

ECN 612 - ASSIGNMENT - FALL 2009
General Equilibrium and Welfare Economics

1. Consider a two person, two good, pure exchange economy. The utility functions and initial endowments of the consumers are

$$u_1(x_{11}, x_{21}) = \min \{2x_{11}, x_{21}\}; \omega_1 = (10, 0)$$

and

$$u_2(x_{12}, x_{22}) = \min \{x_{12}, 2x_{22}\}; \omega_2 = (0, 10).$$

- (a) Represent the contract curve for the economy in an Edgeworth Box.
- (b) Algebraically and graphically determine the competitive price ratio, provided that it exists. Then, if possible, determine the competitive allocation.
- (c) Is the allocation that you determined in (b) Pareto optimal?
- (d) Now assume that #1's utility depends on the amount of the first good that #2 consumes. Specifically, assume that

$$u_1(x_{11}, x_{21}) = \min \{2x_{11} - x_{12}, x_{21}\}.$$

Otherwise everything else is the same. Determine the competitive price ratio. Is the competitive allocation Pareto optimal? Explain.

2. Consider a two person, two good, pure exchange economy. The utility functions and total endowments are given in the following.

$$u_1(x_{11}, x_{21}) = x_{11}^\alpha x_{21}, \alpha > 0$$

$$u_2(x_{12}, x_{22}) = x_{12}^\beta x_{22}, \beta > 0$$

$$\bar{\omega} = (6, 6)$$

- (a) Determine the contract curve for the economy. When is it a straight line?
- (b) Assume that initial endowments are

$$(\omega_{11}, \omega_{21}) = (4, 4); (\omega_{12}, \omega_{22}) = (2, 2).$$

Determine excess demand functions for both goods. Then determine the competitive price ratio and allocations. What happens to the competitive outcome if α increases and β is constant? Explain intuitively. Are the allocations Pareto optimal? Verify this directly, given your result in (a).

- (c) For the allocation of endowments in part (b), determine the offer curves for the individuals. Verify that each individual's competitive allocation is on his/her offer curve.
- (d) Assume that $\alpha = \beta = 1$ and that

$$(\omega_{11}, \omega_{12}) = (2, 4), (\omega_{21}, \omega_{22}) = (4, 2).$$

Assume that the first individual behaves like a monopolist and sets a single price ratio; assume that the second individual takes this ratio as given. What price ratio will the first individual set? What are the resulting allocations? How do the monopoly price and allocations compare with the competitive ones? Explain intuitively. Illustrate your results in an Edgeworth Box.

3. Consider a two person, two good, pure exchange economy. The utility functions and total endowments are given in the following.

$$u_1(x_{11}, x_{21}) = x_{11}^2 x_{21}^2$$

$$u_2(x_{12}, x_{22}) = \frac{1}{2} \log x_{12} + \frac{1}{2} \log x_{22}$$

$$\bar{\omega} = (20, 10)$$

Is the equal division allocation (with obvious meaning) Pareto optimal? Why or why not? Could the allocation be attained as an outcome of the competitive process? If not, explain. If so, then determine an appropriate endowment division and competitive price ratio.

4. Assume an economy with two individuals (Ada and Bill) and two produced goods, tea (t) and jam (j). The individuals have the utility functions

$$u_A(t_A, j_A) = \frac{1}{4} \log t_A + \frac{3}{4} \log j_A$$

and

$$u_B(t_B, j_B) = \frac{1}{2} \log t_B + \frac{1}{2} \log j_B.$$

Tea and jam are produced according to the production functions

$$t = (\min\{K_t, L_t\})^\alpha$$

and

$$j = (K_j L_j)^{\frac{1}{2}},$$

where K is capital, L is labor, and $\alpha > 0$. Assume that Ada owns the endowment of 1 unit of capital in the economy and Bill owns the endowment of 1 unit of labor in the economy.

- (a) Answer each of the following parts ((i)-(iii)): true, false, or uncertain. Thoroughly justify your answers.
- (i) A competitive equilibrium is not defined if $\alpha > 1$.
 - (ii) The competitive price ratios and allocation (if they exist) are independent of the size of Ada's and Bill's ownership shares of the firm which produces tea.
 - (iii) The competitive price ratios and allocation (if they exist) are independent of the size of Ada's and Bill's ownership shares of the firm which produces jam.

Throughout the rest of the problem, assume that Ada owns the firm which produces tea and Bill owns the firm which produces jam. Assume that $\alpha = \frac{1}{2}$.

- (b) Determine the competitive price ratios and competitive allocation.
- (c) Is the competitive allocation you determined in (b) Pareto optimal? Why or why not?
5. Consider a pure exchange economy with two individuals, Ada and Bill, and two goods, x and y . Assume that Ada and Bill have utility functions and endowment vectors

$$u^A(x_A, y_A) = x_A^{\frac{1}{3}} y_A^{\frac{2}{3}}, \quad \omega_A = (0, 10),$$

$$u^B(x_B, y_B) = \min\{x_B, y_B\}, \quad \omega_B = (10, 0),$$

respectively. Answer the following parts.

- (a) Suppose that Bill behaves "monopolistically", while Ada is a price taker. What price would Bill set? What would be the resulting allocation? Is it Pareto efficient? Justify your answer.
 - (b) Now assume that Ada behaves "monopolistically", while Bill is a price taker. What price would Ada set? What would be the resulting allocation? Is it Pareto efficient? Justify your answer.
6. Consider a two consumption good (g , guns, and b , butter), and one input (x , oil) economy, with two consumers, each of whom has the utility function

$$u(b, g) = bg.$$

There is one firm which uses oil to produce the consumption goods according to the relation

$$\alpha g^2 + b = x,$$

where $\alpha > 0$. One consumer is endowed with 1 unit of oil and the other owns all of the firm.

- (a) Determine the competitive price ratios and allocation.
- (b) Is the allocation that you found in (a) Pareto optimal? Thoroughly explain.

- (c) Determine what happens to the competitive allocation and prices when α increases. Show formally and explain intuitively.
7. Consider a two consumption good (g , guns, and b , butter), and one input (x , oil) economy, with one consumer who has the utility function

$$u(b, g) = b^\alpha g^{1-\alpha},$$

where $0 < \alpha < 1$. There is one firm which uses oil to produce the consumption goods according to the relation

$$\max \{g, b\} = x.$$

The consumer is endowed with 1 unit of oil and owns all of the firm.

- (a) Determine the competitive price ratios and allocation.
- (b) Is the allocation that you found in (a) Pareto optimal? Thoroughly explain.
- (c) Determine what happens to the competitive allocation and prices when α increases. Show formally and explain intuitively.
8. MC 16.G.5
9. Assume that both Ada and Bill have the utility function

$$u(x, y) = \min \{x, y\},$$

and that

$$\bar{\omega} = (10, 5).$$

Determine the set of fair allocations.

10. Assume that Ada has the utility function

$$u_A(x_A, y_A) = x_A y_A,$$

Bill has the utility function

$$u_B(x_B, y_B) = 2x_B + y_B,$$

and

$$\bar{\omega} = (10, 10).$$

- (a) Determine the set of fair allocations.
- (b) Determine feasible allocations $((\tilde{x}_A, \tilde{y}_A), (\tilde{x}_B, \tilde{y}_B))$ and $((\hat{x}_A, \hat{y}_A), (\hat{x}_B, \hat{y}_B))$, where $((\tilde{x}_A, \tilde{y}_A), (\tilde{x}_B, \tilde{y}_B))$ is equitable, but $((\hat{x}_A, \hat{y}_A), (\hat{x}_B, \hat{y}_B))$ is not, and $((\hat{x}_A, \hat{y}_A), (\hat{x}_B, \hat{y}_B))$ is Pareto superior to $((\tilde{x}_A, \tilde{y}_A), (\tilde{x}_B, \tilde{y}_B))$, i.e., $(\hat{x}_i, \hat{y}_i) \succsim_i (\tilde{x}_i, \tilde{y}_i)$ for every $i \in \{A, B\}$ and $(\hat{x}_j, \hat{y}_j) \succ_j (\tilde{x}_j, \tilde{y}_j)$ for some $j \in \{A, B\}$.