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HEDONIC MARKETS AND SORTING EQUILIBRIA: BID-FUNCTION ENVELOPES FOR PUBLIC SERVICES AND NEIGHBORHOOD AMENITIES

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Abstract

Hedonic regressions with house value as the dependent variable are widely used to study public services and neighborhood amenities. This paper builds on the theory of household bidding and sorting across communities to derive bid-function envelopes, which provide a form for these regressions. This approach allows for household heterogeneity and multiple amenities, yields estimates of the price elasticity of amenity demand directly from the hedonic without a Rosen two-step procedure, and provides tests of hypotheses about sorting. An application to Cleveland area data from 2000 yields price elasticities for school quality and neighborhood ethnic composition and supports the sorting hypotheses.

Key Words: Hedonics, Capitalization, Sorting, Demand for Public Services

1. Introduction

House-value regressions, also called hedonic regressions, are a central empirical tool of urban economics and local public finance. This tool has been used to study many topics, including the demand for public services and environmental quality, property tax capitalization, the trade-off between housing and commuting costs, and racial prejudice and discrimination. Scholars have long recognized that these regressions capture both bidding by households of a given type and sorting of different household types across locations. Many studies of amenity demand use the two-step method in Rosen's seminal (1974) article to separate these two effects, but this approach runs into an endogeneity problem that has proven to be difficult to solve. This paper draws on the theory of local public finance to derive a practical alternative approach. This new approach facilitates consideration of household heterogeneity and multiple amenities, leads to direct estimates of service and amenity demand elasticities, and makes it possible to test sorting hypotheses.

The foundation of this paper is the theory of household bids for housing in locations with different public services or neighborhood amenities. The bid functions in this paper are based on constant-elasticity demand functions for public services, neighborhood amenities, and housing. These functions involve household heterogeneity from both observable and unobservable factors. The main theorem from the literature on household sorting (namely, that households sort according to the slopes of their bid functions) makes it possible to derive the envelope of the household bid functions across household types and to incorporate it into a house-value regression. Most parametric specifications in previous studies are special cases of the one derived here.

This derivation emphasizes the distinction in Rosen (1974) between a household type's marginal willingness to pay for an amenity and movement along the bid-function envelope, which involves a change in household type. I show how to separate these two effects and to test the sorting theorem. This approach can accommodate cases in which an “amenity” has positive value for some households and negative value for others. In addition, I show that some studies make inconsistent assumptions about the forms of the envelope and of the underlying bid functions.

The second part of the paper estimates this new approach using all house sales in the Cleveland area in 2000. The methods developed here, combined with extensive controls for housing and neighborhood traits, yield estimates of the price elasticity of amenity demand and of the sorting parameters, plus support for the sorting theorem, for a school district’s high school performance and for a neighborhood’s ethnic composition. Bidding and sorting are more difficult to separate for several other school performance measures.

2. Preview of the Literature

This section introduces the literature; a more complete review and comparisons with my approach appear in Section 4. The core studies model how households bid for housing across communities with various levels of public services and property taxes and then sort into communities (Ellickson 1971; Epple et al. 1984; Henderson 1977; and Wheaton 1993). This literature, reviewed in Ross and Yinger (1999), predicts that property taxes and public services will be capitalized into the price of housing. A large empirical literature on capitalization inspired by Oates (1969) has also appeared. Recent contributions on public service capitalization include Bayer et al. (2007), Black (1999), Brasington (2002, 2007); Brasington and Haurin (2006), Clapp et al. (2008), and Kane et al. (2006). Studies on tax capitalization are reviewed in Yinger et al. (1988) and Ross and Yinger (1999).

A related literature on “hedonics” estimates the impact of product attributes on product prices. Housing attributes include features of the house’s location, such as school quality. This literature, reviewed in Sheppard (1999) and Taylor (2008), goes beyond estimating the impact of public services on house values to exploring the underlying demand for these services. Many studies follow the Rosen (1974) two-step procedure: (1) regress product price on product attributes (the hedonic) and (2) find implicit prices of attributes by differentiating the hedonic and estimate household demand for each attribute as a function of its implicit price and other things. In the Rosen framework, a household's marginal willingness to pay for the amenity or MWTP equals the implicit price at the level of the amenity it receives. Because this MWTP applies only to marginal changes in the amenity, however, it is of limited usefulness in welfare analysis; examples in Rosen show that welfare analysis generally requires the demand estimates from his second step.

Epple (1987) explains that with a nonlinear hedonic equation the implicit price depends on the quantity of an attribute consumed by the household and therefore is endogenous. Epple et al. (2010) and Heckman et al. (2010) provide solutions to this problem.[1] This paper builds on standard bidding and sorting models to derive a hedonic equation that allows direct estimation of the service/amenity price elasticities without a second step.

The Alonso/Mills/Muth model of urban residential structure (reviewed in Brueckner 1987), predicts that commuting costs are reflected in housing prices. Polinsky and Shavell (1976) introduce an exogenous amenity such as air pollution into an urban model, and many studies, including Cameron (2006) and Neill et al. (2007), estimate the impact of pollution on property values. Yinger (1976) develops an urban model with neighborhood ethnic composition as an endogenous amenity; empirical studies of this topic include Bayer et al. (2007) and Zabel (2008).

3. The Theory of Bidding and Sorting

This section derives bid-functions with constant elasticity demands, develops a new method to account for household heterogeneity and sorting, and incorporates the results into a house-value regression. The standard model assumes that households maximize utility over a continuous public service quality or amenity, *S*, housing services, *H*, and a composite good, *Z*, with a price of unity. Households bid for housing based on *S* and the effective property tax rate, τ, and households with different incomes and preferences sort into different jurisdictions. Households are assumed to be mobile, so a key equilibrium condition is that all households in an income-taste class achieve the same utility. Households live in an urban area with many local governments financed by a property tax. Everyone who lives in a given jurisdiction receives the same *S*, and the only way to gain access to the *S* in a jurisdiction is to live there. All households are homebuyers, but, depending on assumptions about property tax incidence, this model can be applied to renters, as well. This model can also be extended to multiple public services and neighborhood amenities. The household budget constraint sets household income, *Y*, equal to *Z* plus housing consumption, *PH*, where *P* is the price per unit of *H*, plus property taxes. A household's property tax payment is τ multiplied by its house value, $V = PH/r$, where *r* is a discount rate (and $\tau^* = \tau/r$), so

$$Y = Z + PH + \tau V = Z + PH\left(1 + \frac{\tau}{r}\right) = Z + PH(1+\tau^*)\ . \qquad (1)$$

3.1 Bidding

A straightforward way to derive housing bids is to determine the maximum amount a household would pay per unit of *H* in different locations, holding utility constant (Wheaton 1993). Solving (1) for *P*, this approach leads to the following maximization problem:

$$\underset{H,Z}{\text{Maximize}} \quad P = \frac{Y - Z}{H(1+\tau^*)}, \ \text{subject to } U\{Z, H, S\} = U^0\ , \qquad (2)$$

where U^0 is the utility level obtained by households in this income-taste class and *S* and τ are parameters. Applying the envelope theorem, with subscripts to indicate partial derivatives, yields

$$P_S = \frac{U_S / U_Z}{H(1+\tau^*)} = \frac{MB_S}{H(1+\tau^*)} \qquad (3)$$

$$P_\tau = -\frac{P}{(r+\tau)} = -\frac{P}{r(1+\tau^*)} \ , \tag{4}$$

The numerator of (3) is the marginal rate of substitution between *S* and *Z*, which equals the marginal benefit from *S*, MB_S, because each unit of *Z* costs $1.

The differential equation (4) can be solved using the initial condition that the before-tax price, $\hat{P}$, which depends on *S*, equals the after-tax price, *P*, when τ equals zero. The solution is

$$P\{S,\tau\} = \frac{\hat{P}\{S\}}{(1+\tau^*)} \ . \tag{5}$$

Differentiating (5) with respect to *S* yields another helpful result:

$$P_S = \frac{\hat{P}_S}{(1+\tau^*)} \ . \tag{6}$$

In this context, the demands for *S* and *H* are not observed directly, but they are comparable to other demands and can be expressed in the usual way. More specifically, let us assume that the latent demands for *S* and *H* take the well-known constant-elasticity form. First,

$$S = K_S N^{\delta} Y^{\theta} W^{\mu} e^{\varepsilon_S} \ , \tag{7}$$

where *W* equals price (or tax price), K_S is a constant, *N* is a set of observable factors that influence the demand for *S*, and ε_S is a random error. Second,

$$H = K_H M^{\rho} Y^{\gamma} \left(P(1+\tau^*)\right)^{\nu} e^{\varepsilon_H} = K_H M^{\rho} Y^{\gamma} \hat{P}^{\nu} e^{\varepsilon_H} \ , \tag{8}$$

where K_H is a constant, *M* is factors that influence housing demand, and ε_H is a random error. The presence of these random errors raises theoretical and empirical issues to which we will return.

LaFrance (1986) derives these forms from a model of "incomplete" demand, in which one set of commodities (*Z*) is not observed but affects the observed commodities (*S* and *H*) through a price index, which appears in *N* and *M*. More specifically, LaFrance shows that these forms are consistent with the integrability requirements of a demand system if observed commodities all have the same income elasticity of demand and cross-price elasticities equal zero.[2] In the case of (7) and (8), these conditions require that $\theta = \gamma$, that W not be an element of M, and that $\hat{P}$ not be an element of N. Under these conditions, "there is sufficient information embodied in the structure of the quasi-expenditure, indirect utility, and direct utility functions to permit exact welfare measures for changes in income and prices of the commodities of interest" (p. 551). In other words, parameter estimates based on these assumptions can be used in benefit-cost analysis.

As discussed in Section 3.2, these conditions are not as restrictive as they first appear. Indirect dependence across amenity demands can be introduced, and most parametric estimating equations in the literature emerge as special cases of the form based on these conditions. Moreover, several studies, including Duncombe and Yinger (2011), find that (7) works well for estimating community-level service demand equations, and (8) has been widely used to study housing demand (Zabel 2004).

The inverse demand function associated with (7) is:

$$W = \left(\frac{S}{K_S N^{\delta} Y^{\theta} e^{\varepsilon_S}} \right)^{1/\mu} \equiv MB_S \ . \tag{9}$$

Combining (3), (6), (8), and (9) yields

$$\hat{P}_S \hat{P}^{\nu} = \frac{S^{1/\mu}}{\left(K_S N^{\delta} e^{\varepsilon_S} \right)^{1/\mu} K_H M^{\rho} e^{\varepsilon_H} Y^{(\theta/\mu)+\gamma}} = \psi S^{1/\mu} \ , \tag{10}$$

where

$$\psi = \left(\left(K_S N^{\delta} e^{\varepsilon_S} \right)^{1/\mu} K_H M^{\rho} e^{\varepsilon_H} Y^{(\theta/\mu)+\gamma} \right)^{-1} . \tag{11}$$

With C as a constant of integration, (10) can be solved for the bid function:

$$\hat{P}\{S\} = \left((1+\nu)\left(C + \left(\frac{\psi\mu}{1+\mu} \right) S^{(1+\mu)/\mu} \right) \right)^{1/(1+\nu)} . \tag{12}$$

This equation cannot be estimated for a given value of ψ because only the tangency point with the envelope is observed. Of course, one can regress housing prices on observable demand characteristics interacted with *S*, but these three types of variables are simultaneous outcomes of sorting, and this regression will yield biased estimates of the parameters in (12).

3.2 Sorting

According to the standard model, household types sort into locations based on the slopes of their bid functions for *S*. As shown by Ellickson (1971), Henderson (1977), Yinger (1982, 1995), Epple et al. (1984), and Wheaton (1993), household types with steeper bid functions win the competition for housing where *S* is higher. These studies (and this one) all rely on the single-crossing assumption: if a household type has a steeper bid function than another at one value of *S*, it also has a steeper bid function at other values of *S*. The slope of a household's bid function is the derivative of $\hat{P}$ with respect to *S*, which is given by (10). This slope is the household's MWTP for *S*. A household's relative slope is determined by the variables in (10) that are unique to that household, that is, everything except for *S* and $\hat{P}$. These terms are collected in ψ as defined by (11). It follows that ψ can be interpreted as in index of the steepness of a household's bid function (or of its MWTP) and hence an index of the value of *S* into which the household sorts.

A model of sorting must allocate heterogeneous households, defined by ψ, to locations with different amenity levels, defined by *S*. In other words, the equilibrium can be characterized by the relationship between ψ and *S*. This paper derives the hedonic envelope based on the assumption that this relationship takes the following form:

$$S = (\sigma_1 + \sigma_2\psi)^{\sigma_3} , \tag{13}$$

where the σs are constants. I use this form for four reasons: (a) it is a polynomial form and its parameters can be estimated so it can approximate any equilibrium; (b) it has relatively few parameters, which facilitates estimation; (c) it leads to tractable envelope derivations; and (d) it is consistent with a wide range of assumptions about the distributions of S and ψ. Before turning to the derivation of an envelope with the σs as parameters, let us consider reason (d).

Many distributional assumptions that lead to (13) can be identified in the case of one-to-one matching, which is defined as a sorting equilibrium in which each household type has a unique value for S. In this context, a household type is a set of households with the same ψ. This definition allows for considerable within-type heterogeneity. As in Epple and Platt (1998), for example, a high-income household with a weak preference for S based on unobservable factors might have the same ψ as a low-income household with a strong preference for S. With one-to-one matching, these two households sort into the same location. The advantage of a model based on one-to-one matching is that, under many circumstances, it provides a reasonable simplification of sorting and leads to a specific form for the hedonic. This approach cannot be applied to discrete amenities, however, and it may not provide good approximations when households with widely different values of ψ share the same amenity level.[3] Alternative approaches are discussed in Section 4.

The path from one-to-one matching to (13) begins with the following theorem, which is proved in Yinger (2014): If the sorting equilibrium is characterized by one-to-one matching and if the distribution of a monotonically increasing transformation of S equals the distribution of a (possibly different) monotonically increasing transformation of ψ, then in equilibrium the relationship between S and ψ depends only on the transformations, not on the post-transformation distributions. It is well-known, for example, that a linear transformation can convert one normal distribution into another. If S and ψ both have normal distributions, therefore, then a linear transformation will convert one of these distributions into the other, and this theorem indicates that, with one-to-one matching, the equilibrium relationship between *S* and ψ is linear, which means that it equals (13) with $\sigma_3 = 1$.[4] Yinger also shows that with one-to-one matching many other assumptions about the distributions of *S* and ψ, including cases in which these distributions are not the same, lead to (13). A standard uniform distribution for *S* and a beta distribution for ψ, for example, or an exponential distribution for *S* and a Weibull distribution for ψ, lead to an equilibrium with $\sigma_1 = 0$, $\sigma_2 = 1$ $\sigma_3 > 0$.

This analysis applies to the overall distributions of *S* and ψ, regardless of how these distributions are determined. In some cases, however, a more formal derivation of these distributions is possible. Suppose, for example, that after taking logs, the set of variables on the right side of (7) and the set on the right side of (11), including the εs, each have a multivariate normal distribution. Because a linear combination of the variables in a multivariate normal distribution is also normal, even if the the εs or other variables are correlated, these assumptions imply that $\log\{S\}$ and $\log\{\psi\}$ have normal distributions, as well. Thus, the Yinger theorem indicates that, for the appropriate *a* and *b*, $\log\{S\} = a + b(\log\{\psi\})$. Rearranging the exponential of this equation leads to an equation equivalent to (13) with $\sigma_1 = 0$.

In sum, the assumption of one-to-one matching is neither necessary nor sufficient for (13) to hold, but this assumption leads to many equilibria in the class that (13) describes. These equilibria include cases in which the assumed distribution of *S* can be transformed into that of ψ (or vice versa) using the algebraic form in (13). They also include cases in which this type of transformation can be applied to distributions of *S* and ψ that are derived from assumptions about the distributions of their underlying components.

Now let us turn to the derivation of an envelope based on (12) and (13). In standard notation, an envelope for a one-parameter family of functions must satisfy $F\{\alpha, x, y\} = 0$, which is the family in implicit form, and $\partial F/\partial\alpha = 0$, where α is the parameter. In this paper, $\alpha = \psi$, $y = P$, and $x = S$. Equation (12) includes these three terms but does not have an envelope as written; if all bid functions have the same constant, C, then they never cross. Figure 1 shows how to make C a function of ψ and hence to enable the derivation of an envelope. Two household types whose bid functions cross at $S = S^*$ have different values of ψ and hence different bid-function slopes, but, by the definition of "cross," they also have the same bid, $\hat{P}$, at S^*. The bid function with the steeper slope must have a smaller C. The derivation of an envelope therefore involves finding a formula for C such that $d\hat{P}/d\psi = 0$ when S is held constant at S^*. Applying this condition to (12) yields

$$\left.\frac{dC}{d\psi}\right|_{S=S^*} = \frac{-\mu}{1+\mu} S^{*\frac{1+\mu}{\mu}} . \tag{14}$$

Now $C\{\psi\}$ can be found by substituting (13) into (14) and integrating:

$$C = C_0 - \left(\frac{\mu}{1+\mu}\right)\left(\frac{\mu}{\sigma_3(1+\mu)+\mu}\right)\left(\frac{(\sigma_1+\sigma_2\psi)^{\frac{\sigma_3(1+\mu)+\mu}{\mu}}}{\sigma_2}\right), \tag{15}$$

where C_0 is a constant of integration. Substituting (15) into (12) yields an F function with an envelope. One can easily verify that $\partial F/\partial\alpha \equiv \partial F/\partial\psi$ is given by (13) so it can be solved for ψ and combined with this F function to find the bid-function envelope, identified with superscript E:[5]

$$\hat{P}^E\{S\} = \left((\nu+1)\left(C_0 - \left(\frac{\sigma_1}{\sigma_2}\right)\left(\frac{\mu}{1+\mu}\right)S^{\left(\frac{1+\mu}{\mu}\right)} + \left(\frac{1}{\sigma_2}\right)\left(\frac{1}{\frac{1+\mu}{\mu}+\frac{1}{\sigma_3}}\right)S^{\left(\frac{1+\mu}{\mu}+\frac{1}{\sigma_3}\right)}\right)\right)^{1/(\nu+1)} . \tag{16}$$

Equation (16) shows that the impact of S on housing prices depends on a household's willingness to pay for S, as indicated by μ, and on sorting, as measured by the σs. Moreover, estimates of μ and the σ parameters in (16) identify these two effects and, with the help of (13), lead to a test of the sorting theorem that $\partial S/\partial \psi > 0$. Because (16) is a market relationship, not a bid function, the standard endogeneity problem does not affect an estimate of μ.[6]

Further explanation is provided by Figure 2, which builds on figures in Rosen (1974). Illustrative bid functions and their envelope appear in the top panel; their slopes are plotted in the bottom panel. These bid functions apply to an amenity, *S*, with a fixed distribution, so firms' offer functions are not needed. The slope of a bid function is the associated household's MWTP for *S*. The slope of the envelope reflects both the decline in a household's MWTP as *S* increases and the increase in MWTP as the sorting process allocates people with steeper bid functions to higher-*S* locations. In Figure 2, the first effect is illustrated by the movement down a bid function on the dotted line, and the second effect is illustrated by the vertical shift from one bid function to another. This vertical shift is ψ; its positive value corresponds to the standard sorting theorem. The $\psi\{S\}$ function, which is the inverse of $S\{\psi\}$ (e.g. equation (13)), indicates how ψ changes as *S* increases. In other words, it indicates how much a household's relative MWTP must increase for it to gain access, through sorting, to a higher level of *S*.

After adjusting the constant, (16) can be re-written using Box-Cox forms:

$$\left(\hat{P}^E\right)^{(\lambda_1)} = C_0' - \frac{\sigma_1}{\sigma_2} S^{(\lambda_2)} + \frac{1}{\sigma_2} S^{(\lambda_3)} , \qquad (17)$$

where $\lambda_1 = 1 + \nu$; $\lambda_2 = (1+\mu)/\mu$; $\lambda_3 = \lambda_2 + 1/\sigma_3$; $X^{(\lambda)} = (X^{\lambda} - 1)/\lambda$ if $\lambda \neq 0$; and $X^{(\lambda)} = \log\{X\}$ if $\lambda = 0$. Most functional forms used for hedonic estimation (including linear, semi-log, log-linear, Box-Cox, and quadratic) are special cases of (17). In other words, these forms are based on stronger assumptions than the ones used here

A key insight from (17) is that specifications consistent with sorting generally require two terms on the right side. Consider the one-term specification that arises when $\sigma_3 = \infty$. In this case the two terms on the right side of (17) have the same exponent and collapse into a single term. However, sorting is an outcome in which S varies systematically with ψ. If $\sigma_3 = \infty$, raising both sides of (13) to the $1/\sigma_3$ power reduces the left side to 1, which indicates that S is not related to ψ.[7] A one-term specification consistent with sorting can arise only if $\sigma_1 = 0$, that is, if, in equilibrium, S is proportional to some power of ψ. The coefficient of the first term in (17) tests this possibility.[8]

Equation (17) also reveals the implicit assumptions behind many common hedonic specifications. Linear and quadratic forms, for example, implicitly assume $\mu = -\infty$, which indicates a horizontal demand curve; a linear form also assumes $\sigma_3 = \infty$, whereas a quadratic form is based on a linear sorting equilibrium ($\sigma_3 = 1$). The flexible Box-Cox form introduced into hedonics by Goodman (1978) also implicitly assumes that $\sigma_3 = \infty$, and, ignoring across-amenity interactions, Halvorsen and Pollakowski's (1981) quadratic Box-Cox has two terms for each amenity with exponents λ and 2λ. Equating these exponents with those in (17) leads to one possible interpretation of results from this form, namely, that $\mu = 1/(\lambda - 1)$ and $\sigma_3 = 1/\lambda$. Finally, a semi-log form, which is "[p]erhaps the most commonly used specification" (Taylor 2008, p. 20), implicitly assumes that $\nu = \mu = -1$ and $\sigma_3 = \infty$.

This approach can be generalized to many public services and neighborhood amenities (henceforth "amenities"). The LaFrance (1986) derivation of constant elasticity demands applies to any number of amenities and requires the cross-price elasticities of amenity demand to be zero. With this assumption the bid function for S_i does not depend on any S_j, $j \neq i$, or on its price. Thus, the envelope for S_j can be incorporated into (16) through the constant of integration, C_0, and the multi-S envelope simply sums the right side of (16) or (17) across different Ss.

Although S_j cannot appear in the bid function for S_i, an indirect form of interaction across Ss may arise through the community-level determinants of S. Households may demand more S_i , for example, in a community that has relatively low costs for producing S_j, and is therefore more likely to select a high value of S_j. So long as the impact of these community traits is the same across households,[9] the systematic determinants of S_j can be introduced as a multiplicative term in (7) for S_i. Variation in these determinants therefore leads to different bids across locations with the same S_i but does not affect sorting.

An extension to many amenities must also assume that the table of values is full enough that each amenity can be selected independently. A high correlation between S_i and S_j, for example, might force households to select from a limited number of (S_i, S_j) pairs, so that the marginal conditions derived here do not hold. In this case, S_i and S_j can be combined into a single index, a procedure I follow by averaging passing rates across various state tests (see Section 5.2).

A neighborhood's ethnic composition might influence house values for at least three reasons: ethnic prejudice, a correlation with unobserved neighborhood traits, and ethnic discrimination (Zabel 2008). As explained in detail in Yinger (In Press), these possibilities can be incorporated into the approach in this paper by treating ethnic composition as a neighborhood amenity and adding control variables for locations in and near Cleveland's long-standing black ghetto, defined as a set of contiguous neighborhoods at least 80 percent black. The final specification recognizes that preferences for neighborhood ethnic composition vary widely; some people prefer largely black neighborhoods, for example, whereas others prefer neighborhoods that are largely white. In this case, the envelope for percent minority is U-shaped. Now define S^M as the minimum of the envelope and $(S - S^M)^2$ as the amenity. With this approach, implemented in Section 6, S^M is a parameter to be estimated.

A standard urban model specifies housing bids as a function of distance to a central worksite (Brueckner 1987), and households sort according to the slopes of their bid functions. One key extension of the standard model is to multiple worksites (White 1999). Thus, the regressions include average employment-weighted distance to major worksites, u, as well as u^2.

3.3 Estimating Equation

Following (1) and (5) we can now bring in τ and H, which is a function of structural housing characteristics, X. If β is the degree of property tax capitalization, then

$$V = \frac{P^E H\{X\}}{r} = \frac{\hat{P}^E\{S_1, S_2, ..., S_n, u\}H\{X\}}{(r + \beta\tau)} , \tag{18}$$

where $\hat{P}^E$ reflects (17) and the comparable terms for u. I specify $H\{X\}$ as multiplicative; Sieg et al. (2002) show that this form is consistent with the assumption used here that V is the product of P (a function of locational characteristics) and H (a function of the Xs). [10]

Equations (18) and (11) define a two-step procedure. Estimates of μ and the σs for each amenity can be obtained by estimating (18) (after inserting (17)). These estimates make it possible to solve (13) for ψ and to estimate (11) in log form. This procedure differs from Rosen's (1974) because it obtains μ from the first step and because the coefficient of Y in the second step is $-(\theta/\mu + \gamma)$, not θ. Henderson (1977) shows that "normal" sorting, defined as an equilibrium in which higher-income households live in locations with higher S, requires $(\theta/\mu + \gamma) < 0$, so the coefficient of Y in (11) provides a test (implemented in Section 6.4) of normal sorting.[11]

4. Comparison with Previous Approaches

Scholars face three major challenges when estimating amenity demand or studying sorting. The first challenge, discussed in Section 2, is the endogeneity of the implicit prices in Rosen's (1974) two-step procedure. The most obvious response is to use instrumental variables. As recent reviews (Sheppard 1999; Taylor 2008) indicate, however, no widely applicable instruments have emerged.[12] As a result, scholars have developed a variety of alternative approaches.[13]

Bajari and Benkard (2005) and Bajari and Kahn (2005) assume that that utility is linear in Z

and log$\{S\}$. This form implies that μ = -1, so (3) and (9) yield a second-step with $P_S S$ on the left and nothing endogenous on the right. Although it is convenient, this approach cannot estimate μ, which is, of course, is a key parameter for determining willingness to pay.

Epple et al. (2010) avoid the endogeneity problem by estimating a semi-parametric hedonic derived from a general equilibrium model in which households, which differ based on income and a taste parameter, sort across locations (i.e. amenity levels). This approach has the advantage that it can account for cases of inexact sorting in which one household type lives in multiple locations or in which multiple household types share a single location. It is not well suited to a housing market with many amenities, however, because it assumes households value a single amenity index. Suppose the index is $A = \alpha_1 A_1 + \alpha_2 A_2$. Then $100/\alpha_1$ units of A_1 and no units of A_2 yield the same utility as $100/\alpha_2$ units of A_2 and no units of A_1—or as an appropriately weighted mix of the two.[14]

Another approach is to identify the marginal utility function for the amenity based on the difference in the curvatures of the hedonic and bid functions (Heckman, et al. 2010). This approach has the advantage that it allows "the curvature of the marginal utility for the product attribute, as well as the distribution of the marginal utilities, to vary in general ways across agents with different observed characteristics" (p. 1570). This approach also makes use of an amenity index, however, and is therefore not well suited for housing market applications.

Although it is not based on a hedonic regression, the discrete-choice approach of Bayer et al. (2007) provides another way to shed light on amenity demand. These authors develop a model of sorting in which the choice of a house depends on the traits of the house, the neighborhood, and the household. This model does not limit the number of amenities and can accommodate inexact sorting. A drawback of this model is that it assumes linear utility functions, which lead to linear bid functions and thus, as shown by (12), to the implicit assumption that $\mu = -\infty$.[15]

The semi- or non-parametric procedures used by Bajari and Kahn (2005), Epple et al. (2010), and Heckman et al. (2010) may provide close approximations to the hedonic envelope, but studies estimating parametric forms face a second challenge, namely, to account for the mathematical connection between the hedonic and the amenity demand functions. To be specific, the use of a flexible form for the first-step hedonic and a specific form for the second-step amenity demand functions cannot separate bidding and sorting without relying on implicit assumptions about the nature of the sorting equilibrium, which may be stronger than the explicit assumptions used here. Equation (17) shows, for example, that a Box-Cox form, which has a common exponent and one term for each amenity, assumes that all amenities have the same μ and rules out sorting. Linear and log forms rule out sorting, as well. Moreover, the implicit amenity demand assumptions in the first-step form may be logically inconsistent with the demand functions in the second step. A semi-log form, for example, implicitly assumes that $\mu = -\infty$, so it is inconsistent to estimate μ in the second step based on implicit prices derived from this form. These conclusions are based, of course, on (7), (8), and (13), but similar inconsistencies could arise with other specifications.

Equation (17) provides an alternative to these approaches by deriving a form for the hedonic based on constant elasticity demand functions for housing and the amenities, the standard sorting theorem, and an equation for the sorting equilibrium that is consistent with many assumptions about the distributions of S and ψ. This approach avoids the endogeneity problem discussed above, ensures consistency between the estimated hedonic and the underlying amenity demands, is not restricted to a single amenity index, and is consistent with basic utility postulates.[16] This approach is applicable to continuously measured amenities, as long as a single location (= amenity level) is not inhabited by households with a wide range of bid-function slopes.

The third challenge is to account for possible omitted variable bias in the first-step hedonic regression. Many strategies, including instrumental variables and various fixed effects, are available in some cases (Parmeter and Pope 2013; Nguyen-Hoang and Yinger 2011), but no study has shown how to adapt these strategies for a key objective of this study, namely, to estimate the impact of multiple services or amenities at the school district level.

One strategy pioneered by Black (1999), for example, is to define border fixed effects for elementary school attendance zone boundaries. These fixed effects capture all across-district variation in school quality, however, so they make it impossible to estimate the impact of school-district quality on house values. The distinction between elementary and district quality may be important; parents may not be as concerned about a low-scoring elementary school if their children will eventually attend a high-scoring middle school and high school. Another common strategy is to estimate the impact of changes in amenities on changes in house values. In their application of this method, however, Figlio and Lucas (2004) explain that the estimated coefficients from this approach cannot be interpreted as measures of willingness to pay because they may reflect changes in household types due to re-sorting, not just changes in bids for a given household type.

An alternative strategy, which is followed here, is to greatly expand the set of controls for neighborhood level amenities, such as parks and lake views, and thereby to minimize the possibility that key variables are omitted. One check on the success of this strategy, which is implemented in Section 6.1, is to replace all the school-district level variables with district fixed effects; if this change has little impact on the explanatory power of the regression, we can conclude that unobserved school-district traits are not a source of bias.

Some neighborhood controls may not be appropriate. Black (1999), Chay and Greenstone (2005), Kane et al. (2006), Bayer et al. (2007), and Clapp et al. (2008), include neighborhood “demographic” controls, such as average income, in their hedonic regressions. Because of sorting, these controls serve as proxies for household demand traits, particularly if the neighborhoods are as small as CBGs.[17] Consequently, this approach introduces variables in the amenity demand function into the estimation of the hedonic. As Rosen and many other scholars make clear, however, the inclusion of demand variables transforms the hedonic into a family of bid functions.

Chay and Greenstone and Bayer et al. argue that people care about the demographic character of a neighborhood, so these variables are simply additional amenities. Even if neighborhood demographics serve as amenities, however, they are still demand variables, and adding them transforms the regression from an envelope, (17), to set of bid functions, (12). This distinction is analogous to the estimation of long-run and short-run average cost curves. When plant size (the analog to a demand variable) is included, the regressions yield short-run cost curves, not their long-run envelope (see, e.g., Keeler 1974). Estimation of bid functions is fine in principle, but flounders in practice on the hedonic endogeneity problem: Housing prices and households' choices of amenity levels are simultaneously determined. Thus, one can either estimate an envelope without demand variables, which might be subject to omitted variable bias, or estimate a bid-function regression with these variables, which is subject to endogeneity bias.

To preserve the envelope interpretation while still accounting for income-related amenities, I exclude income and education from my regressions but include many neighborhood traits that potential home buyers may interpret as a sign of neighborhood income, such as proximity to a golf course or to a public housing project. These traits are more readily observed by potential buyers than are residents' incomes and education. As noted in Section 3.2, measured of ethnicity are included. The ethnicity of potential neighbors and the location of restaurants, churches, and other institutions linked to ethnicity often can be observed. Moreover, ethnicity may not affect housing demand (or cause endogeneity bias) once other factors are accounted for, as they are in the derivation of (17). Goodman's (2002), for example, finds no significant difference in housing demand between blacks and whites, all else equal. I assume that the Goodman result also applies to amenity demand, but further research on this topic would be valuable.

5. Estimation Procedures

5.1 Econometric Model

Although (18) could be estimated directly, I estimate it in two stages. First, I estimate (18) in log-linear form using the sample of house sales in neighborhoods (defined as census block groups, CBGs) with at least two sales.[18] The dependent variable is the log of sales price and the explanatory variables are CBG fixed effects, D; housing characteristics, X; and within-neighborhood differences in locational traits, Q. Houses are indexed by i and neighborhoods by n; C is a constant, ε is a random error; and superscripts indicate the regression stage. Hence,

$$\ln\{V_i\} = C^1 + \sum_n \alpha_n D_{in} + \sum_m \eta_m X_{im} + \sum_l \zeta_l Q_{il} + \varepsilon_i^1 \qquad (19)$$

The coefficient of a fixed effect, α_n, indicates the impact on house values of all the neighborhood traits shared by houses in CBG n, whether observed or not. In the second stage, I estimate (17) for the sample of CBGs using the values of α_n as the dependent variable and a wide range of amenities, S_{nj}, as explanatory variables. Some amenities, such as those measured with a dummy variable, do not fit this form; appropriate specifications for these amenities, labeled $\tilde{S}_{nk}$, are given in Section 5.2. This approach is equivalent to using log$\{P^E\}$ as the dependent variable, which, according to (17), assumes that ν = -1. This assumption is tested in Section 6.1.

$$\alpha_n = C^2 + \sum_j \left(-\left(\frac{\sigma_{1j}}{\sigma_{2j}}\right)\left(S_{nj}\right)^{(1+1/\mu_j)} + \left(\frac{1}{\sigma_{2j}}\right)\left(S_{nj}\right)^{(1+1/\mu_j+1/\sigma_{3j})} \right) + \sum_k f_k\left\{\tilde{S}_{nk}\right\} + \beta\tau_n^* + \varepsilon_n^2 \qquad (20)$$

Because border fixed effects are not appropriate here, the possibility of omitted variable bias is minimized by collecting extensive information on the Xs and Qs in equation (19) and on the Ss and the $\tilde{S}$s in equation (20).[19] Equation (20) is estimated using nonlinear least squares. Robust standard errors are calculated using the method recommended by Davidson and MacKinnon (2004, p. 239) for models with heteroskedasticity.[20]

This two-stage approach, which corresponds to Rosen's first step, has two advantages. First, it prevents omitted variable bias in the first-stage coefficients of housing traits due to the correlation between these traits and unobserved neighborhood traits. Extensive controls are still needed, of course, to minimize bias from omitted housing traits. Second, it facilitates consideration of many functional forms for the bid-function envelope. Epple et al. (2010) use an equivalent procedure, although their second stage has a different specification.[21]

5.2 Data and Variable Definitions

This study builds on the data set described in Brasington (2007) and Brasington and Haurin (2006), which consists of all the house sales in Ohio in 2000, and, in particular, on the sub-sample for the Cleveland MSA. This data set indicates sales price, housing traits, housing location, census block group characteristics, school performance, crime rate, and air quality. These data are supplemented with many additional school and neighborhood characteristics.[22]

The explanatory variables in the first-stage regression include seventeen housing traits and nine distance-based variables that vary within a CBG: distance to worksites, to the nearest public elementary school, to the nearest private school, to the nearest environmental hazard, to Lake Erie, to the black ghetto, to the Cleveland airport, to the center of the CBG, and to a high-crime location. Houses close to an airport might be more valuable due to convenient access or less valuable due to noise. Houses located on the outskirts of a neighborhood might be less valuable than those in the center, approximated by the center of a CBG. Because CBG fixed effects are included, these variables are specified to measure only within-CBG variation, which is the difference between the distance to the house and the distance to the center of the CBG. The first-stage regression also controls for differences between a house and the center of the CBG in test scores of the nearest elementary school and in location within a historic district or near public housing projects.

The commuting variable is average employment-weighted distance to the Cleveland area's five major worksites. A worksite is defined as a set of zip codes each with at least 5,000 jobs that are within 6 miles of a central point, contain at least one zip code with 25,000 jobs, and contain at least 45,000 total jobs. Because many jobs are located near a beltway around Cleveland, one worksite is defined as a ring 7 to 11 miles from the center of the downtown site. The radius of the beltway, 8.59 miles, was selected to minimize the squared employment-weighted distance to the associated zip codes. The center of other sites was set by concentrations of business buildings (identified using satellite images on the U.S. Geological Survey web site) near the job-weighted centroid of the zip-code cluster. These five worksites accounted for 75.8 percent of the jobs in the Cleveland area in 2000. Each CBG was assigned to the closest worksite consistent with the requirement that the share of workers assigned to each site (based on CBG data) equals the share of employment located at that site (based on zip code data). Each worksite dummy was then interacted with the difference between house and CBG job-weighted distance to worksites.

Variable definitions and results for the basic hedonic are presented in Table 1. The hedonic is estimated with the "areg" option in Stata using 22,880 observations. The R-squared is 0.7893. The housing characteristics are, except for "One Story," significant with the expected sign. The set of geographic fixed effects is also highly significant. The commuting variables are all significant with similar coefficients and the expected negative sign. Additional small but significant impacts on housing prices come from the distance from a private school (negative), from an environmental hazard (positive), from Lake Erie (negative), from a black ghetto (negative), from the center of the

CBG (negative), and from a high-crime location (positive). In addition, prices are higher near small family public housing projects but lower near large ones.

The second-stage equation is the bid-function envelope. The dependent variable is the set of coefficients for the CBG fixed effects and the number of observations, 1,665, is the number of CBGs with at least two house sales. In unlogged form with the first-stage constant term included, this variable has a mean of 84,836 and a standard deviation of 23,331 (Table 2).

The key amenity variables (Table 2) are the test scores of the neighborhood elementary school relative to the district average (Elementary), the district's passing rate on twelfth grade tests (High School), a measure of a district's elementary value added (Value Added), the share of minority teachers in the district (Minority Teachers), and two measures of neighborhood ethnic composition (Share Non-Black and Share Non-Hispanic). Results are similar using minority shares. Test scores are measured by the passing rate on all five examinations (mathematics, reading, writing, science, and citizenship) in the state's accountability system. Elementary, which refers to fourth grade scores, is the difference between the passing rate in the nearest elementary school and the average score in the district. High School is the share of students who enter the twelfth grade who pass all the state's twelfth grade tests. This variable equals the passing rate on the tests, which reflects only those students who do not drop out, multiplied by the graduation rate, which indicates the share of students who stay in school. Value Added is a school district's sixth grade passing rate in 2000-2001 minus its fourth grade passing rate in 1998-99. Thus, it is a test score change for a given cohort, ignoring movement in or out of the cohort. Minority Teachers is the share of a district's teachers who belong to a minority group. The average value for Elementary is 0.3148 and the average value for High School is 0.320. The minimum value for High School is very low; under 5 percent of students starting twelfth grade in Cleveland pass all five state exams.

In the best districts, over three-quarters of entering students reach this standard. Value Added, which is adjusted to be always positive, ranges from 1 to 50 percent, and Minority Teachers ranges from 0 to 61 percent. The average CBG was 80 percent non-black and 96 percent non-Hispanic.

Cleveland and its neighbor, East Cleveland, are unlikely to exhibit the same relationship between school quality and housing prices as other districts. Starting in 1996-97, children in the Cleveland district, but not in any other district in the area, became eligible for vouchers to help pay private tuition (U.S. Government Accountability Office 2001). Vouchers were granted by lottery, with priority given to families with incomes below 200 percent of the poverty line. For these families, the voucher equaled the lesser of 90 percent of tuition and $2,250. First-time recipients had to be in a grade from kindergarten to third, and recipients were guaranteed vouchers through eighth grade. At the end of 1999-2000, 3,400 students were receiving vouchers, which they used in 52 private schools. In addition, charter schools (called community schools) were authorized in Ohio in 1997. By 2001 Cleveland had 7 elementary charter schools (Center for Education Reform 2012), all of which were within two miles (and three were within 0.62 miles) of a CBG with elementary passing rates more than 10 points below the district average. Only one charter school had opened outside Cleveland. Unlike any other district in the area, East Cleveland received a huge state grant for school construction in 1998-2000 (Ohio Schools Facility Commission 2000). The total grant was almost $96 million, compared to annual own-source revenue of about $36 million. In short, housing prices in these two districts may be higher than expected at a given measured school quality. Moreover, the link between vouchers and income, the placement of charter schools near low-performing schools, and the likely focus of construction on the most distressed schools may alter the link between housing prices and Elementary in these districts. Thus, I include a fixed effect for these two districts and estimate a separate Elementary envelope there.[23] This fixed effect also accounts for the possibility that the form derived in this paper not be a good approximation in locations where an amenity is shared by widely heterogeneous households.

Tax variables are defined in Table 3. Four school districts levy an income tax with a rate of 0.75 or 1.0 percent. Ohio requires assessment at market value, but some slippage exists and nominal property rates are translated into effective rates using assessment-sales data.[24] The average effective rate is 3.1 percent. See Table 2. Most school districts coincide with cities. The regressions also include city property tax rates (net of school taxes) and the reduction in this rate due to exemptions. Indicator variables account for school districts without assessment-sales data, 13 percent of the sample, and for non-city districts, 14 percent.

As discussed above, the commuting variable is average job-weighted distance to five major worksites, entered in quadratic form to approximate sorting. Various specifications are used for neighborhood amenities other than school quality and ethnicity. Household demands for these amenities are undoubtedly heterogeneous but (17) may not apply. Households may care not only about the level of an amenity within their own CBG, but also about its level in nearby CBGs. They also may care about distance from certain amenities—but only up to a point. Specifications for these types of amenities are consistent with household heterogeneity and sorting, in the sense that they allow for variation in bid functions, but they do not yield structural parameters. See Tables 3 and 4. A few other amenities, such as location near a railroad, simply take on values of zero or one.

Households care about crime in their own neighborhood but may also be concerned about living near a crime "hot spot" (Table 3). Following Bowes and Ihlanfeldt (2001), crime rates are defined per unit area. Because property and violent crime are highly correlated ($\rho = 0.8$), I use four indicators of within-neighborhood crime; below-average property and violent crime (the omitted category), below-average property and above-average violent crime, above-average property and below-average violent crime, and above-average property and violent crime. Crime hot spots are above the 95th percentile of the sample on violent and property crime. Distance from these spots, three nearby locations in Cleveland, is measured from their population-weighted centroids.

Households may also care about pollution (Table 3). I identified three clusters of CBGs where a relatively high volume of pollutants was released into the air and found the pollution-weighted centroid of each cluster. The final variables measure whether a CBG was within 20 miles of the nearest cluster and the distance from the cluster if it was less than 20 miles. Another pair of pollution variables indicates distance from a hazardous waste site.

The Cleveland area's "ghettos" were defined by plotting all CBGs with a population at least 80 percent black and then identifying large clusters of these CBGs. Two such clusters were found and Ghetto and Near Ghetto (see Table 4) were defined based on the population-weighted centroids of these clusters. The eastern suburbs of Cleveland receive unusual amounts of snow (Table 4). The "snowbelt" variables make it possible to determine whether housing prices are different in locations with particularly heavy snowfall. Snowfall maps from the years just preceding 2000 indicate that the heaviest snows are east of the town of Pepper Pike and about 10 miles from Lake Erie. The quadratic specification of these variables estimates the distance from the lake at which the snowfall effect in the eastern suburbs is maximized.

5.3 Estimation Strategy

These steps make it possible to estimate the bid function envelope, equation (17) or (20). My strategy is to estimate several models, corresponding to different assumptions about the parameters of (17), and to test whether the more general models are supported by the data. The first two models, linear and quadratic, can be estimated with OLS. As indicated by (17), the linear model assumes that $-\mu = \sigma_3 = \infty$ for each amenity, and the quadratic model assumes that $\mu = -\infty$ and

$\sigma_3 = 1$. The third model retains the assumption that $\sigma_3 = 1$, which indicates a linear sorting equilibrium, and uses nonlinear least squares (NLLS) to estimate μ for each amenity. These estimates all use $\log\{P^E\}$ as the dependent variable; I test the associated assumption that $v = -1$.

As explained in Section 3.2, σ_3 represents an assumption about the distribution of household preferences, ψ, relative to the distribution of S, and many plausible distributional assumptions lead to $\sigma_3 = 1$. Nevertheless, the assumption that $\sigma_3 = 1$ leaves us in the realm of “partial identification,” which “calls for analyzing the sensitivity of our inferences on the parameter of interest to these esoteric [e.g. distributional] assumptions” (Tamer 2010, p. 168). To address this issue, I follow what Tamer (p. 169) calls “a top-down approach,” which begins with “a fully parametric model that point identifies the parameter of interest,” in this case μ, based on the assumption that $\sigma_3 = 1$ (Table 5), and then estimates “different values of the parameter of interest that correspond to” different assumptions about the hedonic equilibrium, i.e., about σ_3.[25]

6. Empirical Results

6.1 Main Results

Table 5 presents results for alternative envelope specifications for Elementary, High School, Share Non-Black, and Share Non-Hispanic. Results for Value Added and Minority Teachers raise additional issues and are discussed in Section 6.3. The results in this table provide strong support for the method developed in this paper, which produces reasonable, highly significant amenity price elasticity estimates from the hedonic (first-step) regression, along with significant parameters for the sorting equilibrium, σ_1 and σ_2, in many cases.

With the linear model (column 1) Elementary, High School, and the two ethnicity variables are all significant with the expected positive sign. The coefficient for Elementary is not significant in Cleveland and Cleveland Heights, however. The hypothesis that $v = -1$ cannot be rejected. In column 2 (quadratic), the results for Elementary change dramatically. The Elementary variables are not significant outside of the Clevelands, but in these two districts Elementary has a significant U-shaped envelope. The downward-sloping portion of this relationship suggests that the clustering of private and charter schools or the promise of new facilities makes locations near low-scoring elementary schools attractive to homebuyers despite their low scores. The upward sloping portion indicates that housing prices increase about 11 percent from the mean to the maximum value of Elementary. In addition, the squared terms for High School and Share Non-Black are positive and significant. The last row shows that first model can be rejected in favor of this one, which suggests the need to account for sorting. Again, one cannot reject v = -1.

Turning to the NLLS estimates in column 3, we find virtually identical results for Elementary. These results indicate that $\mu = -\infty$ for this amenity, which corresponds to linear bid functions. My attempts to estimate this μ did not converge, but specification tests reveal that a model based on a finite μ (-0.5 or -1.0, for example) can be rejected in favor of one in which $\mu = -\infty$.[26] For the other three amenities, the estimates of $\sigma_1$and σ_2 (First and Second Term) are significant, and the estimated μs are significant for High School (-0.76) and Share Non-Hispanic (-0.38). The significant values for σ_1 reject a one-variable specification for the amenity, and the positive, significant values for σ_2 support the sorting theorem. This column also indicates that the quadratic specification can be rejected in favor of this one and that we still cannot reject v = -1.

The model in the last column redefines the ethnicity variables to be $(S - S^M)^2$, where S^M is a parameter to be estimated.[27] This change has virtually no impact on other results, but it leads to reasonable price elasticities and S^M values for both ethnicity variables. The μ for Share Non-Black is -0.8915 (significant at the 9 percent level), and the estimate of S^M, which indicates the Share Non-Black below which neighborhoods contain people who prefer black neighbors, is 25.5 percent and significant. In the case of Share Non-Hispanic, μ is -0.6273 and the estimate of S^M is 52.77 percent; both estimates are significant. In contrast, the estimates of $\sigma_1$and σ_2 are not significant in either case. Although this model has a slightly higher R^2 than the one in column 3, neither model can be rejected in favor of the other. We also cannot reject the hypothesis that v = -1.

As a robustness check, the regression in column 2 of Table 5 was re-estimated with school district fixed effects. This step raised the R^2 by less than 4 percent, so the results in this table do not appear to reflect bias from omitted school district variables.

Table 6 presents results from the "partial identification" approach. I re-estimated the model in column 4 of Table 5 using four values for σ_3: ½, 1, 2, and 3. Because the model estimates envelopes for 4 amenities, there are 4^4 = 256 possible combinations of σ_3. However, 56 of the 64 regressions with σ_3 = ½ and 1 of the regressions with σ_3 = 1 for Share Non-Hispanic, did not converge, leaving 198 comparison regressions (excluding the original). Panel A reveals that varying the σ_3 parameters does not lead to price elasticities outside the 95 percent confidence intervals of those in column 4 of Table 5.[28] At the values of the σ_3 parameters that minimize the sum of squared errors, the estimate of μ rises to -1.0 for High School and drops to -0.48 for Share Non-Black, but the gain in explanatory power compared to setting all σ_3's equal to 1 is small.

We can also test the assumption that σ_3 = 1 for assumed values of the μs in the range initially found. Panel B of Table 6 describes estimates of σ_3 for High School, Share Non-Black, and Share Non-Hispanic for 64 different estimations, corresponding to various combinations of four assumed values of μ (-0.6, -0.75, -0.8, -0.9) for these three amenities.[29] In all cases for the first and last amenity, and in three-quarters of the cases for Share Non-Black, 1.0 is within the 95 percent confidence interval for the estimated σ_3. Moreover, the top of this confidence interval drops only to 0.85 for Share Non-Black when its μ is set at -0.6, which is well below its value in

Table 5. These results provide further support for the assumption that $\sigma_3 = 1$.

6.2 Envelopes

The derivations in this paper lead to new shapes for hedonic envelopes. The estimated shapes for the main school variables are plotted in Figure 3. The nonlinear envelopes are based on column 4 of Table 5. This figure also presents illustrative bid functions implied by this estimation. In the case of Elementary (the first panel), the difference between Cleveland/East Cleveland and all other districts is striking. As discussed in Section 5.2, housing prices decrease with relative elementary quality up to the mean quality and increase thereafter. For districts other than Cleveland and East Cleveland, the estimated envelope is quite flat with insignificant coefficients.

The second panel of Figure 3 plots envelopes for High School, excluding Cleveland and East Cleveland. Both nonlinear and quadratic envelopes are presented for comparison. The nonlinear envelope has an anomaly at low values for this variable, namely a downward slope, but this portion of the envelope applies to only four school districts. Three of these school districts have housing prices less than 1 percent above the minimum price and the fourth has housing prices only 3.2 percent above the minimum. Moreover, the envelope is not significantly higher for this latter district than at its minimum. In contrast, school districts with the highest values for High School have housing prices 29.8 percent above the minimum price.[30] The quadratic envelope indicates an even higher difference between these districts: 32.6 percent.

Envelopes for the ethnicity variables appear in Figure 4. The first panel shows that both full nonlinear and quadratic envelopes for Share Non-Black have a U-shape. The minimum points are 25.1 percent non-black for the full non-linear envelope and 25.6 percent non-black for the quadratic envelope. The full non-linear envelope implies that housing prices are 2.7 percent higher at zero percent non-black and 25.5 percent higher at 100 percent non-black than at this minimum point. These results support the view that households have heterogeneous preferences for neighborhood ethnic composition, with some households preferring an increase and others a decrease in the percent non-black.[31] Figure 4 also presents the envelope that arises when the amenity is re-defined to be $(S - S^M)^2$. This envelope looks similar to the original nonlinear envelope, so the original method does not appear to be misleading if it is interpreted properly.

The second panel of Figure 4 plots envelopes for Share Non-Hispanic. The minimum of the nonlinear envelope is at a share of 0.499, but the quadratic envelope slopes continuously upward. On the nonlinear envelope, housing prices decrease by 11.9 percent from the minimum observed share, 0.367, to a share of 0.499 and then increase by 30.5 percent from a share of 0.493 up to a share of 1.0. These results point to heterogeneity in preferences for Hispanic neighbors. An estimate of the split envelope discussed in Section 6.1 is also plotted in Figure 4. As in the case of Share Non-Black, the split envelope is similar to the nonlinear envelope.

6.3. Results for Other Neighborhood Variables

Results for other variables are presented in Tables 3 and 4. The two commuting variables are significant with the expected signs. The coefficients of the property tax variables reflect both property tax capitalization and the impact of public services that are correlated with the property tax rate but not accounted for by the public service variables. The positive estimated coefficient for the school tax rate indicates that the second of these effects is larger for school services. The signs for the two city tax coefficients point to the first effect but these coefficients are not significant. The income tax coefficient is also not significant.

Compared to houses in CBGs with low property and violent crime, houses in CBGs with high property and low violent crime sell for 3.4 percent less, houses in CBGs with low property and high violent crime sell for 8.7 percent less, and houses in CBGs with high property and violent crime sell for 7.7 percent less. Moreover, houses located within one-half mile of a crime hotspot sell for a huge discount: 20.3 percent. This discount declines with distance, but still equals 9.1 percent for houses one to two miles away. Property values are significantly higher in cities than in villages and townships and places that receive their police services from the county. Outside of Cleveland, property values decline with city population. Compared to Glenwillow (population = 449), the city with the second largest population, Parma (population = 85,655), has prices that are 23.3 percent lower. These effects account for unidentified variation in amenities across cities of different sizes. Table 3 also indicates that home buyers care about pollution. Houses located at the center of an air pollution cluster sell for 23.3 percent less than houses with clean air; this effect declines 0.8 percentage points per mile and disappears after about 20 miles. Location next to an environmental hazard lowers prices by 6.1 percent, an effect that fades out within a mile.

Table 3 also presents result for the additional school variables. The relationships between housing prices and both Value Added and Minority Teachers are hill-shaped, which is not expected for an envelope. Housing prices peak when Value Added equals 0.326, somewhat above its average of 0.240. Thus, as expected, housing prices increase (by 21.2 percent) from the lowest Value Added to 0.326, but then, contrary to expectations, decline (by 5.1 percent) as Value Added goes from 0.326 to its maximum. Because the regressions control for High School, a relatively high Value Added indicates a district with a low fourth-grade test score compared to other districts with the same twelfth-grade score. Home buyers apparently appreciate relatively successful elementary schools but are also concerned about those that start too far behind. Housing prices also reach a maximum when Minority Teachers equals 0.249, which corresponds to an integrated teaching force, but the underlying coefficients are not significant.

Table 4 shows that housing prices are 7.1 percent higher within two miles of Lake Erie, without a significant decline based on distance from the lake. The coefficients of the two snowbelt variables imply a maximum positive impact on housing prices at 11 miles from Lake Erie in the eastern suburbs, which is where snowfall is the greatest. Households apparently value snowfall or some related topographical feature. Housing prices are boosted by neighborhood parks but lowered by location near a railroad or a large family public housing project.

6.4 Results for Normal Sorting

The results in Table 5 can be used to calculate ψ using (13), to estimate (11) (which is analogous to Rosen's second step) as a log-linear regression, and to test the "normal sorting" hypothesis. A direct test is based on the coefficient of $\log\{Y\}$ controlling for other demand determinants. An indirect test recognizes that the correlation between *Y* and *S* depends on both the direct impact of *Y* on *S* and the impact on *S* of other demand determinants correlated with *Y*. According to the theorem for omitted-variable bias, regressing $\log\{\psi\}$ on $\log\{Y\}$ yields a coefficient with an expected value equal to the true coefficient of $\log\{Y\}$ plus the sum across other demand variables of their true coefficients multiplied by their correlations with $\log\{Y\}$. The coefficient of $\log\{Y\}$ therefore provides an indirect test for normal sorting.

The indirect test accounts for unobservable demand factors; the direct test is unbiased if these factors (the two error terms in (11)) are uncorrelated with income. These two error components need not be uncorrelated with each other. If the two error terms have a bivariate normal distribution, for example, then the linear combination of them in (11) also has a normal distribution. These errors could also be correlated across amenities, resulting in inefficient estimators. The usual solution to this problem, seemingly unrelated regressions, is not feasible here, however, because there is no basis for the required exclusion restrictions. Further exploration of control variables and exclusion restricts are subjects for future research.

The first panel of Table 7 presents the indirect tests. The coefficient of log{Y} is positive and significant in all four cases, providing support for indirect normal sorting. Direct tests in the second panel, which control for a range of family and educational characteristics, support normal sorting for High School and Share Non-Black.[32] These estimates imply that a one standard deviation increase in Y leads, purely because of sorting, to a 0.82 standard deviation increase in High School but only a 0.32 standard deviation increase in Non-Black (in locations where the envelope has a positive slope). Because they show that CBG income and education are key determinants of amenity demand, these results also highlight the potential for endogeneity bias in a hedonic regression that includes income and education as neighborhood amenities.

7. Conclusions

In contrast to Taylor (2008), who writes "Because the hedonic price function is an envelope function, there is no theoretical guidance for its specification" (p. 20), this paper takes the view that this functional form can be derived precisely because it is an envelope. Using (a) constant elasticity demand functions for amenities, public services, and housing and (b) a characterization of the sorting equilibrium that draws on the consensus theory of household sorting and the assumption of one-to-one matching, I derive the bid-function envelope with heterogeneous households. This envelope, equation (17), consists of three Box-Cox forms. I also show that specifications with a single right-side variable for each amenity implicitly rule out sorting.

My approach, which applies to continuous amenities, has three main advantages. First, it accounts for both observable and unobservable heterogeneity in the demands for amenities and housing. Second, it yields estimates of the price elasticities of demand for amenities directly from the hedonic estimation and thereby avoids the endogeneity and potential inconsistency in the Rosen two-step procedure. Third, it provides the only extant method for estimating the demand for multiple amenities with assumptions that are consistent with standard utility postulates. Finally, it provides tests of the hypotheses that households sort according to the slopes of their bid functions and that higher-income households live in locations with more desirable amenities.

This approach was implemented using data from the Cleveland area in 2000. The regressions, which control for a wide range of housing and neighborhood characteristics, focus on school quality and neighborhood ethnicity. For three measures (the passing rate on high school exams, neighborhood percent non-black, and neighborhood percent non-Hispanic), my specification leads to precise estimates of the price elasticities of demand (-0.75, -0.89, and -0.63, respectively) and of sorting parameters. Moreover, the ethnicity results confirm what surveys have long indicated, namely, that some households prefer largely white neighborhoods whereas others prefer neighborhoods in which blacks or Hispanics are concentrated.

Another key finding is that the homebuyers care about many dimensions of school quality, some of which do not fit standard forms. Higher elementary value added leads to higher housing prices, except when it indicates a low test-score starting point, and to some degree higher property taxes appear to be associated with unobserved, but valued education programs. Moreover, the unique voucher, charter school, and school construction programs in Cleveland and East Cleveland lead to unusual bidding outcomes for elementary school quality in these districts.

Finally, results for the elementary, high school, and neighborhood ethnicity variables support the theorem that households sort into locations according to the slopes of their bid functions. Moreover, the results confirm that sorting is usually “normal,” with higher-income households sorting into locations where the services and amenities are more desirable.

Challenges for future research include considering other assumptions about sorting equilibria or household heterogeneity and accounting for changes in sorting over time.

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Footnotes

[1] Epple et al. (2010) builds on Epple and Sieg (1999) and Epple et al. (2001); Heckman et al. (2010) is a generalization of Ekeland et al. (2004).

[2] In other words, a 1 percent increase in Y leads to a θ percent increase in both S and H. Alternatively, θ can equal 0 for some observed commodities and 1 for others. Although these restrictions on θ seem severe, other studies that bring amenity demand information into an analysis of bidding and sorting make equally strong assumptions; see Section 4. The indirect utility function in LaFrance, which leads to the demand functions in the text, is:

$$v\{Y, p\} = \left(Y^{1-\theta}\right) / (1-\theta) - \sum \alpha_i \left(p_i^{1+\mu_i}\right) / (1+\mu_i) \; .$$

[3] Differences between one-to-one and inexact sorting are discussed in detail in Yinger (2014).

[4] With a truncated normal, the post-transformation truncation values must also be the same.

[5] I specify $\partial F/\partial \alpha$ based on sorting theory and derive the implied constant in F. Rouwendal (1992) proceeds in the opposite order. He assumes the constant (the analog to (15)) is quadratic, substitutes this form into his analog to (12), and differentiates to find $\partial F/\partial \alpha$. His form is equivalent to a special case of (16) when $-\nu = \sigma_3 = 1$ and $\mu = -\infty$. Another example that leads to a quadratic form for the hedonic can be found in Epple (1987).

[6] As shown by Epple et al. (1984), and Epple et al. (2001), community-level studies must recognize the simultaneity between sorting outcomes and public service choices made by the people who move into a community. This simultaneity does not arise with individual house sales data, because homebuyers bid on the basis of observed traits, not traits that may exist in the future.

[7] The case of $\sigma_3 = 0$ also does not involve sorting because it implies that the right side of (13) equals 1, but this case cannot be estimated because it leads to a undefined second term in (17).

[8] If $\sigma_1 = 0$, the two parameters in the exponent of the second term in (17) cannot be separately identified.

[9] This assumption is needed to preserve the single-crossing condition. An example of indirect interaction is provided in Yinger (In Press); bids for neighborhood ethnicity are affected, in some cases, by factors that predict lower-quality schools, namely, poverty concentration and tax price.

[10] This paper is based on (17) with a multiplicative $H\{X\}$. Unless $\mu = \nu = -1$ and $\sigma_3 = \infty$, the *S*s and the *X*s are treated differently. A less plausible model would assume the *X*s are fixed and allocated through sorting, so (17) would apply to the entire estimating equation.

[11] For more on normal sorting, see Wheaton (1993) and Yinger (1995).

[12] Some scholars have recommended using data for multiple urban areas with construction cost as an instrument for implicit price. As Sheppard (1999, p. 1625) points out, however, "estimation of demand from multiple markets requires the assumption of a common demand structure."

[13] Other approaches to the endogeneity problem exist. Bishop and Timmons (2011) avoid endogeneity by estimating the demand parameters that maximize the likelihood of the observed sorting pattern. However, the utility functions that this approach requires violate the strict quasi-concavity postulate. Kuminoff and Pope (2014) show how identification can be obtained in the case of amenity shocks, at least when the shocks do not cause re-sorting.

[14] By combining unrelated amenities in an additive index, in other words, the model in Epple et al. violates the strict quasi-concavity postulate for utility functions, at least for choices among the index components. An average of highly related amenities, such as test scores for different subjects or grades in the nearest school, simply summarizes a set of amenities that cannot easily be distinguished by households and does not violate this postulate for choices between this set of amenities and other amenities (or amenity sets).

[15] A bid function is the housing price (*P*) that holds utility (*V*) constant when *S* changes. So if $V = aS - bP$, which is equivalent to equation (2) in Bayer et al. (2007), the bid function is $P = (a/b)S$ plus a constant, which is linear in *S*. This linear utility function violates the strict quasi-concavity postulate. Bayer et al. say (note 29) that "Alternative specifications of the indirect utility function that are nonlinear in housing prices could certainly be estimated," However, this step would not alter the constant marginal rate of substitution between two amenities and hence would not result in utility functions that are strictly quasi concave.

[16] My approach and the others discussed in this section also avoid the problem pointed out by Brown and Rosen (1982): without functional form restrictions, amenity demand estimates for a single market may simply reproduce implicit price functions.

[17] Ioannides (2004) finds "that the correlation coefficient between incomes of a randomly chosen individual and her neighbors is, at around 0.3, moderate but statistically very significant" (p. 455). This result refers to current income. He also points out that the correlation is likely to be higher for permanent income, which presumably drives housing choices, for two reasons. First, current income includes a transitory component, which is equivalent to an error in measuring permanent income that lowers the estimated correlation. Second, "neighborhoods may be mixed in terms of people of different ages, whose current incomes differ because they happen to be on different points in their life cycles but whose permanent incomes might differ by less" (p. 437).

[18] In a few cases, a CBG was split because it was crossed by one or more school district boundaries; for conciseness, the text refers to each part of split CBGs as a CBG.

[19] The property tax variables approximate (5) (and 18); $\log\{1-a\} \approx a$ when *a* is small.

[20] This method is Stata's hc3 option. Heteroskedasticity can arise because the number of observations used to estimate CBG fixed-effects coefficients varies across CBGs (Borjas 1987).

[21] Equation (40) in Epple et al. (2010) is the same as equation (19) except that it omits the Q variables. Sieg et al. (2002) and Deng et al. (2003) also use the coefficients of neighborhood fixed effects to measure neighborhood price/quality, but neither study estimates a second stage.

[22] Appendixes explaining data collection procedures, estimating techniques, and detailed results are available on the author's web site: http://faculty.maxwell.syr.edu/jyinger .

[23] Several CBGs in the City of Cleveland are in the Shaker Heights School District, and a few CBGs in the Cleveland School District are in small cities on Cleveland's border.

[24] When assessment-sales ratios were not available, the nominal rates were corrected by the average observed assessment-sales ratio and the No A-to-S dummy was set to 1.0.

[25] In (17), σ_3 is the fourth parameter for each amenity (with σ_1, σ_2, and μ), the impact of a change in σ_3 on P^E can be approximated by changes in the other parameters, and σ_3 is not well identified. See Davidson and MacKinnon (2004, Chapter 6). A partial identification strategy is also pursued by Epple et al. (2010) in their "second-stage" analysis.

[26] One cannot obtain standard errors for $\mu = \infty$, and a specification test against $\mu \neq \infty$ is not feasible because this alternative does not converge. One can estimate [λ(specification with $\mu = \infty$) + (1-λ)(specification with $\mu = \mu_0$)] . With $\mu_0 = -0.5$, this test leads to $\lambda = 1.0008$ with a t-statistic above 500, which supports the hypothesis that $\mu = \infty$.

[27] The slope of (17) reverses in sign when S is close to S^M and is very steep for a few CBGs. As shown in Figure 4, a restriction keeps the slope of the bid function at zero from the reversal point to S^M and holds bids constant in the integrated zone defined by S^M. See Yinger (In Press).

[28] The 64 regressions with $\sigma_3 = 3$ for Share Non-Hispanic all yield price elasticities outside the 95 percent confidence interval, but never by more than 7.05 percent. The lack of convergence for some values of σ_3 for Share Non-Hispanic does not appear to have a substantive interpretation.

Slightly simpler regressions, which set S^M for Non-Hispanic at 0.5 and drop Elementary, converge with estimated values of μ for Non-Hispanic within the bounds in Table 7.

[29] To simplify the estimation, Elementary is entered as a quadratic (interacted with the Cleveland variable), the value of S^M is set at 0.25 for Share Non-Black and at 0.5 for Share Non-Hispanic, and the constraint discussed in footnote 27 is not imposed.

[30] Starting at the mean, a one standard deviation increase in High School raises housing prices by 10.2 percent, which is similar to previous studies (Brasington and Haurin 2006; Nguyen-Hoang and Yinger 2011). In contrast to most previous studies, however, this effect is not constant.

[31] Bayer et al. (2007, p. 626) also find across-group differences in neighborhood ethnic preferences but, unlike equation (17), do not allow these preferences to vary within a group.

[32] The discrete-choice estimates in Bayer, et al. (2007, Table 8) indicate that willingness to pay for a whiter neighborhood increases with income for both blacks and whites.

Table 1. Results for First-Stage Hedonic with Neighborhood Fixed Effects

Variable	Definition	Coefficient	Std. Error
One Story	House has one story	- 0.0072	0.0050
Brick	House is made of bricks	0.0153***	0.0052
Basement	House has a finished basement	0.0308***	0.0050
Garage	House has a garage	0.1414***	0.0067
Air Cond.	House has central air conditioning	0.0254***	0.0055
Fireplaces	Number of fireplaces	0.0316***	0.0038
Bedrooms	Number of bedrooms	- 0.0082***	0.0028
Full Baths	Number of full bathrooms	0.0601***	0.0042
Part Baths	Number of partial bathrooms	0.0412***	0.0041
Age of House	Log of the age of the house	- 0.0839***	0.0032
House Area	Log of square feet of living area	0.4237***	0.0086
Lot Area	Log of lot size	0.0844***	0.0037
Outbuildings	Number of outbuildings	0.1320***	0.0396
Porch	House has a porch	0.0327***	0.0073
Deck	House has a deck	0.0545***	0.0053
Pool	House has a pool	0.0910***	0.0180
Date of Sale	Date of house sale (January 1=1, December 31=365)	0.0002***	0.0000
Commute 1[a]	Employment wtd. commuting dist. (house-CBG), worksite 1	- 0.0952***	0.0272
Commute 2[a]	Employment wtd. commuting dist. (house-CBG), worksite 2	- 0.0991***	0.0321
Commute 3[a]	Employment wtd. commuting dist. (house-CBG), worksite 3	- 0.1239***	0.0302
Commute 4[a]	Employment wtd. commuting dist. (house-CBG), worksite 4	- 0.1012***	0.0295
Commute 5[a]	Employment wtd. commuting dist. (house-CBG), worksite 5	- 0.0942***	0.0344
Dist. to Pub. School[a]	Dist. to nearest pub. elem. school in district (house-CBG)	- 0.0032	0.0061
Elem. School Score[a]	Average math and English test scores of nearest pub. elem. school relative to district (house-CBG)	0.0170	0.0197
Dist. to Pri. School	Distance to nearest private school (house-CBG)	- 0.0168***	0.0057
Distance to Hazard	Dist. to nearest environmental hazard (house-CBG)	0.0332***	0.0082
Distance to Erie[a]	Dist. to Lake Erie (if < 2; house-CBG)	- 0.0021**	0.0010
Distance to Ghetto[a]	Dist. to black ghetto (if < 5; house-CBG)	- 0.1020***	0.0331
Distance to Airport[a]	Dist. to Cleveland airport (if < 10; house-CBG)	0.0259**	0.0122
Dist. to CBG Center	Distance from house to center of CBG	- 0.0239***	0.0074
Historic District[a]	In historic district on national register (house-CBG)	0.0120	0.0178
Elderly Housing[a]	Within 1/2 mile of elderly housing project (house-CBG)	- 0.0327*	0.0194
Family Housing[a]	Within 1/2 mile of small family hsg. project (house-CBG)	0.0836**	0.0403
Large Hsg Project[a]	Within 1/2 mile of large family housing project (>200 units; house-CBG)	- 0.0568**	0.0257
High Crime	Distance to nearest high-crime location (house-CBG)	0.0701***	0.0246

Notes: Dependent variable = log of transaction amount; 22,880 observations; R2 = .7893; F(31, 21180) = 369.17 (significant at 0.000 level); 1,665 fixed effects with F(1664, 21,180) = 8.534 (significant at 0.000 level); estimated with "areg" command in Stata. Distances are measured in miles. A * (**) [***] indicates statistical significance at the 10 (5) [1] percent level.

[a]Variable added to the original Brasington data set.

Table 2. Descriptive Statistics for Key Variables

	Mean	Std. Dev.	Minimum	Maximum
CBG Price per unit of Housing	84835.68	23331.25	32215.83	345162.50
Relative Elementary Score[a]	0.3148	0.0894	0.0010	0.6465
High School Passing Rate	0.3197	0.2040	0.0491	0.7675
Elementary Value Added[a]	0.2400	0.0942	0.0100	0.4960
Share Minority Teachers[b]	0.1329	0.1548	0.0010	0.6146
Share Non-Black in CBG[b]	0.8022	0.3226	0.0010	1.0000
Share Hispanic in CBG	0.9623	0.0810	0.3673	1.0000
Weighted Commuting Distance	13.2046	7.4567	7.2660	39.5236
Income Tax Rate[c]	0.0091	0.0012	0.0075	0.0100
School Tax Rate	0.0309	0.0083	0.0172	0.0643
City Tax Rate[d]	0.0578	0.0140	0.0227	0.1033
Tax Break Rate[d]	0.0330	0.0121	0.0047	0.0791
No A-to-S	0.1339	0.3407	0.0000	1.0000
Not a City	0.1393	0.3464	0.0000	1.0000
Crime Lowhigh	0.0252	0.1569	0.0000	1.0000
Crime Highlow	0.1291	0.3354	0.0000	1.0000
Crime Highhigh	0.1934	0.3951	0.0000	1.0000
Crime Hotspot1	0.0126	0.1116	0.0000	1.0000
Crime Hotspot2	0.0354	0.1849	0.0000	1.0000
Crime Hotspot3	0.0847	0.2785	0.0000	1.0000
Crime Hotspot4	0.2667	0.4423	0.0000	1.0000

[a.] Constant added to make all values positive.

[b.] 0.001 added to avoid zero values (unless initial value is 1.0).

[c.] Statistics apply to the 27 observations with a positive income tax rate.

[d.] Statistics apply to the 1433 observations in a city.

Table 3. Results for Tax, Commuting, Crime, Pollution, and Ancillary School Variables

Variable	Definition	Coefficient	Robust Std. Error
Income Tax Rate	School district income tax rate.	1.8818	5.1607
School Tax Rate	School district effective property tax rate.	3.8335***	1.2968
City Tax Rate	Effective city property tax rate beyond school tax.	- 1.9647	1.3896
Tax Break Rate	Exemption rate for city property tax	4.0874*	2.1146
No A-to-S	Dummy: No A/V data	- 0.0422	0.0353
Not a City	CBG not in a city	0.0673*	0.0385
Commute 1	Job-weighted distance to worksites	- 0.0265***	0.0072
Commute 2	(Commute 1) squared	0.0004***	0.0002
Crime Lowhigh	Low property, high violent crime	- 0.0869***	0.0333
Crime Highlow	High property, low violent crime	- 0.0341**	0.0149
Crime Highhigh	High property and violent crime	- 0.0773***	0.0214
Crime Hotspot1	CBG within ½ mile of crime hot spot	- 0.2030***	0.0500
Crime Hotspot2	CBG ½ to 1 mile from crime hot spot	- 0.0689*	0.0392
Crime Hotspot3	CBG 1 to 2 miles from crime hot spot	- 0.0908***	0.0341
Crime Hotspot4	CBG 2 to 5 miles from crime hot spot	- 0.0410*	0.0392
Village	CBG receives police from a village	- 0.1056**	0.0493
Township	CBG receives police from a township	- 0.1118***	0.0431
County Police	CBG receives police from a county	- 0.1758***	0.0452
City Population	Population of city (if CBG in a city)	-2.09E-05***	0.0000
City Pop. Squared	City population squared/10000	5.69E-06***	0.0000
City Pop. Cubed	City population cubed/10000^2	-4.89E-07***	0.0000
City Pop. to Fourth	City pop. to the fourth power/10000^3	7.69E-09***	0.0000
Smog	CBG within 20 miles of air pollution cluster	- 0.2331***	0.0701
Smog Distance	(Smog)×Distance to cluster (not to the NW)	0.0080**	0.0039
Near Hazard	CBG is within 1 mile of a hazardous waste site	- 0.0607***	0.0169
Distance to Hazard	Distance to nearest hazardous waste site (if <1)	0.0749***	0.0229
Value Added 1	School district's 6th grade passing rate on 5 state tests in 2000-2001 minus its 4th grade passing rate in 1998-99	1.1913***	0.3924
Value Added 2	(Value Added 1) squared	- 1.8283***	0.6998
Minority Teachers 1	Share of district's teachers from a minority group	0.3260	0.2169
Minority Teachers 2	(Minority Teachers 1) squared	- 0.7161*	0.3921
Cleveland SD	Dummy for Cleveland & E. Cleveland Schl. Dists.	0.4755	0.3269
Near Public	CBG is within 2 miles of public elem. school	- 0.0133	0.0220
Distance to Public	(Near Public) ×Distance to public school	- 0.0167	0.0108
Near Private	CBG is within 5 miles of a private school	0.0209	0.0229
Distance to Private	(Near Private) ×Distance to private school	- 0.0031	0.0050

Note: Except for "Near Private" and "Distance to Private," these variables were added to the Brasington data set using CBG latitudes and longitudes and (for Smog) other information in that data set. These results are based on the nonlinear regression in the 4th column of Table 5; results are similar for the regressions in other columns. Standard errors are estimated using the vce(hc3) option in Stata. The regression also includes the variables in Tables 4 and 5. A * (**) [***] indicates statistical significance at the 10 (5) [1] percent level.

Table 4. Results for Other Geographic Controls

Variable	Definition	Coefficient	Robust Std. Error
Lakefront	Within 2 miles of Lake Erie	0.070***	0.0239
Distance to Lake	(Lakefront) ×(Distance to Lake Erie)	- 0.0297	0.0192
Snowbelt 1	(East of Pepper Pike) ×(Distance to Lake Erie)	0.0291***	0.0066
Snowbelt 2	(Snowbelt 1) squared	- 0.0013***	0.0004
Ghetto	CBG in the black ghetto	- 0.0309	0.0439
Near Ghetto	CBG within 5 miles of ghetto center	- 0.0033	0.0246
Near Airport	CBG within 10 miles of Cleveland airport	0.0292	0.0329
Airport Distance	(Near Airport) ×(Distance to airport)	- 0.0017	0.0038
Local Amenities	No. of parks, golf courses, rivers, or lakes within ¼ mile of CBG	0.0189**	0.0081
Freeway	CBG within ¼ mile of freeway	0.0123	0.0116
Railroad	CBG within ¼ mile of railroad	- 0.0305***	0.0104
Shopping	CBG within 1 mile of shopping center	- 0.0161*	0.0094
Hospital	CBG within 1 mile of hospital	0.0117	0.0098
Small Airport	CBG within 1 mile of small airport	0.0295	0.0219
Big Park	CBG within 1 mile of regional park	0.0037	0.0108
Historic District	CBG within an historic district	0.0072	0.0183
Near Elderly PH	CBG within ½ mile of elderly public housing	- 0.0040	0.0237
Near Small Fam. PH	CBG within ½ mile of small family public housing	- 0.0256	0.0267
Near Big Fam. PH	CBG within ½ mile of large family public housing (>200 units)	- 0.0909**	0.0420
Worksite 2	Fixed effect for worksite 2	0.0437**	0.0184
Worksite 3	Fixed effect for worksite 3	0.0889**	0.0354
Worksite 4	Fixed effect for worksite 4	0.0631*	0.0354
Worksite 5	Fixed effect for worksite 5	- 0.0095	0.0246
Geauga County	Fixed effect for Geauga County	- 0.0199	0.0497
Lake County	Fixed effect for Lake County	0.2448***	0.0353
Lorain County	Fixed effect for Lorain County	0.1493***	0.0325
Medina County	Fixed effect for Medina County	0.0122	0.0467
Constant		0.0437**	0.0184

Note: These variables were added to the Brasington data set using the CBG latitudes and longitudes other information in that data set. These results are based on the nonlinear regression in the 4th column of Table 5; results are similar for the regressions in other columns. Standard errors are estimated using the vce(hc3) option in Stata. The regression also includes the variables in Tables 3 and 5. A * (**) [***] indicates statistical significance at the 10 (5) [1] percent level.

Table 5. Specification Tests and Results for Key School and Ethnicity Variables

Variable	Linear	Quadratic	Nonlinear Estimation of μ's, $\sigma_3 = 1$	Nonlinear Est. of μ's, $\sigma_3 = 1$, Split Ethnicity Vars.
Relative Elementary Score				
First Term	0.1268**	0.2448	0.5676	0.6032
	(0.0595)	(0.2480)	(0.3912)	(0.5057)
Second Term	-	- 0.2086	- 2.2281	- 2.5101
		(0.3584)	(3.5516)	(4.5470)
First Term Cleveland	- 0.0606	- 1.2976***	0.3903***	0.3908***
	(0.0722)	(0.3911)	(0.0230)	(0.0229)
Second Term Cleveland		1.6740***	0.3004***	0.2979***
		(0.4876)	(0.0885)	(0.0875)
μ	-∞	-∞	-∞	-∞
High School Passing Rate				
First Term	0.4826***	- 0.0862	0.2177***	0.2166***
	(0.0587)	(0.2631)	(0.0338)	(0.0342)
Second Term	-	0.6049**	1.3139**	1.3255**
		(0.2849)	(0.5328)	(0.5366)
μ	-∞	-∞	- 0.7555***	- 0.7511***
			(0.2752)	(0.2704)
Share Non-Black in CBG				
First Term	0.2633***	- 0.2120*	0.2494***	0.0136
	(0.0291)	(0.1247)	(0.0940)	(0.0544)
Second Term	-	0.4114***	1.2571***	2.6556*
		(0.1031)	(0.3476)	(1.4193)
μ	-∞	-∞	-34.6629	- 0.8915*
			(605.0691)	(0.5300)
S^M				0.2554**
				(0.1193)
Share Non-Hispanic in CBG				
First Term	0.4849***	- 0.2440	0.5001***	0.0014
	(0.0656)	(0.4306)	(0.0211)	(0.0076)
Second Term	-	0.4832*	0.8544***	3.9924
		(0.2871)	(0.2590)	(3.4529)
μ	-∞	-∞	- 0.3773***	- 0.6273***
			(0.1406)	(0.1558)
S^M				0.5277***
				(0.0330)
R-Squared	0.7046	0.7156	0.7167	0.7169
SSE	34.7383	0.7156	33.3169	33.2932
Test of Hypothesis that v = -1				
Estimate of v	-1.0433	-1.0272	-0.0183	-0.0322
Test statistic	0.7500	0.3000	-0.0400	-0.0800
P-value	0.3850	0.5830	0.9640	0.9350
Test of Hypothesis that Model Adds to Explanatory Power				
Test statistic	-	6.66	2.04	n.a.
P-value		0.000	0.042	n.a.

Notes: Dependent variable is log{P} in the CBG; number of observations is 1,665. Coefficients are estimated with OLS (first two columns) or NLLS (last two columns). Parentheses contain t-statistics based on robust standard errors (using the hc3 option in Stata). A * (**) [***] indicates statistical significance at the 10 (5) [1] % level. The last two columns estimate σ_1 ("First Term") and σ_2 ("Second Term") directly. The "First Term" estimate is $(1-\sigma_1)/\sigma_2$ in column one and $-\sigma_1/(2\sigma_2)$ in column 2. The "Second Term" estimate in column 2 is $1/(2\sigma_2)$. The underlying structural parameters in column 2 (σ_1 for "First Term" and σ_2 for "Second Term") usually have the same statistical significance as the reported parameters. The hypothesis that v = -1 is tested with Box-Cox regression (1st two columns) or NLLS (last two columns). The hypothesis that the model adds explanatory power (compared to the model in the previous column) is tested with an F-statistic (with 6 and 1593 degrees of freedom) in column 2 and with the Davidson-MacKinnon test (a t-statistic based on a robust standard error) in the other columns. The D-M test indicates that neither column 3 nor column 4 can be rejected in favor of the other. The regressions also include the variables in Tables 3 and 4.

Table 6: Key Parameter Estimates with Alternative Assumptions

	Starting Values	Values that Minimize SSE	Average	Minimum	Maximum	Standard Deviation
Panel A: Assumed Values for σ_3						
Coefficient						
μ for High School	-0.7511	-1.0007	-0.8021	-1.0041	-0.5402	0.1777
μ for Share Non-Black	-0.8915	-0.4821	-1.1080	-1.7035	-0.4821	0.4592
μ for Share Non-Hispanic	-0.6273	-0.6219	-0.8093	-0.9986	-0.3884	0.1689
S^M for Share Non-Black	0.2554	0.2488	0.2553	0.2488	0.2587	0.0028
S^M for Share Non-Hispanic	0.5277	0.5278	0.5273	0.5269	0.5279	0.0003
t-Statistic						
μ for High School	-2.78	-2.05	-2.79	-4.08	-2.04	0.7931
μ for Share Non-Black	-1.68	-4.02	-1.82	-4.03	-0.67	1.3293
μ for Share Non-Hispanic	-4.03	-4.12	-2.82	-8.91	-1.57	1.6065
S^M for Share Non-Black	2.14	2.09	2.13	2.08	2.17	0.0215
S^M for Share Non-Hispanic	15.97	16.36	15.65	15.05	16.54	0.3919
SSE	33.2240	33.2227	33.2985	33.2227	33.3490	0.0426
Panel B: Assumed Values for the μs						
Coefficient						
σ_3 for High School	0.8281	2.1216	1.2411	0.6550	2.1364	0.5756
σ_3 for Share Non-Black	1.6506	0.7928	1.0693	0.6658	1.6558	0.3865
σ_3 for Share Non-Hispanic	1.5093	0.8895	1.4001	0.8793	2.2020	0.4816
t-Statistic						
σ_3 for High School	1.51	0.54	1.22	0.54	1.97	0.5552
σ3 for Share Non-Black	2.79	6.18	5.09	2.79	7.32	1.7763
σ3 for Share Non-Hispanic	2.76	5.83	3.72	1.62	5.94	1.6387
SSE	33.1108	33.0823	33.0967	33.0823	33.1154	0.0083

For Panel A: Column 1 comes from column 4 in Table 5; all σ_3s =1. The other columns are based on 199 replications of the methodology in that column using different values for σ_3 for the 4 key amenity variables. Four values for σ_3 are used: 1/2, 1, 2, and 3. With 4 variables and 4 values, there are $4^4 = 256$ possible combinations. However, 56 of the 64 regressions with σ_3 = 1/2 for Share Non-Hispanic (SNH), along with 1 of the regressions with σ_3 = 1 for SNH, did not fully converge, leaving (256 - 56 - 1) = 199 regressions. The values of σ_3 that result in the minimum SSE are 1/2 for Elementary (El), 3 for High School (HS), 1/2 for Share Non-Black (SNB) and 1 for SNH. For Panel B: These results assume various values for the 3 μs in Panel A and estimate the associated σ_3s. They also assume S^M = 0.25 for SNB and 0.5 for SNH and specify El as a simple quadratic. The 1st column assumes μ = -0.75 for HS, -0.9 for SNB and 0.6 for SNH; the values for μ in the second column are -0.9, -0.75, and -0.6 for HS, SNB, and SNH, respectively. The other columns are based on 64 replications with the values of μ for HS, SNB, and SN set at all possible combinations of -0.6, -0.75, -0.8, and -0.9.

Table 7. Tests for Normal Sorting

Type of Test	Relative Elementary Score	High School Passing Rate	Share Non-Black	Share Non-Hispanic
Indirect Test				
Income Coefficient	1.7699***	1.0189***	0.3795***	0.2515***
Standard Error	(0.5793)	(0.0564)	(0.0521)	(0.0316)
R-squared	0.0849	0.2028	0.0280	0.0362
Observations	142	1113	1417	1649
Conclusion	Support	Support	Support	Support
Direct Test				
Income Coefficient	1.3540*	0.6426***	0.4575***	0.0393
Standard Error	(0.8059)	(0.0906)	(0.1052)	(0.0433)
R-squared	0.2236	0.3026	0.2171	0.4335
Observations	142	1113	1417	1649
Conclusion	Weak Support	Support	Support	Inconclusive

Notes: Tests are conducted with OLS using robust standard errors (hc3 option in Stata) and all observations with a positively sloped envelope. The 1st column only includes observations in Cleveland and Cleveland Heights. Indirect tests regress $\log\{\psi\}$ (based on column 4 of Table 5) on $\log\{Y\}$ (median owner income in CBG). Direct tests control for the CBG's percent of households that have children, are headed by a married couple, speak English at home, are Asian, are headed by an elderly person; five education categories for adults (all for the CBG), and the share of households in the tract that moved during the last year. In all columns except the first, most variables are significant at the 5% level. Results are similar using other sets of controls or the results in column 3 of Table V. A * (**) [***] indicates statistical significance at the 10 (5) [1] percent level.

Figure 1: Deriving the Bid-Function Envelope

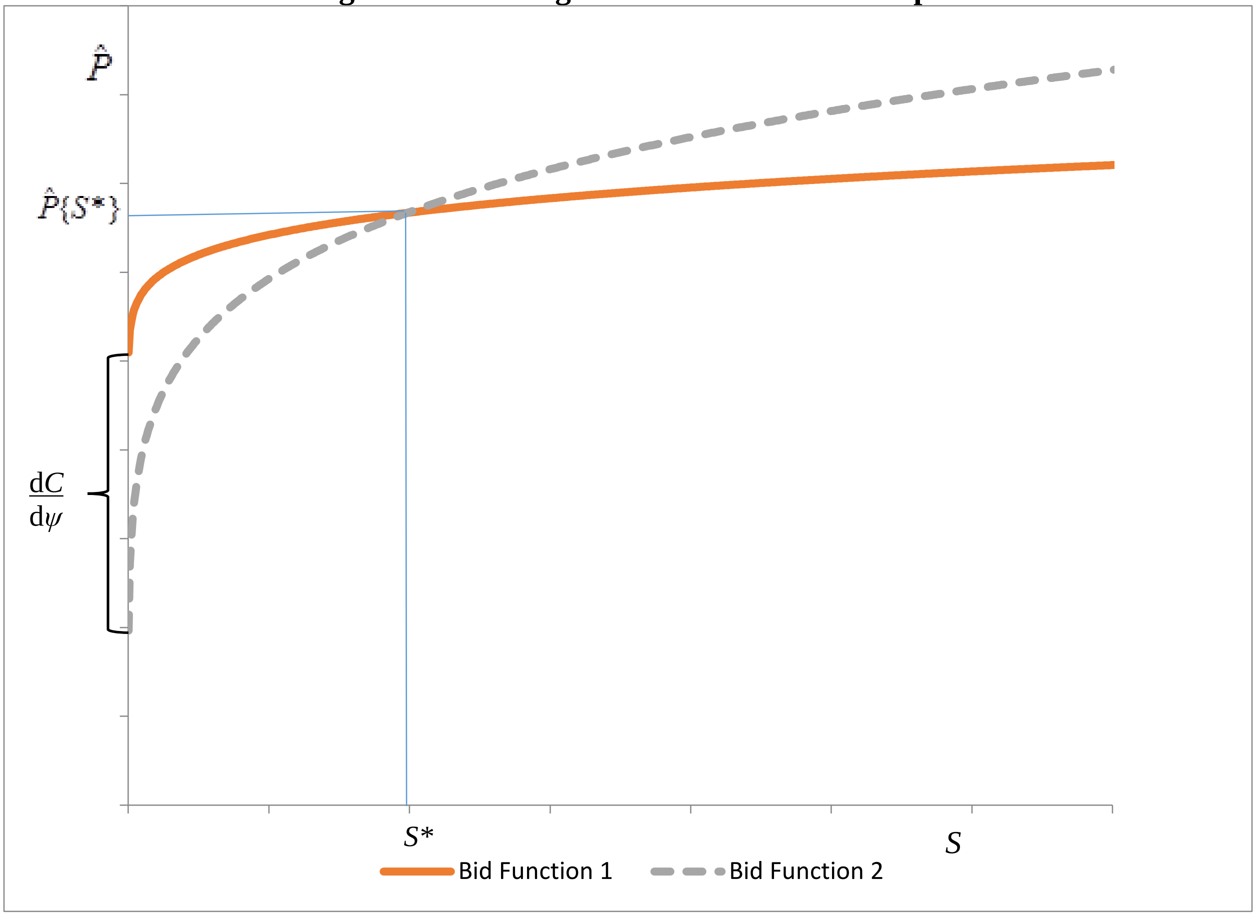

Figure 2. Bidding and Sorting

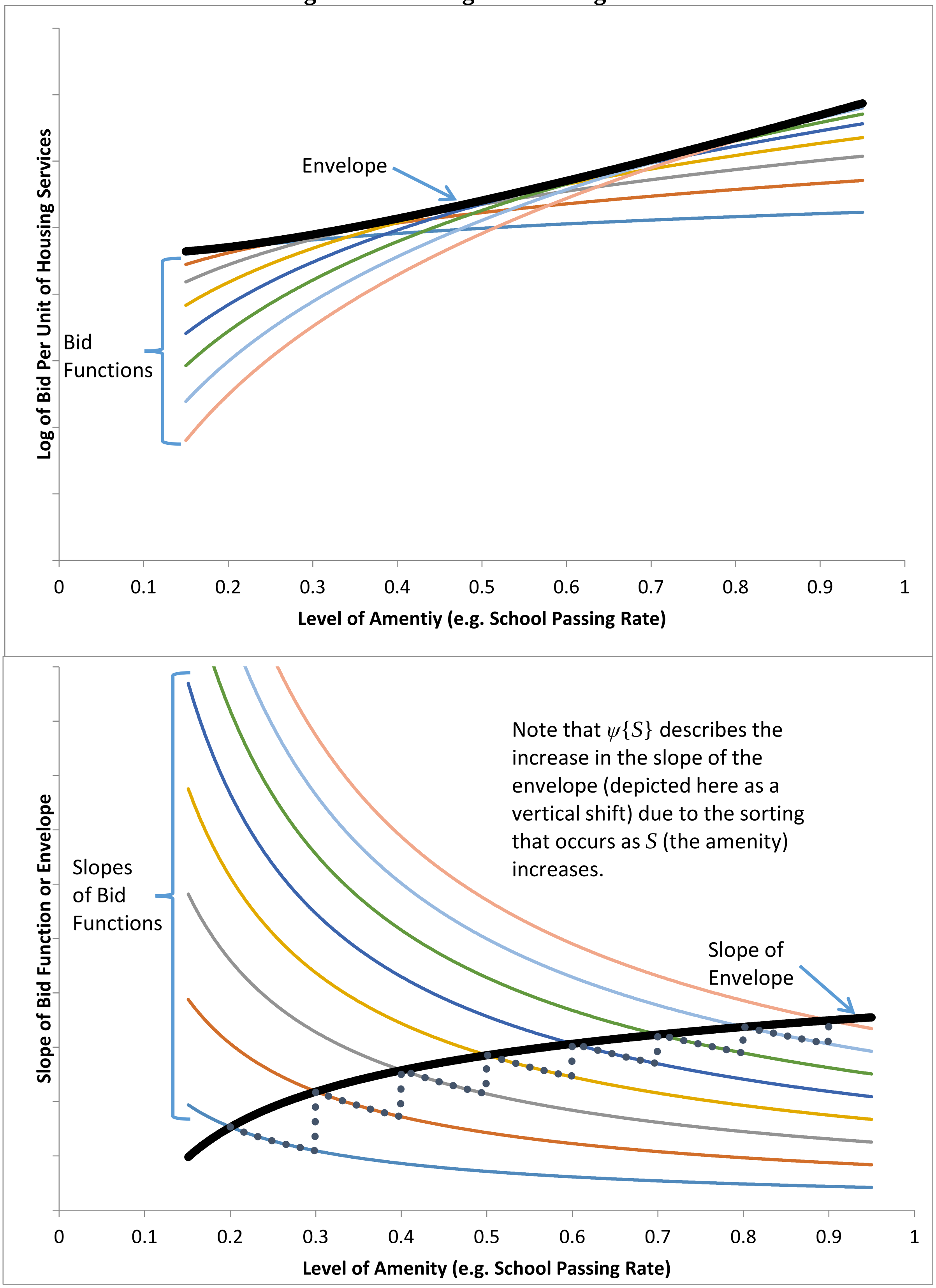

Figure 3: Estimated Bid-Functions and Envelopes for School Variables

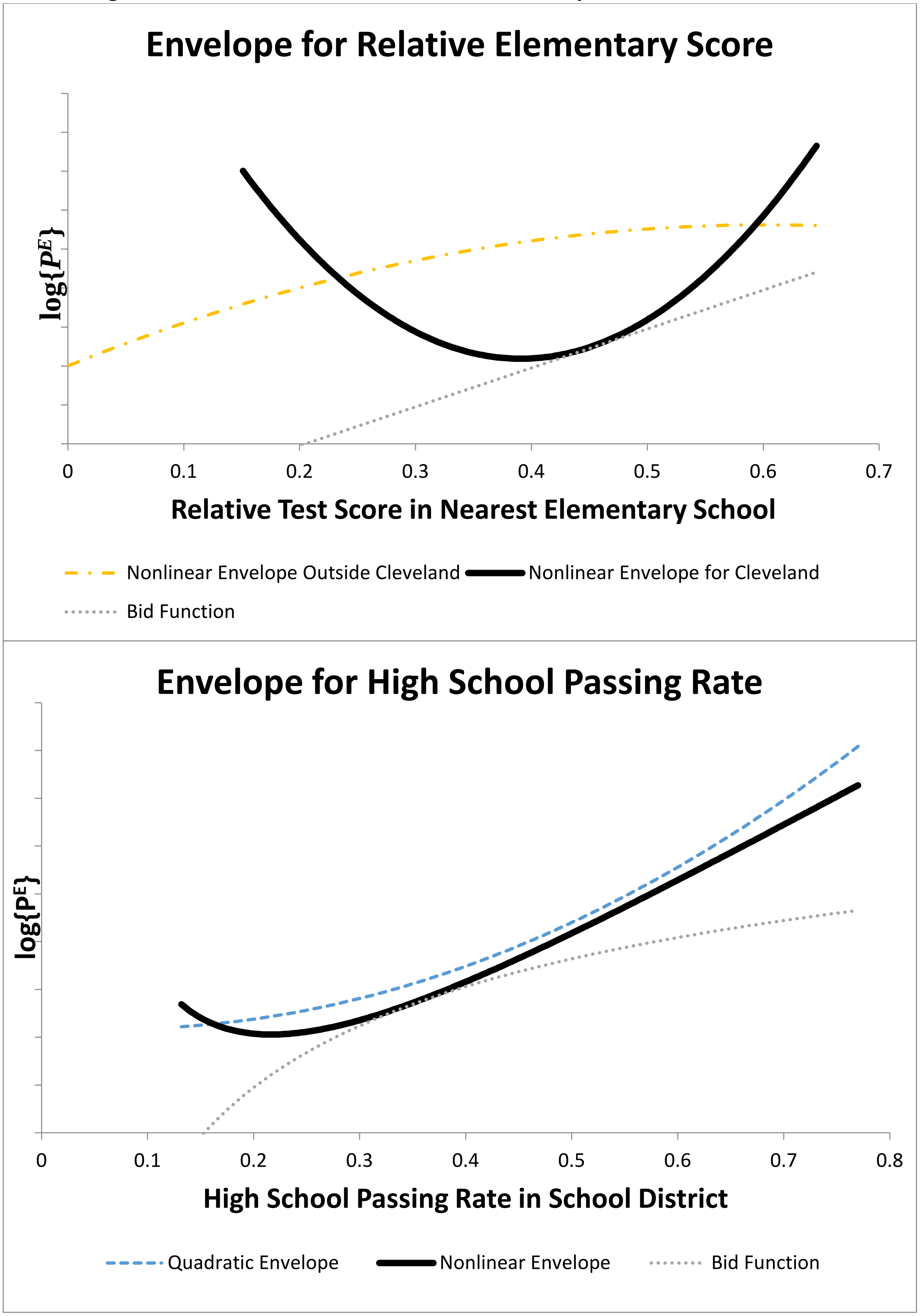

Figure 4: Estimated Bid-Functions and Envelopes for Ethnicity Variables