

ECN 612 - EXAM 1 - FALL 2009

NAME: _____

INSTRUCTIONS

- Please **word-process** all answers, with the exception of graphs and technical expressions.
- Turn in at the beginning of class on Wed, Oct 28.
- You may use any books or your notes. You may consult with me about any questions, but do not consult with others.
- Be sure to read each question thoroughly. Explain and justify all of your work. Aside from your algebra, supply diagrams illustrating your answers whenever possible.
- Submit your answers in order. Staple your copy of the exam to the front of your answers.

1. (20 pts.) Is the competitive mechanism incentive compatible? That is, do individuals have the incentive to declare their preferences truthfully? If so, prove. If not, prove by counterexample, i.e., show an example assuming the usual type of preferences. You may confine your attention to two person, two good, pure exchange economies.
2. (20 pts.) Consider a world in which there are two individuals, two private goods, no public goods, and no production. The individuals' utility functions and initial endowment vectors are

$$u^1(x_1, y_1) = \delta_1 x_1 + y_1, \quad \omega_1 = (0, 1)$$

and

$$u^2(x_2, y_2) = \delta_2 x_2 + y_2, \quad \omega_2 = (1, 0),$$

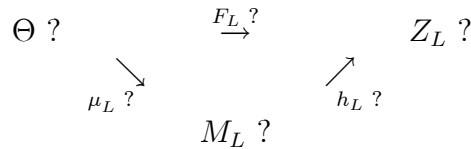
where $(\delta_1, \delta_2) \gg 0$. Assume that each individual reports $\tilde{\delta}_i > 0$ and that the outcome is $\omega = (\omega_1, \omega_2)$ unless $\tilde{\delta}_1 > \tilde{\delta}_2$, in which case the outcome is $((1, 0), (0, 1))$. Answer the following parts and thoroughly justify your answers.

- (a) Is this mechanism privacy preserving? Individually rational? Pareto satisfactory?
 - (b) Is this mechanism incentive compatible? In what sense? Strong? Weak?
3. (20 pts.) Consider all of the mechanisms dealing with the provision of public goods that we covered in class.
 - (a) Which are (complete) revelation mechanisms? Which are not? Justify thoroughly.
 - (b) Which are privacy preserving? Which are not? Justify thoroughly.

4. (20 pts.) Assume a two individual, two good (one public good x and one private good y) world. The utility functions are

$$u^i(x, y_i) = \alpha_i x^{\beta_i} + y_i,$$

where $\alpha_i > 0$, $0 < \beta_i < 1$, $i = 1, 2$. (Assume initial endowment vectors are all $(0, 1)$ and that the $MRT = 1$.) Consider the Lindahl Mechanism defined in class. This question asks you to deal with the mechanism in a more formal way, according to ideas laid out in Hurwicz and Reiter. You are asked to fill in the following diagram. You should follow a method analogous to the development of the Walrasian goal function and mechanisms which realize it.



- (a) First, define a Lindahl goal function F_L . Make the dimension of the space Z as small as possible.
- (b) Define two distinct mechanisms which realize your Lindahl goal function. For at least one of the mechanisms, if possible, use the rectangles method in your approach. If possible, include a mechanism with the smallest possible (dimensional) size for its message space.
5. (20 pts.) For the following questions, you are to work with the National Forest Example of Chapter 1, Hurwicz and Reiter.
- (a) Is the mechanism incentive compatible? Why or why not? Illustrate your answer with very specific numbers, illustrations, etc.
- (b) Change the example by assuming a new goal function $F(a_1, a_2, b_1, b_2) = \frac{1}{2}$, where the goal is the intensity of logging. Determine the mechanism with the smallest possible message space (with respect to the dimension) which realizes the goal function. Is the mechanism incentive compatible? In what sense? Strong? Weak?