

ECN 612 - ASSIGNMENT - FALL 2009
Public Goods

1. Assume that $MRT_{xy} = 1$. Assume that consumers have the utility functions

$$u^i(x, y_i) = \beta_i \log x + y_i,$$

$i = 1, \dots, I$. Show that the i th consumer's marginal rate of substitution is

$$\frac{\beta_i}{x}$$

and the optimal level of public good is

$$x^* = \sum_{j=1}^I \beta_j.$$

2. Assume that consumers have the utility functions

$$u^i(x, y_i) = 2\beta_i x^{\frac{1}{2}} + y_i,$$

$i = 1, \dots, I$.

(a) Show that the optimal level of public good is $x^* = \left(\sum_{j=1}^I \beta_j \right)^2$.

(b) Assume that the government knows the form of the v_i 's, but does not know the β_i values. Show that for individual i submitting a truthful β_i is a dominant strategy.

(c) Show that the Clarke mechanism can yield deficits.

For the rest of the problems, assume throughout that $MRT_{xy} = 1$, $p_y = 1$. For questions 3 through 7, assume a three consumer world and assume that $\omega_i = 1$, $i = 1, 2, 3$.

For questions, 3 through 6, assume that consumers have the utility function

$$u^i(x, y_i) = 4\beta_i x^{\frac{1}{4}} + y_i,$$

$i = 1, 2, 3$.

3. (a) Determine the Lindahl prices.
(b) Show that Consumer 1 has an incentive to lie.
4. (a) Determine the Clarke taxes.
(b) Can the mechanism ever yield a deficit? A surplus?
5. (a) Determine the Groves taxes (as laid out on the handout).
(b) Does the mechanism yield a surplus?
6. (a) Determine the Groves-Ledyard taxes (as laid out on the handout).
(b) Can you show that the mechanism is not individually rational?
7. Assume that consumers have the utility function

$$u^i(x, y_i) = -\frac{1}{2}x^2 + \theta_i x + y_i$$

$i = 1, 2, 3$. Also, assume that

$$S_i = \frac{1}{6} \left(\sum_{\ell \neq i} \tilde{\theta}_\ell - 1 \right)^2 + \frac{1}{6} \sum_{\substack{j \neq k \\ j, k \neq i}} \tilde{\theta}_j \tilde{\theta}_k - \frac{1}{18},$$

where $\tilde{\theta}_i$ is individual i 's declared value for θ_i , $i = 1, 2, 3$.

Assume that the mechanism has the same structure as that of the Clarke Tax Mechanism, except that the Clarke tax is supplemented by S_i .

- (a) Is truth a dominant strategy?
- (b) Does the system yield a surplus?

8. Prove that the Clarke and Groves Mechanisms are Individually Rational.