ECN 612 - HANDOUT - FALL 2009
General Equilibrium and Welfare Economics

PURE EXCHANGE ECONOMY

NOTATION

- Assume L goods and I consumers.
- $\omega_i = (\omega_{1i}, \ldots, \omega_{Li})$ denotes the initial endowment vector for consumer i, $i = 1, \ldots, I$ and $\omega = (\omega_1, \ldots, \omega_I)$ is the endowment vector. $\bar{\omega} = (\bar{\omega}_1, \ldots, \bar{\omega}_L)$ denotes the vector of total amounts of the goods available in the economy, where $\sum_{i=1}^{I} \omega_i = (\bar{\omega}_1, \ldots, \bar{\omega}_L) = \bar{\omega}$.
- Let $x_i(p) = (x_{1i}(p), \ldots, x_{Li}(p))$ denote a vector of demands for consumer i, given p and ω_i. Then $z(p) = \sum_{i=1}^{I} x_i(p) - \bar{\omega}$ is the vector of (aggregate) excess demands.
- $p \cdot z(p) = p \cdot \left(\sum_{i=1}^{I} x_i(p) - \bar{\omega}\right) = \left\{\sum_{\ell=1}^{L} p_\ell \left[\sum_{i=1}^{I} (x_{\ell i}(p) - \omega_{\ell i})\right]\right\}$ is the value of excess demand.

RESULTS

Proposition 1 (Walras' Law) *If each consumer satisfies his/her budget constraint with equality, the value of excess demand is zero (for any set of prices).*

Notes
1. If $p \gg 0$ and $(L-1)$ of the markets clear, then the Lth market clears.
2. If $p \gg 0$ and if there is excess demand in some market, there must be excess supply in another market.

Definition 2 p^* *defines a Walrasian (competitive) equilibrium if*

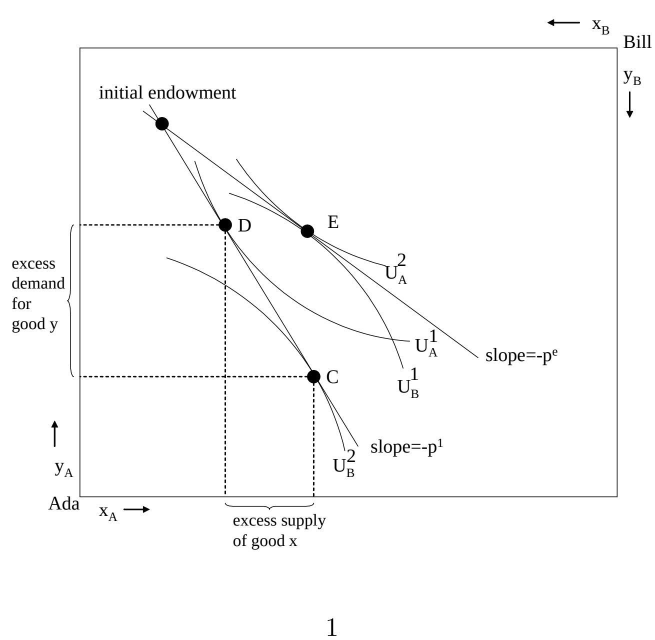
$$\sum_{i=1}^{I} x_i(p^*) = \sum_{i=1}^{I} \omega_i,$$

or

$$z(p^*) = \sum_{i=1}^{I} x_i(p^*) - \sum_{i=1}^{I} \omega_i = 0.$$

$(x_1(p^*), \ldots, x_I(p^*))$ *is a competitive allocation;* p^* *is a competitive price vector.*

Note If p^* is a competitive price vector, then so is tp^* for any $t \in \mathbf{R}_{++}$.



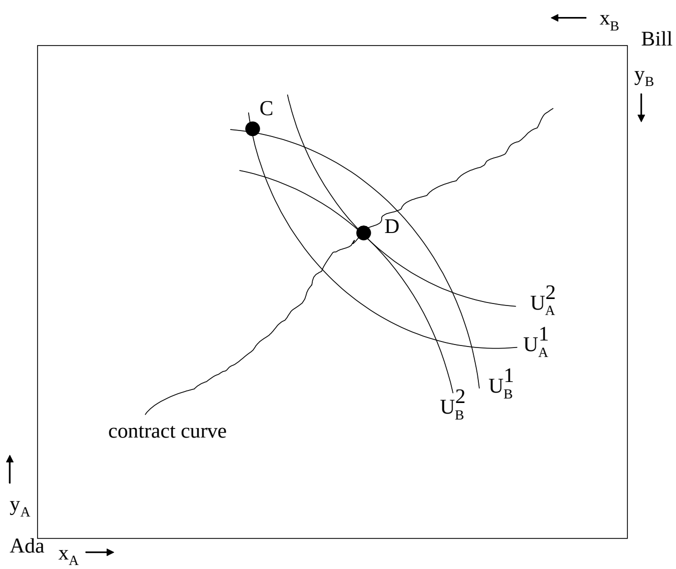
WELFARE PROPERTIES OF COMPETITIVE EQUILIBRIUM

Definition 3 *An allocation* $(\hat{x}_1, \ldots, \hat{x}_I)$ *is feasible (attainable) if*

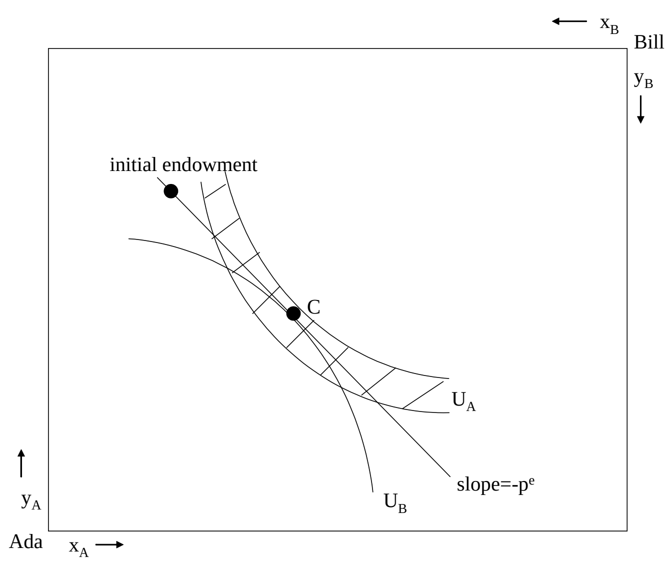
$$\sum_{i=1}^{I} \hat{x}_i = \sum_{i=1}^{I} \omega_i.$$

Note that we adopt the Chapter 16 feasibility definition of Mas-Colell et al., as opposed to the Chapter 15 definition.

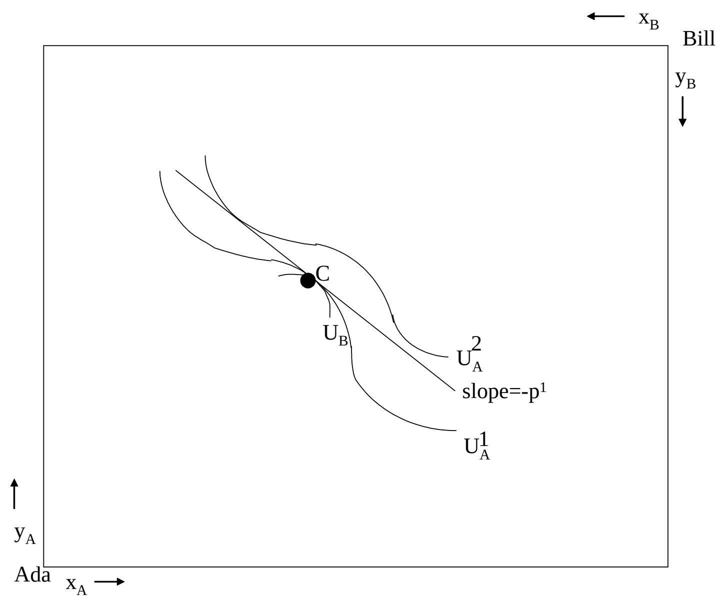
Definition 4 *A feasible allocation* $(\hat{x}_1, \ldots, \hat{x}_I)$ *is Pareto optimal (efficient) if there does not exist another feasible allocation* $(\tilde{x}_1, \ldots, \tilde{x}_I)$ *such that* $\tilde{x}_i \succsim_i \hat{x}_i$, $i = 1, \ldots, I$, *with strict preference for at least one* i.

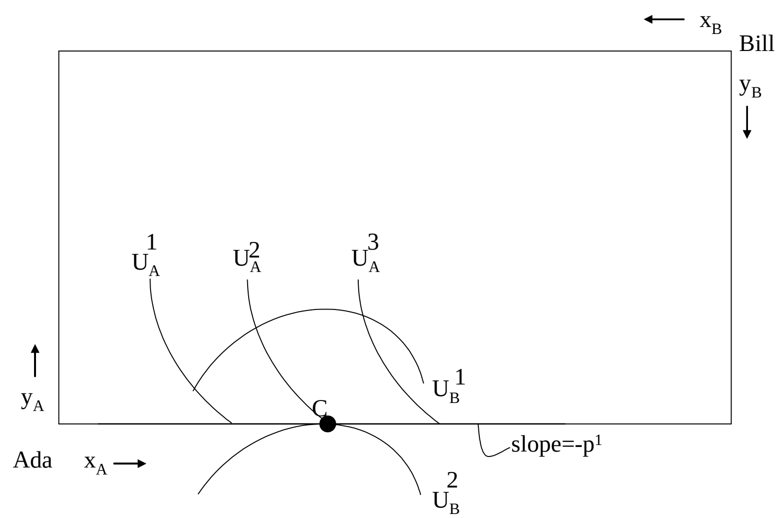


Proposition 5 (First Fundamental Theorem) *Assume that each consumer exhibits local nonsatiation. If* $(x_1^*, \ldots, x_I^*)$ *is a competitive allocation, it must be Pareto optimal.*



Proposition 6 (Second Fundamental Theorem) *Suppose that* $(x_1^*, \ldots, x_I^*)$ *is a Pareto efficient allocation with* $x_i^* \gg 0$, $i = 1, \ldots, I$, *and preferences are convex, continuous, and locally nonsatiated. Then* p^* *exists such that* $(x_1^*, \ldots, x_I^*; p^*)$ *is a competitive equilibrium for the initial endowment* $(\omega_1, \ldots, \omega_I) = (x_1^*, \ldots, x_I^*)$.





Problem for You. Consider the possibility of interdependent utilities (or the case where there are externalities in consumption). Do the conclusions of the First and Second Fundamental Theorems of Welfare Economics necessarily hold? Specifically, consider an exchange economy with two goods and two consumers where

$$u_A = u_A(x_A, x_B), \ u_B = u_B(x_A, x_B).$$

Derive the conditions associated with Pareto optimality and with Walrasian equilibrium. Is a competitive equilibrium necessarily Pareto optimal?

ADDITIONAL WELFARE ECONOMICS

Preliminary Definitions

Assume throughout that $X_i = \mathbf{R}^L_+$ and that $\succsim_i$ is a weak order, $i = 1, \ldots, I$.

Definition 7 *Let $x = (x_1, \ldots, x_I)$ be a feasible allocation. If $x_j \succ_i x_i$ for some $i, j \in \{1, \ldots, I\}$, individual i envies individual j. If there are no envious individuals at allocation x, then x is equitable, i.e., x is equitable if $x_i \succsim_i x_j$ for all $i, j \in \{1, \ldots, I\}$.*

Definition 8 *A feasible allocation x is Pareto optimal (efficient) if there is no feasible allocation y such that $y_i \succsim_i x_i$ for every $i \in \{1, \ldots, I\}$ and $y_j \succ_j x_j$ for some $j \in \{1, \ldots, I\}$.*

Definition 9 *If an allocation is both equitable and Pareto efficient, then it is fair.*

Results

Proposition 10 *Assume that preferences are monotonic. If (x, p) is a competitive equilibrium with $p \cdot x_i = p \cdot x_j$ for all i and j, then x is fair.*

Proposition 11 *If preferences are convex and monotonic, then fair allocations exist.*

Proposition 12 *If individuals i and j have strictly convex, identical preferences, and if x is a fair allocation, then $x_i = x_j$.*

THE GENERAL CASE

Notation

- I consumers, J firms, L commodities
- Preferences of the consumers: $\succsim_i$, $i = 1, \ldots, I$
- Initial endowment vectors: $\omega_i \in \mathbf{R}^L$, $i = 1, \ldots, I$
- Consumption sets: $X_i \subseteq \mathbf{R}^L$, $i = 1, \ldots, I$
- Production (technology) sets: $Y_j \subseteq \mathbf{R}^L$, $j = 1, \ldots, J$. An element of Y_j is of the form $(y_{1j}, \ldots, y_{Lj})$, where $y_{\ell j} \leq (\geq) 0$ if the ℓth commodity is a net input (output).
- Aggregate technology set:

$$Y = \sum_{j=1}^{J} Y_j = \left\{ \sum_{j=1}^{J} y_j : y_j \in Y_j,\ j = 1, \ldots, J \right\}$$

- Distribution of profits of the firms: θ_{ij} = share of the ith consumer in the jth firm, where $\theta_{ij} \geq 0$, $i = 1, \ldots, I$; $j = 1, \ldots, J$; and $\sum_{i=1}^{I} \theta_{ij} = 1$, $j = 1, \ldots, J$.
- Firm's profit: $\pi_j = p \cdot y_j$, $j = 1, \ldots, J$
- Consumer's budget constraint:

$$p \cdot x_i \leq p \cdot \omega_i + \sum_{j=1}^{J} \theta_{ij} \pi_j,\ i = 1, \ldots, I$$

Definitions

Definition 13 $\xi = (\{(X_i, \succsim_i)\}, \{Y_j\}, \{\omega_i\}, \{\theta_{ij}\})$ *is a private ownership economy if the following conditions hold.*

1. *For all $i = 1, \ldots, I$,*
   - (a) *X_i is the consumption set for the ith consumer.*
   - (b) *$\succsim_i$ is the weak order of the ith consumer.*
   - (c) *ω_i is the initial endowment vector of the ith consumer.*
2. *Y_j is the production set of the jth firm, $j = 1, \ldots, J$.*
3. *$\theta_{ij} \geq 0$ is the share of the ith consumer in the jth firm, $i = 1, \ldots, I$, $j = 1, \ldots, J$, such that $\sum_{i=1}^{I} \theta_{ij} = 1$, $j = 1, \ldots, J$.*

Definition 14 *Assume that we have an economy where consumption sets, initial endowments, and production sets are given. The allocation (state)* $(x_1, \ldots, x_I; y_1, \ldots, y_J)$ *is feasible (attainable) if* $x_i \in X_i$, $i = 1, \ldots, I$, $y_j \in Y_j$, $j = 1, \ldots, J$, *and* $\sum_{i=1}^{I} x_i = \sum_{j=1}^{J} y_j + \sum_{i=1}^{I} \omega_i = \sum_{j=1}^{J} y_j + \bar{\omega}$.

Definition 15 $(x_1^*, \ldots, x_I^*; y_1^*, \ldots, y_J^*; p^*)$ *is a competitive equilibrium for a private ownership economy if all of the following conditions are satisfied.*

1. $(x_1^*, \ldots, x_I^*; y_1^*, \ldots, y_J^*)$ *is a feasible state.*
2. *For each* j, $j = 1, \ldots, J$, $p^* \cdot y_j^* \geq p^* \cdot y_j$ *for every* $y_j \in Y_j$. *Let*

$$\pi_j(p^*) = p^* \cdot y_j^*,$$

   $j = 1, \ldots, J.$
3. *For all* i, $i = 1, \ldots, I$,
   (a) $p^* \cdot x_i^* \leq p^* \cdot \omega_i + \sum_{j=1}^{J} \theta_{ij} \pi_j(p^*)$.
   (b) $x_i^* \succsim_i x_i$ *for all* $x_i \in X_i$ *such that*

$$p^* \cdot x_i \leq p^* \cdot \omega_i + \sum_{j=1}^{J} \theta_{ij} \pi_j(p^*).$$

Welfare Theorems

Theorem 16 (First Fundamental Theorem) *If preferences are locally nonsatiated and* $(x_1^*, \ldots, x_I^*, y_1^*, \ldots, y_J^*)$ *is a competitive allocation, then it is Pareto optimal.*

Theorem 17 (Second Fundamental Theorem) *Suppose that* $(x_1^*, \ldots, x_I^*, y_1^*, \ldots, y_J^*)$ *is Pareto efficient where* $x_i^* \in \mathrm{Int}(X_i)$, $i = 1, \ldots, I$. *Assume that preferences are convex, continuous, and locally nonsatiated, and that* Y *is convex. Then* p^* *exists such that* $(x_1^*, \ldots, x_I^*, y_1^*, \ldots, y_J^*; p^*)$ *is a competitive equilibrium.*

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Debreu. *Theory of Value: An Axiomatic Analysis of Economic Equilibrium.*

Feldman. *Welfare Economics and Social Choice Theory.*

Mas-Colell et al.

Varian, Equity, Envy, and Efficiency, *Journal of Economic Theory* 9 (1974), 63-91.