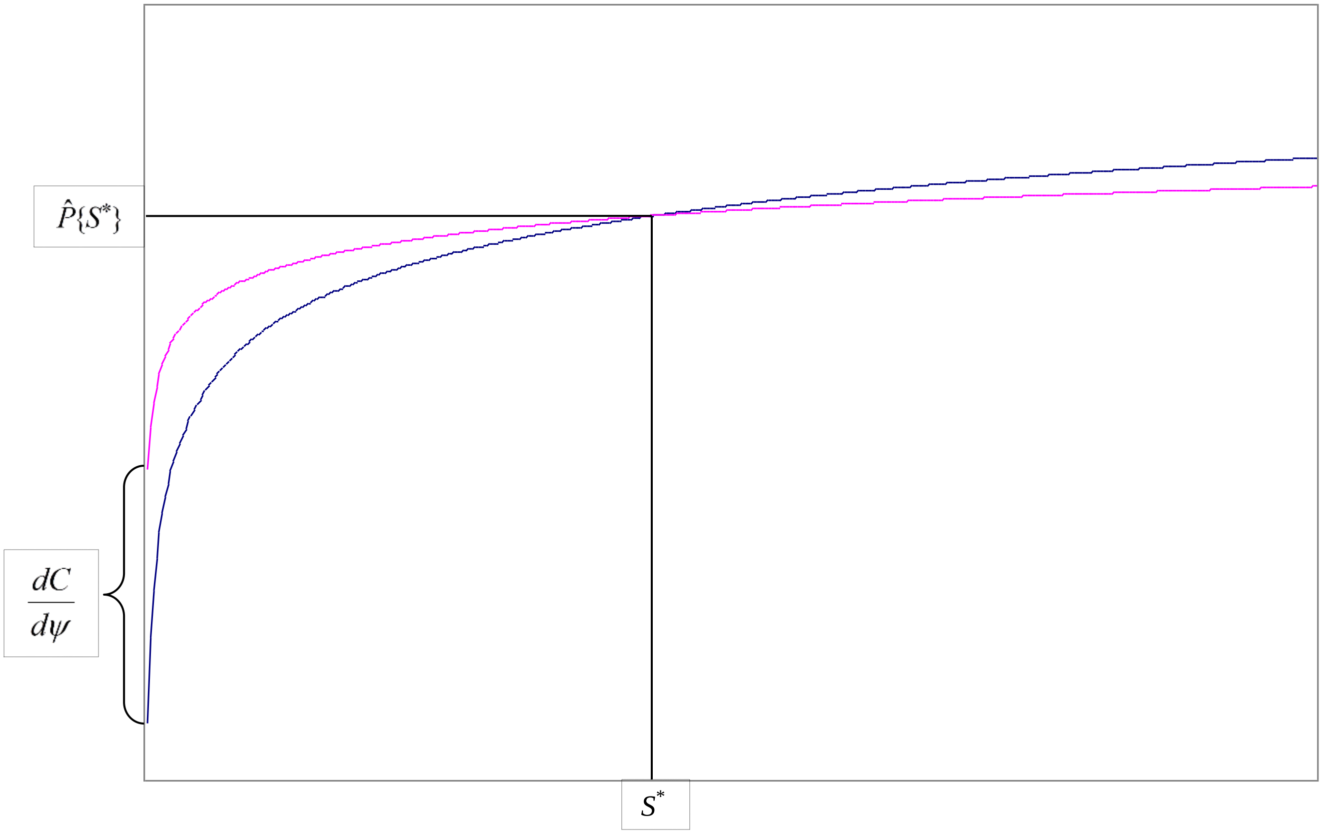


Figure 2. Supply and Demand for School Quality

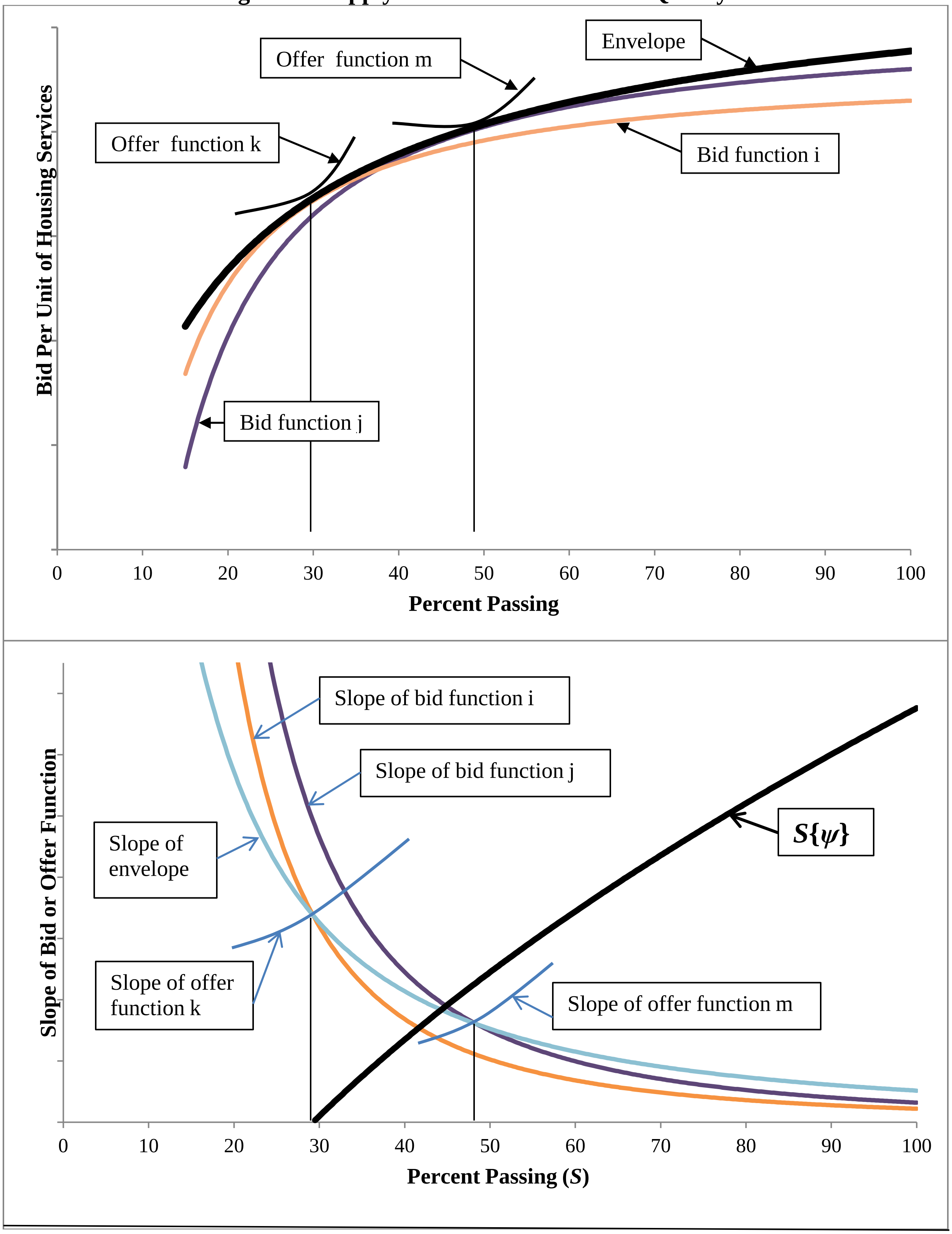
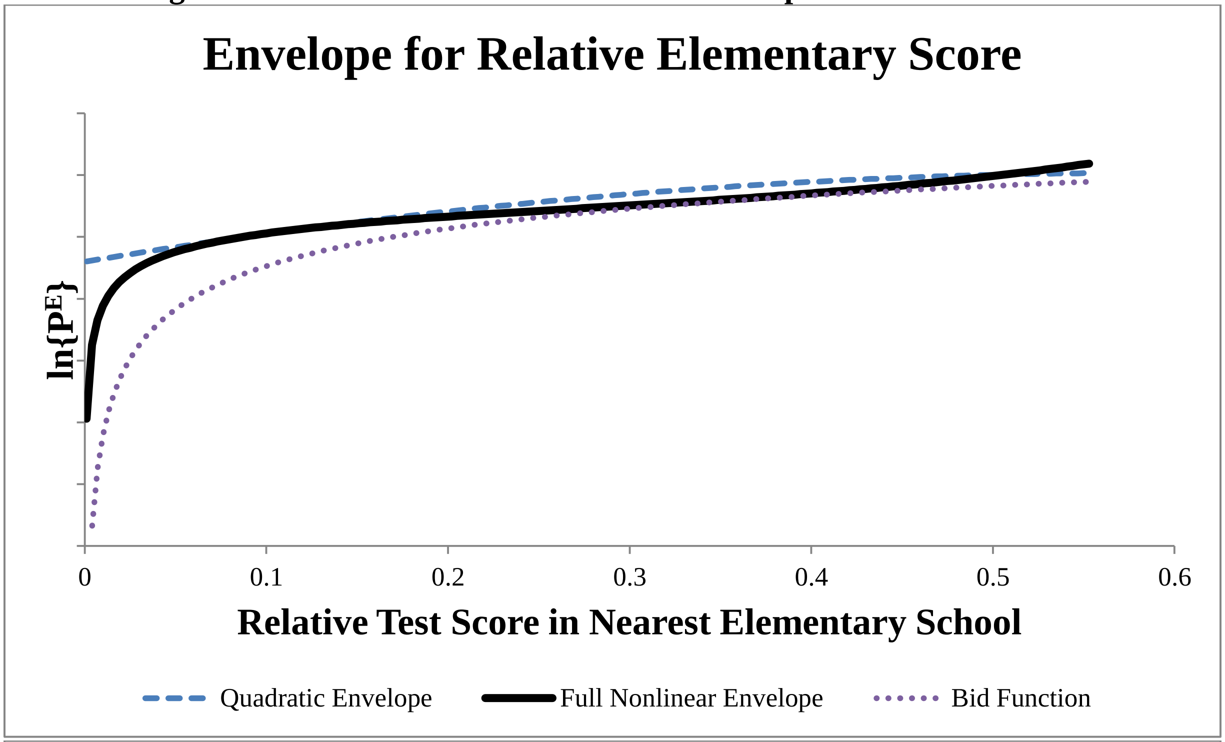


Figure 3: Estimated Bid-Functions and Envelopes for School Variables



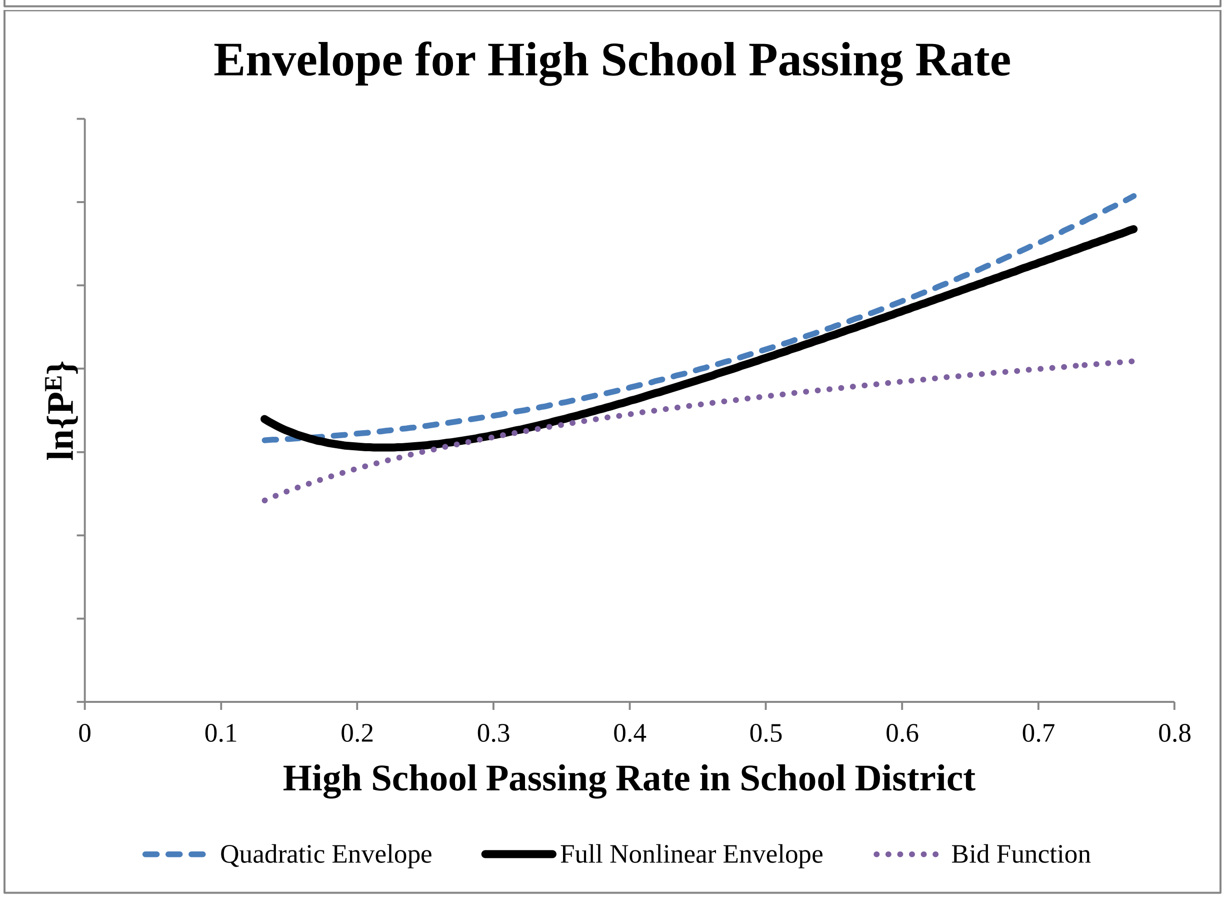
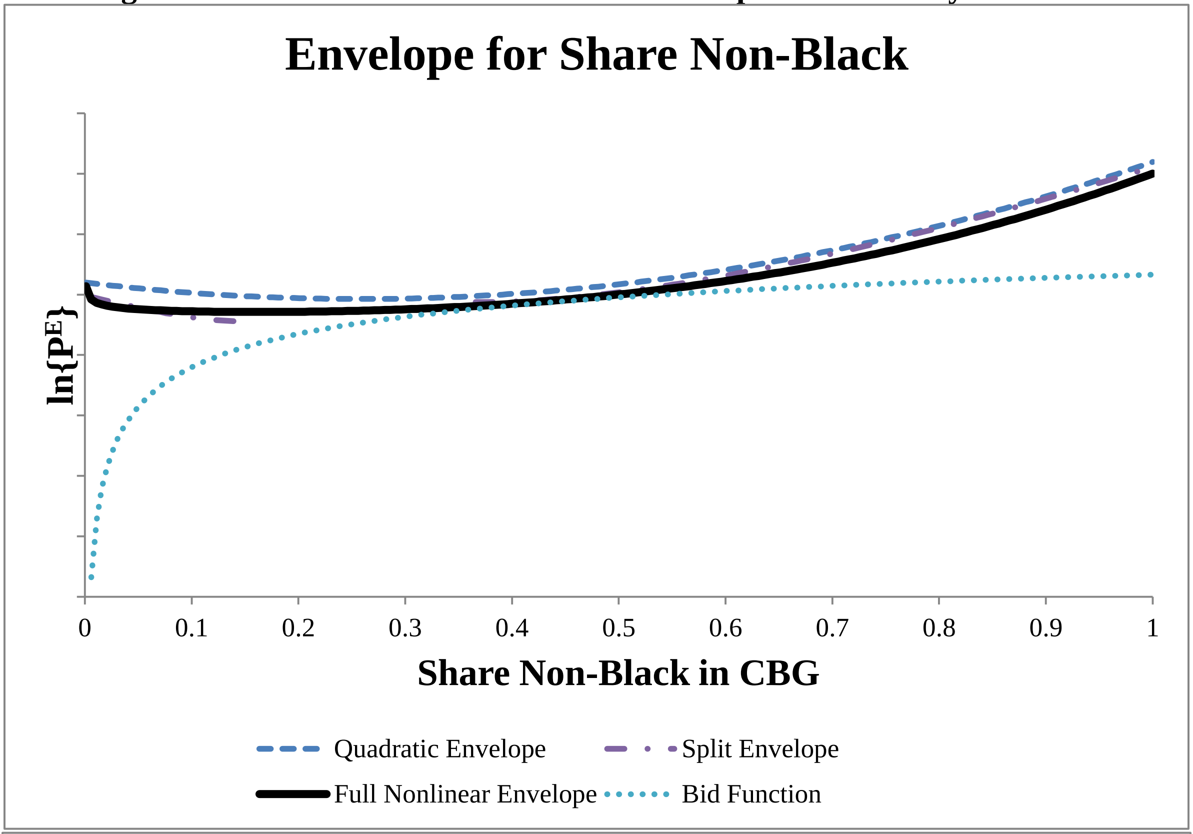


Figure 4: Estimated Bid-Functions and Envelopes for Ethnicity Variables



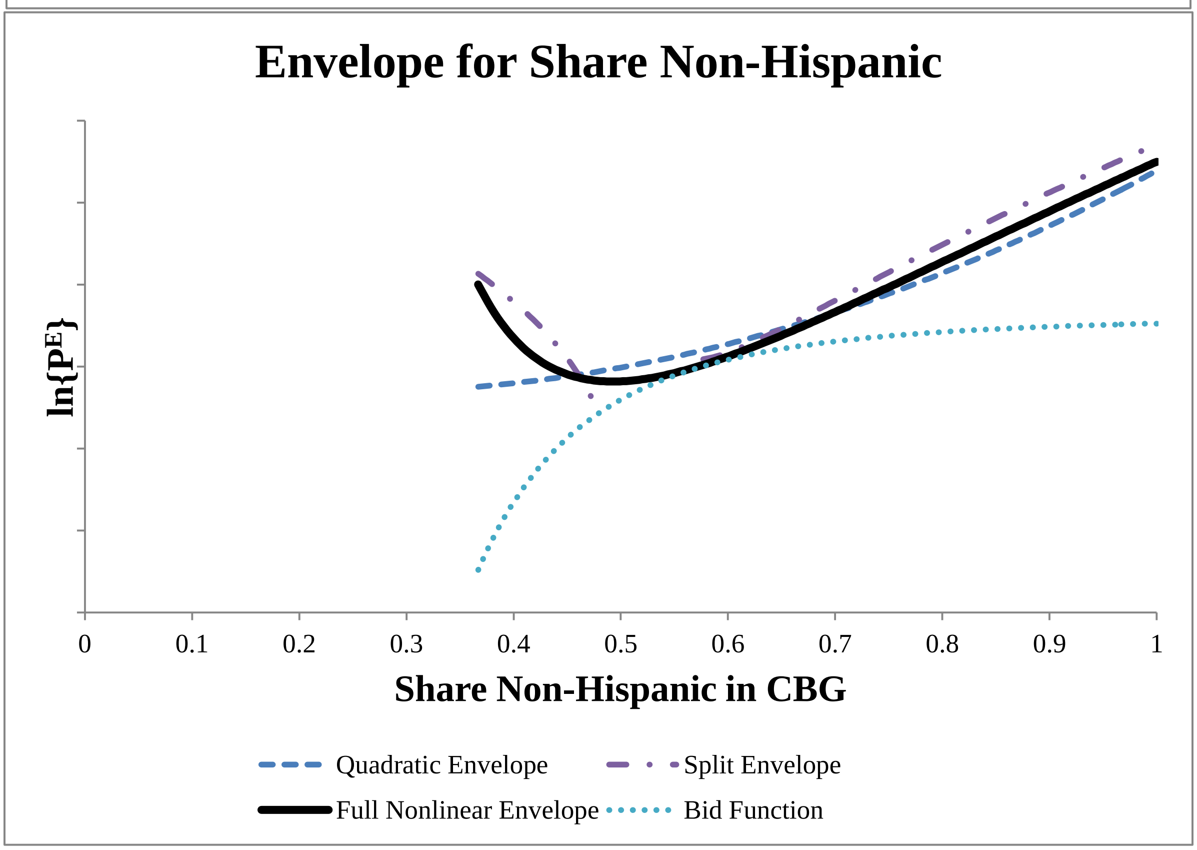


Table 1. Variable Definitions and Results for Basic Hedonic with Neighborhood Fixed Effects

Variable	Definition	Coefficient	Std. Error
One Story	House has one story	- 0.0072	0.0050
Brick	House is made of bricks	0.0153***	0.0052
Basement	House has a finished basement	0.0308***	0.0050
Garage	House has a garage	0.1414***	0.0067
Air Cond.	House has central air conditioning	0.0254***	0.0055
Fireplaces	Number of fireplaces	0.0316***	0.0038
Bedrooms	Number of bedrooms	- 0.0082***	0.0028
Full Baths	Number of full bathrooms	0.0601***	0.0042
Part Baths	Number of partial bathrooms	0.0412***	0.0041
Age of House	Log of the age of the house	- 0.0839***	0.0032
House Area	Log of square feet of living area	0.4237***	0.0086
Lot Area	Log of lot size	0.0844***	0.0037
Outbuildings	Number of outbuildings	0.1320***	0.0396
Porch	House has a porch	0.0327***	0.0073
Deck	House has a deck	0.0545***	0.0053
Pool	House has a pool	0.0910***	0.0180
Date of Sale	Date of house sale (January 1=1, December 31=365)	0.0002***	0.0000
Commute 1[a]	Employment wtd. commuting dist. (house-CBG), worksite 1	- 0.0952***	0.0272
Commute 2[a]	Employment wtd. commuting dist. (house-CBG), worksite 2	- 0.0991***	0.0321
Commute 3[a]	Employment wtd. commuting dist. (house-CBG), worksite 3	- 0.1239***	0.0302
Commute 4[a]	Employment wtd. commuting dist. (house-CBG), worksite 4	- 0.1012***	0.0295
Commute 5[a]	Employment wtd. commuting dist. (house-CBG), worksite 5	- 0.0942***	0.0344
Dist. to Pub. School[a]	Dist. to nearest pub. elementary school in district (house-CBG)	- 0.0032	0.0061
Elem. School Score[a]	Average math and English test scores of nearest pub. elementary school relative to district (house-CBG)	0.0170	0.0197
Dist. to Private School	Distance to nearest private school (house-CBG)	- 0.0168***	0.0057
Distance to Hazard	Dist. to nearest environmental hazard (house-CBG)	0.0332***	0.0082
Distance to Erie[a]	Dist. to Lake Erie (if < 2; house-CBG)	- 0.0021**	0.0010
Distance to Ghetto[a]	Dist. to black ghetto (if < 5; house-CBG)	- 0.1020***	0.0331
Distance to Airport[a]	Dist. to Cleveland airport (if < 10; house-CBG)	0.0259**	0.0122
Dist. to CBG Center	Distance from house to center of CBG	- 0.0239***	0.0074
Historic District[a]	In historic district on national register (house-CBG)	0.0120	0.0178
Elderly Housing[a]	Within 1/2 mile of elderly housing project (house-CBG)	- 0.0327*	0.0194
Family Housing[a]	Within 1/2 mile of small family housing project (house-CBG)	0.0836**	0.0403
Large Hsg Project[a]	Within 1/2 mile of large family housing project (>200 units; house-CBG)	- 0.0568**	0.0257
High Crime	Distance to nearest high-crime location (house-CBG)	0.0701***	0.0246

Notes: Dependent variable = log of transaction amount; 22,880 observations; R2 = .7893; F(31, 21180) = 369.17 (significant at 1% level); 1,665 fixed effects with F(1664, 21,180) = 8.534 (significant at 1% level); estimated with "areg" command in Stata; distances in miles. A * (**) [***] indicates statistical significance at the 10(5)[1]% level.
[a]Variable added to the original Brasington data set.

Table 2. Descriptive Statistics for Key Variables

	Mean	Std. Dev.	Minimum	Maximum
CBG Price per unit of Housing	84835.68	23331.25	32215.83	345162.50
Relative Elementary Score[a]	0.3148	0.0894	0.0010	0.6465
High School Passing Rate	0.3197	0.2040	0.0491	0.7675
Elementary Value Added[a]	24.0021	9.4164	1.0000	49.6000
Share Minority Teachers[b]	0.1329	0.1548	0.0010	0.6146
Share Non-Black in CBG[b]	0.8022	0.3226	0.0010	1.0000
Share Hispanic in CBG	0.9623	0.0810	0.3673	1.0000
Weighted Commuting Distance	13.2046	7.4567	7.2660	39.5236
Income Tax Rate[c]	0.0091	0.0012	0.0075	0.0100
School Tax Rate	0.0309	0.0083	0.0172	0.0643
City Tax Rate[d]	0.0578	0.0140	0.0227	0.1033
Tax Break Rate[d]	0.0330	0.0121	0.0047	0.0791
No A-to-S	0.1339	0.3407	0.0000	1.0000
Not a City	0.1393	0.3464	0.0000	1.0000
Crime Lowhigh	0.0252	0.1569	0.0000	1.0000
Crime Highlow	0.1291	0.3354	0.0000	1.0000
Crime Highhigh	0.1934	0.3951	0.0000	1.0000
Crime Hotspot1	0.0126	0.1116	0.0000	1.0000
Crime Hotspot2	0.0354	0.1849	0.0000	1.0000
Crime Hotspot3	0.0847	0.2785	0.0000	1.0000
Crime Hotspot4	0.2667	0.4423	0.0000	1.0000

Note: More detailed variable definitions are in in the text of Table 3.

[a.] Constant added to make all values positive.

[b.] 0.001 added to avoid zero values (unless initial value is 1.0).

[c.] Statistics apply to the 27 observations with a positive income tax rate.

[d.] Statistics apply to the 1433 observations in a city.

Table 3. Definitions and Results for Tax, Commuting, Crime, Pollution, and Ancillary School Variables

Variable	Definition	Coefficient	Robust Std. Error
Income Tax Rate	School district income tax rate	1.9621	5.1627
School Tax Rate	School district effective property tax rate	3.9007***	1.2979
City Tax Rate	Effective city property tax rate beyond school tax	- 1.9737	1.3926
Tax Break Rate	Exemption rate for city property tax	3.9998*	2.1290
No A-to-S	Dummy: No A/V data	- 0.0420	0.0352
Not a City	CBG not in a city	0.0629	0.0387
Commute 1	Job-weighted distance to worksites	- 0.0259***	0.0072
Commute 2	(Commute 1) squared	0.0004**	0.0002
Crime Lowhigh	Low property, high violent crime	- 0.0882***	0.0336
Crime Highlow	High property, low violent crime	- 0.0344**	0.0150
Crime Highhigh	High property and violent crime	- 0.0776***	0.0213
Crime Hotspot1	CBG within ½ mile of crime hot spot	- 0.2044***	0.0497
Crime Hotspot2	CBG ½ to 1 mile from crime hot spot	- 0.0696*	0.0392
Crime Hotspot3	CBG 1 to 2 miles from crime hot spot	- 0.0916***	0.0341
Crime Hotspot4	CBG 2 to 5 miles from crime hot spot	- 0.0408***	0.0157
Village	CBG receives police from a village	- 0.1050**	0.0492
Township	CBG receives police from a township	- 0.1119**	0.0437
County Police	CBG receives police from a county	- 0.1740***	0.0448
City Population	Population of city (if CBG in a city)	-2.10E-05***	0.0000
City Pop. Squared	City population squared/10000	5.72E-06***	0.0000
City Pop. Cubed	City population cubed/10000^2	-4.92E-07***	0.0000
City Pop. to Fourth	City pop. to the fourth power/10000^3	7.75E-09***	0.0000
Smog	CBG within 20 miles of air pollution cluster	- 0.2383***	0.0698
Smog Distance	(Smog) × Distance to cluster (not to the NW)	0.0084**	0.0039
Near Hazard	CBG is within 1 mile of a hazardous waste site	- 0.0623***	0.0169
Distance to Hazard	Distance to nearest hazardous waste site (if <1)	0.0757***	0.0228
Value Added 1	School district's 6th grade passing rate on 5 state tests in 2000-2001 minus its 4th grade passing rate in 1998-99	0.0120***	0.0039
Value Added 2	(Value Added 1) squared	- 0.0002***	0.0001
Minority Teachers 1	Share of district's teachers from a minority group	0.3379	0.2177
Minority Teachers 2	(Minority Teachers 1) squared	- 0.7334*	0.3967
Rel. Elem. Cle. 1	Average 4th grade passing rate on 5 state tests in nearest elementary school minus district average (1998-99 and 1999-2000) for Cleveland and E. Cleveland only	- 1.5045***	0.4066
Rel. Elem. Cle. 2	(Rel. Elem. Cle. 1) squared	1.8398***	0.4951
Cleveland SD	Dummy for Cleveland & E. Cleveland School Districts	0.1414	0.2232
Near Public	CBG is within 2 miles of public elem. school	- 0.0118	0.0219
Distance to Public	(Near Public) × Distance to public school	- 0.0178*	0.0108
Near Private	CBG is within 5 miles of a private school	0.0217	0.0230
Distance to Private	(Near Private) × Distance to private school	- 0.0032	0.0050

Note: Except for "Near Private" and "Distance to Private," these variables were added to the Brasington data set using CBG latitudes and longitudes and (for Smog) other information in that data set. These results are based on the nonlinear regression in the 3rd column of Table 5; results are similar for the regressions in other columns. Standard errors are estimated using the vce(hc3) option in Stata. The regression also includes the variables in Tables 4 and 5. A * (**) [***] indicates statistical significance at the 10 (5) [1] % level.

Table 4. Definitions and Results for Other Geographic Controls

Variable	Definition	Coefficient	Robust Std. Error
Lakefront	Within 2 miles of Lake Erie	0.0714***	0.0240
Distance to Lake	(Lakefront) × (Distance to Lake Erie)	- 0.0305	0.0192
Snowbelt 1	(East of Pepper Pike) × (Distance to Lake Erie)	0.0289***	0.0065
Snowbelt 2	(Snowbelt 1) squared	- 0.0013***	0.0004
Ghetto	CBG in the black ghetto	- 0.0314	0.0447
Near Ghetto	CBG within 5 miles of ghetto center	- 0.0027	0.0246
Near Airport	CBG within 10 miles of Cleveland airport	0.0322	0.0326
Airport Distance	(Near Airport) × (Distance to airport)	- 0.0019	0.0037
Local Amenities	No. of parks, golf courses, rivers, or lakes within ¼ mile of CBG	0.0187**	0.0081
Freeway	CBG within ¼ mile of freeway	0.0126	0.0116
Railroad	CBG within ¼ mile of railroad	- 0.0163*	0.0094
Shopping	CBG within 1 mile of shopping center	0.0121	0.0098
Hospital	CBG within 1 mile of hospital	0.0340	0.0220
Small Airport	CBG within 1 mile of small airport	0.0034	0.0108
Big Park	CBG within 1 mile of regional park	- 0.0307***	0.0103
Historic District	CBG within an historic district	0.0072	0.0182
Near Elderly PH	CBG within ½ mile of elderly public housing	- 0.0259	0.0267
Near Small Fam. PH	CBG within ½ mile of small family public housing	- 0.0912**	0.0417
Near Big Fam. PH	CBG within ½ mile of large family public housing (>200 units)	- 0.0882***	0.0336
Worksite 2	Fixed effect for worksite 2	0.0423**	0.0184
Worksite 3	Fixed effect for worksite 3	0.0924***	0.0351
Worksite 4	Fixed effect for worksite 4	0.0618*	0.0355
Worksite 5	Fixed effect for worksite 5	- 0.0105	0.0247
Geauga County	Fixed effect for Geauga County	- 0.0233	0.0496
Lake County	Fixed effect for Lake County	0.2437***	0.0351
Lorain County	Fixed effect for Lorain County	0.1456***	0.0327
Medina County	Fixed effect for Medina County	0.0074	0.0469
Constant		11.8887***	0.1272

Note: These variables were added to the Brasington data set using the CBG latitudes and longitudes other information in that data set. These results are based on the nonlinear regression in the 3rd column of Table 5; results are similar for the regressions in other columns. Standard errors are estimated using the vce(hc3) option in Stata. The regression also includes the variables in Tables 3 and 5. A * (**) [***] indicates statistical significance at the 10 (5) [1] % level.

Table 5. Specification Tests and Results for Key School and Ethnicity Variables

Variable	Linear	Quadratic	Nonlinear Estimation of μ's with σ_3 = 1	Nonlinear Estimation of μ's with Various Values for σ_3
Relative Elementary Score				
First Term	0.1268**	0.2448	0.0022***	- 0.0130
	(0.0581)	(0.2480)	(0.0008)	(0.2283)
Second Term	-	- 0.2086	89.4295	1.0114
		(0.3584)	(163.9870)	(130.8000)
μ	-∞	-∞	- 0.3694***	- 0.8371*
			(0.1342)	(0.4565)
σ_3	∞	1	1	1/4
High School Passing Rate				
First Term	0.4826***	- 0.0862	0.2168***	0.6836***
	(0.0600)	(0.2631)	(0.0339)	(0.0268)
Second Term	-	0.6049** ~	1.3087**	0.5582***
		(0.2849)	(0.5294)	(0.1977)
μ	-∞	-∞	- 0.7564***	- 1.0496**
			(0.2762)	(0.4860)
σ_3	∞	1	1	4
Share Non-Black in CBG				
First Term	0.2633***	- 0.2120* ~	0.2496***	0.0044
	(0.0342)	(0.1247)	(0.0952)	(0.0097)
Second Term	-	0.4114***~	1.2515***	1.5285***
		(0.1031)	(0.3469)	(0.2956)
μ	-∞	-∞	-55.9610	- 0.8072***
			(1607.4140)	(0.2751)
σ_3	∞	1	1	1/3
Share Non-Hispanic in CBG				
First Term	0.4849***	- 0.2440	0.5003***	0.0590
	(0.0689)	(0.4306)	(0.0210)	(2.4368)
Second Term	-	0.4832*	0.8546***	1.5812
		(0.2871)	(0.2593)	(3.8819)
μ	-∞	-∞	- 0.3760***	- 0.2171
			(0.1395)	(0.9323)
σ_3	∞	1	1	1/4
Number of Observations	1665	1665	1665	1665
R-Squared	0.7046	0.7156	0.7170	0.7173
SSE	34.7383	33.4405	33.2754	33.2418
Test of Hypothesis that v = -1				
Estimate of v	-1.0433	-1.0272	-1.0308	-1.0289
Test statistic	0.75	0.3	-0.21	-0.19
P-value	0.385	0.583	0.834	0.852
Test of Hypothesis that Model Adds to Explanatory Power				
Test statistic	-	6.66	2.62	0.83
P-value		0.000	0.009	0.409

Notes: The dependent variable is ln{ $\hat{P}$ } in the CBG. Coefficients are estimated with OLS (first two columns) or NLLS (last two columns). Parentheses contain t-statistics based on robust standard errors (using the Stata hc3 option in the first three columns and a jackknife procedure in the last column). A * (**) [***] indicates statistical significance at the 10 (5) [1] % level. In column 2, a ~ indicates that the underlying structural parameter (σ_1 for "First Term" and σ_2 for "Second Term") is significant at the 5% level. The last two columns estimate σ_1 and σ_2 directly. The "First Term" estimate in column 1 corresponds to $(1-\sigma_1)/\sigma_2$. Columns 1-3 assume a value for σ_3; column 4 selects the SSE-minimizing value of σ_3 from the set (1/4, 1/3, 1/2, 1, 2, 3, 4). The hypothesis that v = -1 is tested with Box-Cox regression (χ-squared statistic, first two columns) or NLLS (t-test, last two columns). The hypothesis that the model adds explanatory power (compared to the previous model) is tested with an F-statistic (with 6 and 1593 degrees of freedom) in column 2 and the Davidson-MacKinnon test (a t-statistic based on a robust standard error) in the remaining columns. The regressions also include the variables in Tables 3 and 4.

Table 6. Tests for Normal Sorting

Type of Test	Relative Elementary Score	High School Passing Rate	Share Non-Black	Share Non-Hispanic
Indirect Test				
Income Coefficient	0.0535	0.8385***	0.510***	0.2316***
Standard Error	(0.0349)	(0.0478)	(0.0680)	(0.0267)
Observations	1222	1113	1468	1651
Conclusion	Inconclusive	Support	Support	Support
Direct Test				
Income Coefficient	0.0199	0.5177***	0.4067***	0.0355
Standard Error	(0.0622)	(0.0771)	(0.1345)	(0.0379)
R-squared	0.0264	0.2802	0.2711	0.4554
Observations	1222	1113	1468	1651
Conclusion	Inconclusive	Support	Support	Inconclusive

Notes: Tests are conducted with OLS using robust standard errors (hc3 option in Stata) and all observations with a positively sloped envelope. Indirect tests regress log{ψ} (based on column 4 of Table 5) on log{Y} (median owner income in CBG). Direct tests control for the CBG's percent of households that have children, are headed by a married couple, speak English at home, are Asian, are headed by an elderly person; five education categories for adults (all for the CBG), and the share of households in the tract that moved during the last year. In all columns except the first, most variables are significant at the 5% level. Results are similar using other sets of controls or the results in column 3 of Table 5. A * (**) [***] indicates statistical significance at the 10 (5) [1] % level.