**Strategic Substitutes or Complements?**
**The Game of Where to Fish**

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**Abstract**
The "global game with strategic substitutes and complements" of Karp et al. (2007) is used to model the decision of where to fish. A complete information game is assumed, but the model is generalized to $S > 1$ sites. In this game a fisherman's payoff depends on fish density in each site and the actions of other fishermen which can lead to congestion or agglomeration effects. Stable and unstable equilibria are characterized, as well as notions of equilibrium dominance. The model is applied to the Alaskan flatfish fishery by specifying a strategic interaction function (response to congestion) that is a non-linear function of the degree of congestion present in a given site. Results suggest that the interaction function may be non-monotonic in congestion.

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## 1. Introduction

Understanding congestion externalities is important in many settings but is particularly relevant to natural resource economics. For example, Timmins and Murdock (2007) consider valuation of recreational sites while taking into account the effects of congestion on an individual's preferences over those sites. In the commercial fishing industry, vessels may respond to congestion in their decision of "where to fish." In other words, vessel captains behave strategically, and their location decisions are influenced by their interaction with other participants in the fishery. Vessels may agglomerate to increase safety as nearby vessels respond more quickly to distress signals (safety in numbers), while severe congestion may decrease safety as fishing gear become tangled. In the former scenario the payoff to fishing a site is increasing in the number of vessels selecting the same site (strategic complements), while in the latter the payoff is decreasing in the number of vessels (strategic substitutes). The existence of both of these strategic responses (complements or substitutes) implies a strategic *interaction function* that is non-monotonic in congestion. The purpose of this paper is to specify a complete information game with strategic substitutes and complements (Karp et al., 2007), to characterize the equilibria that may exist, and to use vessel location data from the Bering Sea and Aleutian Island (BSAI) flatfish fishery to estimate a non-monotonic interaction function within a random utility model framework (Bayer and Timmins, 2007; Timmins and Murdoch, 2007). Results suggest a non-monotonic interaction function exists within the fishery.

The departure point for models of strategic complements or substitutes is the binary-action, game of complete information (Bulow et al., 1985). If payoffs are monotonically increasing in congestion (strategic complements), then a unique equilibrium generally exists, while if payoffs are monotonically decreasing in congestion (strategic substitutes), then multiple equilibria may exist.

Complete information holds when players possess full-knowledge of some market signal, and players generally take the action when the market signal is strong. Karp et al. (2007) extend the binary-action model in interesting ways. First, they allow for an interaction function that is quasi-concave (non-monotonic) in congestion. That is, a player's payoff and interaction function are first increasing then decreasing as more players choose the crowd-inducing action.[1] Also, theirs is a *global game* in which each player receives a noisy realization of the market fundamental (market signals are heterogeneous across players). They find that when the noise is small and the congestion effect is weak, there exists a pure strategy equilibrium that is monotone in the market signal. That is, as the market signal improves, agents are more likely to take the action. However, if the congestion effect is strong, strategies are non-monotonic in the market signal. That is, a better signal may lead to the *expectation* of greater crowding, causing agents to take the action with lower probability.

In the context of fishing, we extend the Karp et al. (2007) model to the case where agents select one of *many* potential actions (multiple locations where vessels may fish) and where each action has a different market signal.[2] In commercial fisheries the market signal is the density of fish or expected catch in each location. However, complete information of the market signal is assumed. (Although this is a strong assumption it may be reasonable in this case, since all commercial fishing vessels are equipped with devices to detect fish and other vessels.) Therefore, the model is neither a generalization nor a special case of the Karp et al. (2007) model, but the notions of equilibria are similar. Equilibria are characterized (e.g., stable vs. unstable) as are notions of equilibrium dominance, but existence is presupposed. Consequently, the assumption of

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[1] Karp et al. discuss the binary decision to "go to a bar" or "stay home," where players have incentives to go to the bar if it has a good crowd (complements) but is not too crowded (substitutes) and where the market signal is the additively separable quality of the bar. Non-monotonicity of interaction only exists in the "go to the bar" (crowd-inducing) state.

[2] This is analogous to "multiple bars" in the world of Karp et al. (2007).

quasi-concavity of the interaction function of Karp et al. (2007) can be relaxed and equilibria are characterized when the interaction function is more generally non-monotonic. Existence of multiple equilibria when the interaction function is non-monotonic in congestion is discussed in Bayer and Timmins (2005). They develop a sorting model of location choice, but their focus is existence of unique equilibria when the interaction function is monotonic in congestion (in fact, linear). The game presented here is effectively a sorting model of location choice, but the focus is actual characterization of the equilibria in the non-monotonic case.

This model also serves as an intellectual framework for estimation of a random utility model with an interaction function that is non-monotonic in congestion. Bayer and Timmins (2007) develop a random utility model of site choice in the style of Berry et al. (1995) that is linear (positive or negative) in site congestion. In the present paper, their procedure is generalized empirically, allowing utility to be a non-monotonic function of congestion (i.e., the interaction function is non-monotonic).[3] The non-monotonic interaction function and other model parameters are estimated for a panel of BSAI flatfish vessels. Results indicate that vessels may have a non-monotonic response to congestion, and that the general shape of the function is a third-order polynomial.

The paper is organized as follows. The next section develops a complete information game with strategic substitutes and complements and provides a discussion of stable and unstable equilibria, interior, semi-interior and corner equilibria, and notions of equilibrium dominance. Section 3 contains an empirical analysis of the BSAI flatfish fishery using the sorting equilibrium model of Bayer and Timmins (2005, 2007). Evidence that the interaction function in this fishery is non-monotonic in congestion is provided. Section 4 summarizes and concludes.

[3] Per Bayer and Timmins (2005, p. 436), their estimation strategy "at no point requires a unique equilibrium for estimation purposes." Also, it is the particular form of their likelihood function that determines the final equilibrium among many (Bayer and Timmins, 2007, p. 359). Therefore, their strategy is well-suited to the purpose at hand.

## 2. A complete information game with strategic substitutes and compliments

Our basic set-up follows Karp et al. (2007) with the restriction of complete information but generalized for several actions. An individual is considering which of $S$ actions to take, $a \in \{1,...,S\}$. We couch our discussion in terms of a visit to a fishing site $s = 1,...,S$ and associate the action $a = s$ with "visiting site *s*". We assume, without loss of generality, that "not fishing" (staying in port) is not an option.[4] The individual is small and behaves non-strategically, and the mass of individuals is 1. The utility that each individual receives from choosing "visit site $s$" depends on two factors: an underlying state (fish biomass, expected catch and other site-specific variables), denoted $\theta_s \in \Re$; and the fraction (share) of individuals who undertake the action, $\alpha_s \in [0,\ 1]$. The payoff function $U_s(\alpha_s)$ from choosing "visit site *s*" satisfies:

**Assumption 1.** $U_s(\alpha_s) = \theta_s + f(\alpha_s)$, where $f : [0,\ 1] \to \Re$.

**Assumption 2.** $f$ is a bounded, differentiable function on $[0,\ 1]$ with $f(0) = 0$ and $f'(0) > 0$.

Assumption 1 is identical to assumption 1 of Karp et al. (2007) and implies that visiting location $s$ increases with the underlying state (market signal). Assumption 2 ensures that the derivative of profits is well-defined. It also ensures that there are agglomeration effects (increasing profits) in any site that contains zero (or very few) vessels. This is not needed in what follows; it is there to ensure that any vacant site with $\theta_s \geq 0$ has positive probability of a visit.[5] Save for non-monotonicity of the interaction function, these assumptions are consistent with those maintained in the sorting model of Bayer and Timmins (2005, p.465), which form the basis of the empirical model in section 3. Therefore, the theoretical model is well-suited to serve as an intellectual framework for the empirical analysis. In what follows, existence of equilibria is presumed.

[4] This assumption may be problematic if the goal is to inform conservation policy, where fishing zone closures may lead "not fishing" being optimal. The goal here is to develop notions of equilibria in this setting and not to inform policy.

[5] Ultimately, if a site is not visited, then it is viewed as not contained in the set of actions and is eliminated.

*Characterization of Equilibria*

Since the profit function is non-monotonic there may be multiple equilibria (Bayer and Timmins, 2007). Let equilibria be described by the $S$-tuple $[\alpha_1^*,...,\alpha_S^*]$, where $\sum_s \alpha_s^* = 1$. We now develop basic notions of equilibria and stability. In particular, we define interior equilibria, semi-interior equilibria, and corner equilibria. First, when an equilibrium exists at a point on a payoff function that is increasing in congestion, $U_S'(\alpha_S^*) > 0$, we say that in equilibrium the location exhibits an *agglomeration effect*, otherwise we say it exhibits a *congestion effect*. Obviously the former creates incentives for vessels to move (movers) into the site exhibiting the effect, while the later has an opposite effect. Our notion of stability is based on a process of *tâtonnement* or *grouping*. When an individual deviates from the equilibrium action (moves from an equilibrium site to a different site to achieve higher utility), the effect is negligable because the individual possess zero mass. However, groups of individuals can affect an equilibrum in the same sense that groups of buyers and sellers in a Walrasian auction effect equilibrium prices. Logically, groups will only deviate from equlibria into sites exhibiting agglomeration effects. However, individuals possess complete information (instrumentation to detect the location of fish and other vessels), so movers may realize that their former (equilibrium) site exhibited congestion effects which were relieved upon their moving, thus increasing utility for stayers. If the gain in utility for stayers exceeds the gain in utility for movers, then movers will *return* to the equilibrium site, so the equilibrium point is considered stable. The actions of the groups are not coordinated (despite the "grouping" concept), so our notion of stability can produce equilibria that are suboptimal (coordination failure). Indeed, the previous example implied that both movers and stayers were made better off by the deviation of movers, yet the movers had an incentive to return, garnering lower utility for each group. Additionally, since there exists the potential for multiple equilibra, the system may "get stuck" at a low payoff equilibrium, even though there may exist stable equilibria where all players can be made better off. These are precisely the notions of equilibria and stability that Karp et al. (2007) employ in the complete information case.

**Definition 1.** Interior Equilibrium. An interior equilibrium is the $S$-tuple $[\alpha_1^*,...,\alpha_S^*]$, where $\sum_s \alpha_s^* = 1$, $\alpha_s^* \in (0,\ 1)$, and $U_1(\alpha_1^*) = ... = U_S(\alpha_S^*)$.

**Definition 2.** Strong Stability. A *strongly stable* interior equilibrium is one with congestion effects at each site. That is, $U_s'(\alpha_s^*) \leq 0$ for all $s = 1,...,S$ in equilibrium.

The salient feature of a strongly stable equilibrium is that global congestion effects create disincentives to switch sites in equilibrium. No mover can ever be made better off.

**Definition 3**. Instability. An *unstable* interior equilibrium is one for which there exists some $s$ with agglomerations effects, $U_s'(\alpha_s^*) > 0$, and $U_s(\alpha_s^* + \varepsilon) > U_j(\alpha_j^* - \varepsilon)$ for all $j \neq s$ and for $\varepsilon$ small and positive.

The idea of instability here is that there is an agglomeration effect in some location $s$, creating incentives to move out of $j \neq s$ into $s$. However, the movers $(+\varepsilon)$ are made better off, and stayers $(\alpha_j^* - \varepsilon)$ are made worse off, so everyone has an incentive to move into $s$. Sites $j \neq s$ can exhibit either agglomeration or congestion effects in equilibrium. If all $j \neq s$ exhibit congestion effects, then for the equilibrium to be unstable the benefits of leaving, $U_s(\alpha_s^* + \varepsilon)$, must outweigh the benefits of staying when others leave, $U_j(\alpha_j^* - \varepsilon)$. A sufficient condition for an equilibrium to be unstable is that at least two sites exhibit agglomeration effects.

Figure 1a contains an example for the case $S = 2$ and $f$ concave. In the figure, the payoff in site 1 is given by the thin curve starting at the point $(0,\ \theta_1)$ for which $\alpha$ is increasing as we move from west to east along the domain. The payoff in site 2 is given by the thick curve starting at the point $(1, \theta_2)$ for which $1-\alpha$ is increasing as we move from east to west along the domain. Therefore, in this figure $\alpha = \alpha_1$ and all points along the $\alpha$-axis satisfy the adding up condition $\alpha_2 = 1-\alpha$ (the axis is a 1-simplex). Point C is a strongly stable interior equilibrium, because it is interior and each site exhibits congestion effects in equilibrium (i.e., $U_1'(\alpha_C)$ and $U_2'(1-\alpha_C)$, are negative). If a group of vessels switched from site 1 to site 2 at point C, then their payoff would be lowered by moving southwest along the thick curve, $U_2(1-\alpha)$. Similarly, If a group of vessels switched from site 2 to site 1 at point C, then their payoff would be lowered by moving southeast along the thin curve, $U_1(\alpha)$. Hence, there is no incentive to deviate from C. Point B is an unstable interior equilibrium, because there are agglomeration affects at site 1, $U_1'(\alpha_B) > 0$, and congestion effects at site 2, such that movers are made better off, relative to stayers. That is, $U_1(\alpha_B + \varepsilon) > U_2(1-\alpha_B - \varepsilon)$. Vessels in site 2 have an incentive to move to site 1 (a positive $\varepsilon$ move to the northeast along the thin curve), and they receive greater payoff than those remaining in site 2 (a negative $\varepsilon$ move to the northeast along the thick curve ). As such, the system tends to move towards point C, a strongly stable equilibrium.

**Definition 4.** Corner Equilibrium. A corner equilibrium is the $S$-tuple $[\alpha_1^*,...,\alpha_S^*]$, where $\alpha_s^* = 1$ for some $s$, $\sum_s \alpha_s^* = 1$, and $U_s(1) > \theta_j$ for all $j \neq s$.

At a corner equilibrium, payoffs in each site are not necessarily equal. In figure 1a, point A (all vessels fish in site 2) is a corner equilibrium because $U_2(1) > \theta_1$ so that there is no incentive for vessels to switch from site 2 to site 1. Notice that there is no sense in which corner equilibria can be strongly or weakly stable, since the payoffs across sites may not be equal. The point D (all vessels in site 1) is *not* an equilibrium, because $U_1(1) < \theta_2$ and vessels can achieve higher payoff by switching to site 2. This moves the system from D toward the stable equilibrium C. Notice that at point A in the figure, payoff is lower than at point C, yet the system can still be "stuck" at A. This is a coordination failure.

The corner equilibrium gives rise to notions of locational dominance. Let $\overline{f} = \max_\alpha f(\alpha)$ and $\underline{f} = \min_\alpha f(\alpha)$.

**Definition 5.** Strong Dominance. Site $s$ *strongly dominates* site $j$ if $\theta_s - \theta_j > \overline{f} - \underline{f}$.

If *any* site $j$ is strongly dominated by any site $s$, then site $j$ will not be fished in equilibrium. That is, $\alpha_j^* = 0$. The result follows from the fact that any vessel in site $j$ will *always* have a higher payoff if it switches to site $s$.

**Definition 6.** Weak Dominance. Site $s$ *weakly dominates* site $j$ if $\theta_s - \theta_j > f(\alpha) - f(1-\alpha)$ for all $\alpha \in [0, 1]$.

If site $s$ strongly dominates site $j$, then it weakly dominates $j$. This result follows from the fact that $\overline{f} - \underline{f} \geq f(\alpha) - f(1-\alpha)$ on $[0, 1]$. If $S = 2$ and site 1 weakly dominates 2, then $\alpha_1^* = 1$ and $\alpha_2^* = 0$ in equilibrium.

**Definition 7**. Weak Stability. A *weakly stable* interior equilibrium has for each $s = 1,..,S$ with $U_s'(\alpha_s^*) > 0$, some $j \neq s$ such that $U_j(\alpha_j^* - \varepsilon) > U_s(\alpha_s^* + \varepsilon)$ for $\varepsilon$ small and positive.

At a weakly stable interior equilibrium there appears to be an incentive to deviate into $s$, because $U_s'(\alpha_s^*) > 0$. However, once vessels move into $s$ from any $j \neq s$, the stayers gain higher payoff than the movers, so there is an incentive for movers to return to $j \neq s$. Weak stability is a fairly strong condition, because no matter where vessels leave, staying vessels are *always* better off regardless of where they are. Weak stability means there are incentives to deviate but stronger incentives to return after deviation.

Consider the situation in figure 1b. The graph is read the same as figure 1a, except here point B is an *unstable* interior equilibrium, because at least two locations exhibit agglomeration effects (a sufficient condition). There is an incentive to move to the northeast on the thin curve toward C. There are even stronger agglomeration effects moving to the northwest along the thick curve, since it appears that $U_1'(\alpha_B) > U_2'(1-\alpha_B) > 0$. At point C, site 2 exhibits agglomeration effects, but site 1 exhibits congestion effects. Point C is weakly stable because there is an incentive to move to the northwest along the thick curve toward point B, but along this path stayers are made better off that movers, $U_1(\alpha_C - \varepsilon) > U_2(1-\alpha_C + \varepsilon)$, as congestion in site 1 is mitigated. Point A is a corner equilibrium.

The primary difference between figures 1a and 1b is that $\overline{f}$ occurs to the left of $\alpha = 1/2$ (mostly congestion effect) in figure 1a, while $\overline{f}$ occurs to the right of $\alpha = 1/2$ (mostly agglomeration effect) in figure 1b. In figure 1a, if the interior equilibrium C is obtained, congestion precludes the switching of locations. In figure 1b there are agglomeration effects at the interior equilibrium B. Vessels may be drawn away from their current locations to seek the benefits of these effects. In general, it appears that congestion effects tend to produce stable equilibria while agglomeration effects tend to produce unstable equilibria. Indeed, for a concave interaction function and $\theta_s$ approximately equal across sites, $\overline{f}$ far to the left of $\alpha = 1/S$ (congestion) produces stable interior equilibria, while $\overline{f}$ far to the right of $\alpha = 1/S$ (agglomeration) produces unstable interior equilibria. In fact, if the $\theta_s$ are exactly equal there will be an equilibrium at $\alpha_s^* = 1/S$ for all sites, which will be stable or unstable depending on the location of $\overline{f}$ relative to $1/S$ on the unit interval. This argument **only** holds for concave interaction functions. If the interaction function is more complicated, then these arguments may not hold globally, but our definitions of stable and unstable equilibria are local definitions (in the mathematical sense) so they hold regardless of the nature of the non-monotonicity. It is in this sense that our discussion generalizes the discussion in Karp et al. (2007), which assumes a quasi-concave interaction function,

Given the previous discussion, it should be clear that if the interaction function is monotonically increasing in congestion (pure agglomeration), then any interior solution will always be unstable. That is, if $f'(\alpha_s^*) > 0$ for all $\alpha_s^* \in [0, 1]$ and $s = 1,...,S$, then the conditions for stability of an interior equilibrium will never be met, and the only stable equilibrium will be for all vessel to go to the location with the largest $\theta_s$, a corner equilibrium. Similarly, if the interaction function is monotonically decreasing (pure congestion), then any interior solution will necessarily be strongly stable. This result will always hold as long as assumptions 1 and 2 hold and the interaction function is monotonic. Indeed, we could relax the assumption that the interaction function, $f$, be the same across locations, and the pure congestion/agglomeration results will hold. Even if the payoff functions were multiplicatively (weakly) separable, the results will hold.

It should be noted that if "not fishing" were an option and garnered, say, zero payoff, then the system in figure 1a would be reduce to only those equilibria with non-negative payoff. This drives home the notion that with additive separability of the payoff function and no option to "not fish" only the relative differences of the $\theta_s$ matter for a given response function Therefore, we could vary the magnitude of the non-negative difference $\theta_2 - \theta_1$ and see how it affects equilibria in the $S = 2$ concave system of figure 1a. We do this by fixing $\theta_1$ and increasing $\theta_2$. As $\theta_2 - \theta_1$ increases from zero, there are mixing equilibria at point C. Over this interval equilibrium payoff is increasing and the share of vessels in site 1 is decreasing. ("More fish" is always welfare improving regardless of congestion effects.) There is also a pure strategy equilibrium a point A, where all vessels fish site 2 and the payoff is $U_2(1) > \theta_1$. As we continue to increase $\theta_2 - \theta_1$, C will eventually become weakly stable as $U_1'(\alpha_C) > 0$ but $U_1(\alpha_C + \varepsilon) < U_2(1 - \alpha_C - \varepsilon)$. Eventually points B (unstable) and C (weakly stable) become a single, weakly stable point at which $U_1$ and $U_2$ are tangent. Therefore, over the non-negative range of $\theta_2 - \theta_1$, there is one pure strategy equilibrium at point A and an interior equilibrium at point C, which starts out strongly stable but then becomes weakly stable.

> **Definition 8**. Semi-interior Equilibrium. Let $S^* \subset \{1,...,S\}$ such that $U_s(\alpha_s^*) = U_j(\alpha_j^*)$ and $\alpha_s^* \in (0,\ 1)$ for all $j, s \in S^*$ and $\sum_s \alpha^*_{s \in S^*} = 1$, then the $S$-tuple $[\alpha_1^*,...,\alpha_S^*]$ is a semi-interior equilibrium.

Essentially there may exist equilibria where some sites are not visited, $s \notin S^*$, and those sites that are visited, $s \in S^*$, possess an interior solution. An example of such an equilibria might occur in figure 1a if we added a third site, whose $\theta_3$ was so small that it were strongly dominated by sites 1 and 2. In this case point C along with $\alpha_3 = 0$ would be a semi-interior solution. A semi-interior solution will be stable or unstable based on definitions 1, 2 and 7, but for $S^*$ and not $\{1,...,S\}$. Definition 8 leads to a third concept of dominance.

> **Definition 9**. Equilibrium Dominance. A site $j$ is dominated by a stable semi-interior equilibrium if the equilibrium exists and $j \notin S^*$.

The point here is that a site may neither be strongly nor weakly dominated by any or all other sites individually, but it may still be dominated by a particular stable equilibrium point. In other words, a site that is not strongly or weakly dominated may remain unvisited in equilibrium. This is a different form of dominance than those previously defined, so it is difficult to make direct comparisons.

## 3. Empirics

In the fishery economics literature the theory of congestion externalities was initially proposed by Brown (1974), however to the best of our knowledge, empirical work has not been conducted within commercial fisheries. There have been a few empirical papers that investigate congestion in the recreational demand literature (McConnell 1977; Timmins and Murdock 2007). Empirical modeling of preferences in the presence of congestion externalities is challenging because congestion, $\alpha$, is endogenously determined by an agent's selection of where to locate which is, in turn, a function of congestion. While this endogeneity is certainly present in the equilibrium model of section 2, the assumption of a continuum of representative agents with zero mass greatly simplifies that analysis, and of course parameter identification and estimation is not an issue. However, the endogeneity issue becomes the central challenge for any empirical implementation of this equilibrium model, particularly since agents are (in reality) discrete, have positive mass (in the distribution of agents), and have heterogeneous tastes and technology. Of course most empirical implementations of equilibrium models of strategic behavior would suffer from these same issues, so we are not alone. That being said, there remains a clear and strong connection between the concepts of the last section and those of the empirics that follow, so the appropriateness of the theory for providing an intellectual framework for the empirical model should be evident.

Consider a vessel (fisherman) deciding among $S$ sites, each having payoffs $R_s$ and costs of access equal to $C_s$. Based on the game-theoretic model of section 2, suppose that the agent’s payoff in site $s$ is impacted by share of the fleet at site $s$ via the interaction function $f(\alpha_s)$. Following a standard random utility model, the vessel chooses site $s$ over all other $j$ sites if:

$$\beta_1 R_s + \beta_2 C_s + f(\alpha_s) + \varepsilon_s > \beta_1 R_j + \beta_2 C_j + f(\alpha_j) + \varepsilon_j \ \forall j, s \in S . \quad (1)$$

The site-specific error terms are observed by vessels but unobserved by the econometrician. If site $s$ is selected, it may be due to high expected revenues, low costs, or a high value of $\varepsilon_s$. We note here that the observables and unobservables in equation 1 are not vessel-specific. While this is in keeping with the theoretical model of section 2, ultimately we incorporate vessel-specific information into the empirical model and, hence, deviate from the theory.[6] If other fishermen are aware of this unobservable desirable attribute of site $s$, then congestion is also likely to be high at the site. Consequently, there is an unavoidable correlation between the share of the fleet visiting the site, $\alpha_s$, and unobserved site-specific attributes, $\varepsilon_s$. This makes identification of $f$ challenging. Bayer and Timmins (2007) propose a two-stage instrumental variable approach to identify a monotonic $f$. That is, they let $f(\alpha_s) = \beta_3 \alpha_s$, say, in equation 1 and consistently estimate $\beta_1$, $\beta_2$ and $\beta_3$. The sign of $\beta_3$ determines if players strategies are purely complements or substitutes. As discussed in Timmins and Murdock (2007), the model is similar to a static, simultaneous-move Nash bargaining model where agents make decisions conditional on their expectations regarding competing agents. Their model is particularly relevant in our case, since the underlying equilibrium of the spatial distribution of economic activity allows the parameters in equation 1 to be estimated. However, we moderately extend the empirical model of Bayer and Timmins (2007) by assuming that a sorting equilibrium across sites occurs *each day* throughout the season.[7] Thus, we incorporate a time dimension in our estimation methodology, and the payoff vessel $i$ derives from visiting site $s$ at time $t$ is,

$$V_{ist} = \delta_{st} + G_{ist}^{'}\gamma + \varepsilon_{ist} \quad i = 1,...,N\ ;\ s = 1,...,S\ ;\ t = 1,...,T \tag{2}$$

[6] In this setting it may be possible to solve a global game in the style of Karp et al. (2007) where agents receive an individual market signal, thereby allowing for the incorporation of agent-specific observables and unobservables. This is currently being considered by the second author.
[7] In the Alaskan Yellowfin Fishery during the years 2000-2004, fishing occurs on 957 unique days across 71 sites.

where $\delta_{st}$ is given by the partial linear model

$$\delta_{st} = X_{st}^{'}\beta + f(\alpha_{st}) + \eta_{st} \,. \tag{3}$$

The matrix $X_{st}$ contains site-specific data (e.g., revenues, by-catch, etc.) , $G_{ist}$ contains information that varies across vessels (distance traveled to arrive at the location), $\delta_{st}$ (an unobservable constant) represents the baseline payoff from visiting site $s$ before accounting for vessel-specific information, and $\eta_{st}$ is the unobservable portion of site-specific payoff. Equation 3 corresponds to the site-specific payoff function used to characterize equilibria in assumption 1. The $X_{st}^{'}\beta + \eta_{st}$ portion of equation 3 is the time-varying equivalent of $\theta_s$, and we are effectively evaluating vessel interaction over daily equilibria in assumption 1. This is the sense in which our theoretical model is tied to the empirical analysis. Essentially, we estimate $\theta_s$ and $f$ using the partial linear model of equation 3. We may then examine the functional form of $f$ to understand how strategic responses to congestion may change with the level of congestion.

Each vessel maximizes its utility over $S$ spatial alternatives, given their expectations on the actions of the others which influences $\alpha_{st}$. Assuming that $\varepsilon_{ist}$ is distributed *i.i.d.* extreme value, the probability that the $i^{th}$ vessel selects site $s$ in period $t$ is,

$$p_{ist}(\delta,\gamma) = \frac{\exp\{\delta_{st} + G_{ist}^{'}\gamma\}}{\sum_{j=1}^{S}\exp\{\delta_{jt} + G_{ijt}^{'}\gamma\}} \,. \tag{4}$$

Then the predicted share of vessels visiting site $s$ in period $t$ is,

$$\hat{\alpha}_{st}(\delta_t,\gamma) = \frac{1}{N}\sum_{i=1}^{N} p_{ist}(\delta_t,\gamma) \,. \tag{5}$$

The maximum likelihood estimates $\hat{\gamma}$ and $\hat{\delta}$ are obtained from the optimization,

$$\max_{\delta,\gamma} L(\delta,\gamma \mid G_{ist}) = \prod_{i=1}^{N}\prod_{s=1}^{S}\prod_{t=1}^{T}\left(\frac{\exp\{\delta_{st} + G_{ist}^{'}\gamma\}}{\sum_{j=1}^{S}\exp\{\delta_{jt} + G_{ijt}^{'}\gamma\}}\right)^{Y_{iSt}} \tag{6}$$

$$s.t.\quad \alpha_{st}^{*} = \frac{1}{N}\sum_{i=1}^{N} p_{ist}(\delta_t,\gamma) \quad \forall s,t\,, \tag{7}$$

where $\alpha_{st}^{*}$ are the observed equilibrium share of vessels in site $s$ in period $t$, and $Y_{ist}$ equals 1 if vessel $i$ selects site $s$ in period $t$, and zero otherwise.[8] For any estimated value of $\gamma$, a vector of site- and time-specific baseline payoff estimates ($\delta_{st}$) is recovered that preserves the observed sorting equilibrium (i.e., preserves $\alpha_{st}^{*}$ in equation 7). Estimating the $\delta_{st}$ in equations 4-7 is challenging given their large number (more than 5,000) and the requirement that they satisfy the equality constraint in equation 7. Berry (1994) proposes a contraction mapping that simplifies the search over the parameter space and guarantees that the equilibrium shares satisfy equation 7. For each iteration, $\ell$, over the parameter space of the likelihood function, define contraction mapping

$$\hat{\delta}_{st}^{(\ell+1)} = \hat{\delta}_{st}^{(\ell)} - \ln\left[\alpha_{st}^{*} - \hat{\alpha}_{st}(\hat{\delta}_{t}^{(\ell)},\hat{\gamma}^{(\ell)})\right]. \tag{8}$$

The mapping ensures that site-specific constants move towards the observed sorting equilibrium for any value of $\hat{\gamma}$. The technique is amenable to problems having large numbers of constants.

The estimation algorithm proceeds in two-stages. In the first stage equation 6 is maximized subject to 7 with iterations based on 8. In the second stage, the $\hat{\delta}_{st}$, are treated as dependent variables in equation 3, and $f(\alpha_{st})$ is estimated. In what follows the first stage is always the same, however in the second stage a variety of estimation strategies are employed, based on different assumptions on the functional form of $f$ and on the existence of endogeneity through $\alpha_{st}$.

[8] Given that our data are at the "haul" level (individual deployments of fishing gear) *i* is actually defined as the haul. Vessels rarely take more than one haul in each location.

*Data Description*

Data for this analysis come from the Alaska Fisheries Science Center's Observer Program Database, which contains production information on vessels operating within the BSAI with 100% of all spatial production observed for vessels greater than 125 feet in length and with 30% coverage for vessels less than 125 feet. The data were collected during 2000-2004, using federal observers who are aboard each vessel while at sea and who record information on the GIS coordinates of each fishing haul (deployment of gear), the composition of the catch and other biological information relevant to fisheries management. Over this period there were 975 days when vessels fished. Because we do not perfectly observe the spatial behavior of vessels less than 125 in length, we focus the analysis on only those vessels with 100% spatial coverage.[9]

Using the GIS coordinates of each haul, spatial identifications are assigned that are consistent with the statistical reporting zones used by the Alaska Department of Fish and Game. Zones divide the BSAI into grids with each cell's dimension being ½ degree latitude by 1 degree longitude, of which there are 71 unique spatial sites within our data set. Distances are calculated using the centroids of each cell within the grid defining the spatial sites within the BSAI. Price data for the fishery are from the Commercial Fisheries Entry Commission (CFEC) fish ticket and Commercial Operator Annual Report (COAR) and represents the ex-vessel value of fish landed.

The focus is the yellowfin sole fishery operating within the BSAI, which is part of the larger head-and-gut fleet operating within the fishery. Targeting designations are based on National Marine Fisheries Service guidelines, which specify that a vessel is targeting yellowfin sole if more than 50% of its catch is flatfish (yellowfin sole, rock sole, flathead sole, rex sole, etc.) and if more than 70% of that 50% consists of yellowfin sole. In addition, our analysis focuses solely on catcher-

[9] This may cause problems for estimating congestion equilibria, as smaller boats may be more capable of "squeezing" into already congested waters. Excluding them may understate crowding and overstate reactions to congestion.

processors within the BSAI and does not include the smaller catcher vessels operating within the fishery. Although this may seem limiting, the 43 vessels contained in our data set harvest over 80% of the annual yellowfin sole catch each year.

*Estimation*

In equation 4, $G_{ist} = DISTANCE_{ist}$, the distance traveled by vessel $i$ on day $t$ from its last spatial location to location $s$. This distance is a proxy for marginal costs (e.g., fuel, labor, opportunity cost) which are vessel specific.[10] Estimates $\hat{\delta}_{st}$ and $\hat{\gamma}$ are recovered from equations 4-8. The negative and significant (at the 95% level) value for $\hat{\gamma} = \text{-0.0280}$ suggests that, all things being equal, vessel payoffs are lower at more distant sites. The specification selected for equation 3 is,

$$\hat{\delta}_{st} = \beta_{st} + \beta_1 REV_{st} + \beta_2 STDV_{st} + \beta_3 BYCATCH_{st} + f(\alpha_{st}) + \eta_{kt}, \tag{10}$$

where $REV_{st}$ is site- and time-specific revenue (average fleet-wide catch over the last 30 days in each location in each day multiplied by the landed price), $STDV_{st}$ is the standard deviation of those revenues, and $BYCATCH_{st}$ is the expected bycatch (the incidental catch of non-target and non-marketable species also based on average fleet-wide catch over the last 30 days). What follows are a several attempts to estimate $f(\alpha_{st})$ in equation 10.[11]

As a first pass, endogeneity is ignored, so $E[\eta_{st} \mid \alpha_{st}] = 0$, the interaction function is parameterized as the third order polynomial,

$$f(\alpha_{st}) = \lambda_1 \alpha_{st} + \lambda_2 \alpha_{st}^2 + \lambda_3 \alpha_{st}^3, \tag{11}$$

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[10] DISTANCE is the only observable in the analysis that could be constructed to vary over vessels. This is due to limitations in the variability of the other observables once they were allocated to different spatial locations, given our predetermined definition of the spatial resolution.

[11] The specification could also include time-invariant location dummies, but we chose to exclude them. (More on this refinement in subsequent revision.)

and equation 10 is estimated using OLS.[12] In this exercise the $\beta_{st}$ are not identified, so they are dropped. However, this is unimportant if the goal is merely to get a sense the curvature of $f$. Parameter estimate are in the first few columns of table 1, and the estimate of the reaction function is in figure 2. Standard errors are not corrected for any potential temporal or spatial correlations, but again we are only trying to get a sense of the curvature of $f$. Based on table 1, the market signal ($\hat{\delta}_{st}$) is increasing in expected revenues from location $s$, decreasing in the variability of those revenues, and increasing in the expected bycatch. This last result seems odd since vessels tend to avoid bycatch at certain times during the fishing season, however in the final specification the coefficient on bycatch is insignificant. Figure 2 suggests that the interaction function is monotonic in vessel shares when endogeneity is ignored.

Based on the assumption that $E[\eta_{st} \mid \alpha_{st}] = 0$, equation 10 is semi-parametrically estimated using the partial-linear estimator of Robinson (1988) with an arbitrarily selected Gaussian kernel and optimal bandwidth based on least-squares cross validation.[13] The idea is that, even though endogeneity is ignored, the semi-parametric estimator of $f$ will provide a sense of how well the parametric polynomial of equation 11 fits the data. The results of the semi-parametric estimation are in the last few columns of table 1. The coefficients are insignificant, but the plot of the interaction function in figure 3 appears to confirm that the interaction function is close to a third-order polynomial.[14] In fact, the shapes of the functions in figures 2 and 3 are surprisingly similar. The magnitudes are different, but the curvatures are close. In what follows, equation 10 is

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[12] The third-order polynomial was selected based on a series of F-tests . The results are available from the authors.

[13] The bandwidth starting value was $N^{-1/5}$ per Lee (1996), as referenced in the appendix of Van Heeder et al. (2001).

[14] Indeed, the centered curve is a fifth-order polynomial based on the MATLAB Basic Fitting algorithm. However, it matches the third-order polynomial in figure 1 well. The parametric model rejected a fifth-order polynomial.

estimated under the assumption of endogenous shares, so $E[\eta_{st} \mid \alpha_{st}] \neq 0$. Only the parametric approach is considered, since it appears to fit the data quite well.[15]

*Instrumental Variables Estimation*

Bayer and Timmins (2007) develop an instrumental variables approach for identifying congestion/agglomeration effects that leverages the exogenous data in the model as well as the spatial and, in our case, temporal variation to obtain instruments that are correlated with share but are uncorrelated with the error term, $\eta_{st}$. Introduction of non-linear $f$ presents no additional difficulties to this identification approach (Bayer and Timmins, 2007, footnote 18). Theirs is a fully parametric approach, so for the remainder of the paper we assume (based on F-tests) that $f(\alpha_{st})$ is the third order polynomial in equation 11. First, equation 10 is estimated with the restriction $\lambda_1 = \lambda_2 = \lambda_3 = 0$ to recover estimates $\{\hat{\beta}_{st}, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\}$.[16] Then instrumented vessel shares are

$$\tilde{\alpha}_{st} = \frac{1}{N}\sum_{i=1}^{N} \tilde{p}_{ist} \ ,$$

where,

$$\tilde{p}_{ist} = \frac{\exp\left\{\hat{\beta}_{st} + \hat{\beta}_1 REV_{st} + \hat{\beta}_2 STDV_{st} + \hat{\beta}_3 BYCATCH_{st} + \hat{\gamma} DISTANCE_{ist}\right\}}{\sum_{j=1}^{S} \exp\left\{\hat{\beta}_{jt} + \hat{\beta}_1 REV_{jt} + \hat{\beta}_2 STDV_{jt} + \hat{\beta}_3 BYCATCH_{jt} + \hat{\gamma} DISTANCE_{ijt}\right\}}, \qquad (16)$$

and where $\hat{\gamma}$ is the estimate from the original likelihood function in equation 6. Then OLS is used to estimate equation 10 with $\tilde{\alpha}_{st}$ substituted for $\alpha_{st}$ in equation 11.

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[15] Additionally, it is not clear that that the function can be identified semi-parametrically under the assumption of endogenous shares. I has been suggested that a control function approach might help with identification, but this is left for future research.

[16] In their linear setting, Timmins and Murdoch (2007) impose the restriction $\lambda_2 = \lambda_3 = 0$, letting endogenous congestion enter into the first-stage regression. Our restriction does not affect consistency of their procedure, but letting $\lambda_1$ be a free parameter in our first-stage produced nonsensical results. Our sense is that congestion endogeneity is severe in this setting, and including it in the first stage is possibly biasing our results.

Results of the above procedure are in table 2. All three curvature parameters are significant and imply a non-monotonic interaction function, which is illustrated in figure 4 for the range of instrumented shares used in the regression. The thick curve is the conditional mean of the shares and the dashed curves are the 95% upper and lower confidence bounds, based on the standard errors in table 2. There are two points in the curve where the estimated $f'(\tilde{\alpha}_{st}) = 0$: when $\tilde{\alpha}_{st}$ equals 0.3565 and when it equals 0.6066. When less than 35.65% of vessels visit a site on a given day, there are agglomeration effects, but, when 35.65% to 60.66% of vessels visit, there are congestion effects. Finally, when $\tilde{\alpha}_{st}$ is beyond 60.66% (but below 80%), vessels will, again, agglomerate. [17]

Figure 5 shows the distribution of $\tilde{\alpha}_{st}$ resulting from the sorting equilibria. The majority of the distributional mass, approximately 95% of it, lies well below the 0.3565 threshold of agglomeration within the fishery, while the remaining 5% lie primarily between 0.3565 and 0.6066. In fact, only 0.3% of the distributional mass lies above 0.6066. It is for this small number of observations that the agglomeration effect reoccurs in figure 4. Although this last segment of behavior is inconsistent with a (quasi-) concave interaction function, it is only realized for a small number of observations (this is reflected in the downward slope of the lower confidence bound in the figure). Furthermore, these observations all share the common trait that only two sites where visited on that day and therefore one can reasonably treat these observations as outliers. Consequently, the interaction function may be (quasi-) concave. Alternatively, these extreme observations could be the outcome of complex herding behavior equilibria (Bikhchandani et al. 1992) in which the private vessel's signal of stock abundance is outweighed by the public

[17] While it is technically an inconsistent estimation procedure due to the non-linearity of *f*, estimation of the semi-parametric model using instrumented (vs. actual) shares produces a similarly shaped, non-monotonic function that is not presented here.

information provided by the observed behavior of the other vessels. However, private information is inconsistent with our complete information model.

In summary, the empirical results suggest a non-monotonic interaction function. Therefore, both agglomeration and congestion effects are present in the strategic behavior of vessels in the yellowfin sole fishery. There are a number of reasons why vessels may wish to fish in regions where other vessels are located. Safety was mentioned in the introduction as a potential motivation for agglomeration. Additionally, if vessels tend to fish in the same regions, it reduces their exposure to the economic risk of deviating from the behavior of everyone else in the fishery and potentially earning less relative to others within the fleet. Therefore, agglomeration is consistent with risk-adverse spatial fishing behavior.[18] In addition, the concave portion of $f(\alpha_{st})$ suggests that the benefits of agglomeration are eventually mitigated. This may be consistent with the notion of serial depletion in fisheries. Serial depletion arises when repetitive fishing in a site lowers catch on subsequent hauls. Therefore, although there are agglomeration effects, they may be diminishing, because too many fish are likely to be taken.

As a (simplistic) thought experiment, the nature of equilibria arising from a similarly shaped interaction function are discussed. Suppose $S = 2$ as in figures 1a and 1b, then the estimated reaction function in figure 4 for arbitrary $\theta_s = X_s^{'}\beta$ might be approximated as the curves in figure 6. In the figure we have extrapolated the function beyond the sample maximum value of instrumented shares (beyond $\alpha = 0.8$) based on the third-order polynomial. This may be a mistake (in a statistical sense), but it is what the data suggest the function may look like above this point. Also, the curve is clearly only an approximation of figure 4. In this figure, points A and E are stable corner equilibria,

[18] Knowing that other vessels are fishing in a location may also provide a positive signal on the abundance of fish within that region, but this argument is not consistent with our model, since this is less of a response to crowding and more of a response to market signal. If this were true it would suggest the market signal is observed with uncertainty.

B and D are unstable interior equilibria, and C is a strongly stable interior equilibrium (i.e., there are congestion effects at each site). However, C is inferior to both A and E. Also, if $\theta_1 > \theta_2$, then E dominates A, otherwise A dominates E. We cannot rule out the possibility that weakly stable equilibria may arise in this situation, but there are none in this particular figure. Of course, based on the lower bound in figure 4, we cannot reject the hypothesis that the interaction function is (quasi-) concave, and we have seen that weakly stable equilibria may arise in such a situation. Finally, since we are only examining 2 of the 71 possible locations to fish, figure 6 necessarily implies that there are 69 locations that are not visited. This situation did actually arise in the data set several times. If this is the case, then the equilibria actually are semi-interior per definition 8.

**4. Conclusions**

This research expands the global game with strategic substitutes and complements of Karp et al. (2007) to a spatial decision model of where to harvest a resource when non-monotonic congestion and agglomeration effects may be present. The theoretical game presented is empirically investigated using data from the BSAI flatfish fishery to determine the extent of strategic substitutes and complements present. Result suggest non-monotonicity of the interaction function. There appear to be agglomeration effects within the BSAI flatfish fishery when the share of vessels in a site is below 40% and congestion effect when the site visitation shares increase beyond that point. While our empirical results suggest that agglomeration may exist later in the profile of the interaction function, confidence bounds indicate that congestion effects may persist. Therefore, our general results indicate that the interaction function is non-monotonic in congestion within the fishery and, perhaps, a (quasi-) concave function of $\alpha_{st}$.

Although this research has expanded spatial choice theory and the empirical characterization of sorting equilibrium modeling, it does leave a number of unanswered questions which will be investigated in future research. One of the more important unanswered questions is how one might precisely characterize equilibria within an empirical model with 71 locational choices. Given the structure of the empirical model, it is not feasible to characterize all the equilibria in the data, however our (simplistic) thought experiment for two locations has suggested a few. Finding a practical way to do this for all sites in the analysis might strengthen the connection between the theory and the empirics. Furthermore, a more complete understanding of these equilibria would inform resource management policy, because regulations based on spatial location are commonly enforced in many U.S. fisheries. However, as previously mentioned, this may require a generalization of the theory to the case where "not fishing" is a possible action.

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**Table 1:** Preliminary Estimates Ignoring Endogeneity of Vessel Shares

| | Parametric | | Semi-Parametric | | |
|---|---|---|---|---|---|
| Parameter | Coefficient | Standard Error | Coefficient | Standard Error | Bandwidth |
| $\beta_1$ | 0.0005 | 0.0005 | -0.0026 | 0.0023 | 0.0671 |
| $\beta_2$ | -0.0012* | 0.0002 | -0.0001 | 0.0008 | 0.0016 |
| $\beta_3$ | 0.0014* | 0.0006 | 0.0009 | 0.0033 | 0.0364 |
| $\lambda_1$ | 17.23* | 0.0920 | --- | --- | --- |
| $\lambda_2$ | -33.10* | 0.3091 | --- | --- | --- |
| $\lambda_3$ | 21.47* | 0.2752 | --- | --- | --- |

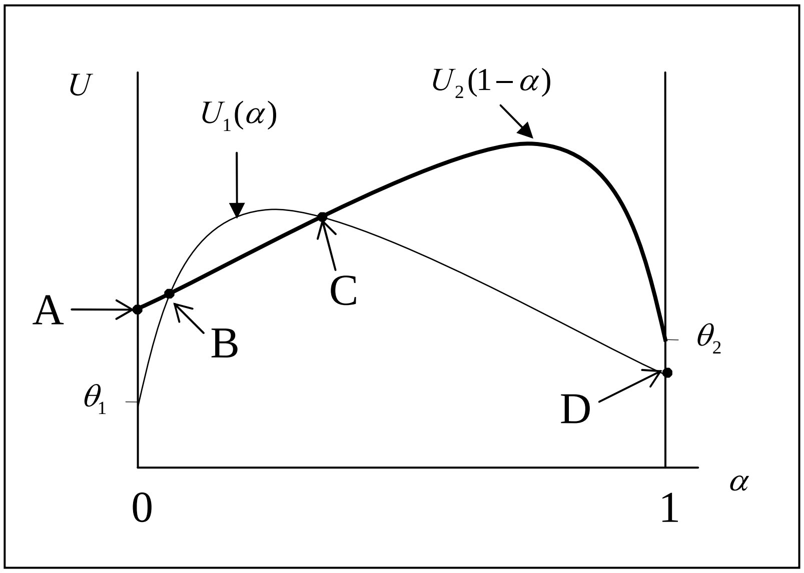
* indicates significance at the 95% level.

**Table 2:** Final Parametric Estimates.

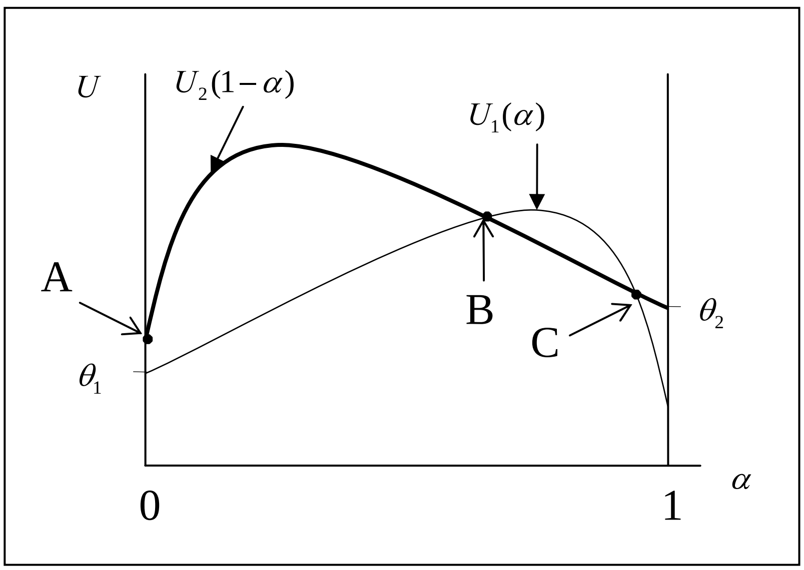
| Parameter | Coefficient | Standard Error |
|---|---|---|
| $\beta_1$ | 0.0061* | 0.0029 |
| $\beta_2$ | -0.0075* | 0.0024 |
| $\beta_3$ | 0.0009 | 0.0032 |
| $\lambda_1$ | 10.77* | 2.4794 |
| $\lambda_2$ | -23.99* | 6.9262 |
| $\lambda_3$ | 16.61* | 6.3025 |

* indicates significance at the 95% level.

**Figure 1. Non-Monotonic Interaction Functions.**




**(a)**




**(b)**

**Figure 2:** Third-Order Polynomial $f(\alpha)$, Endogeneity Ignored.

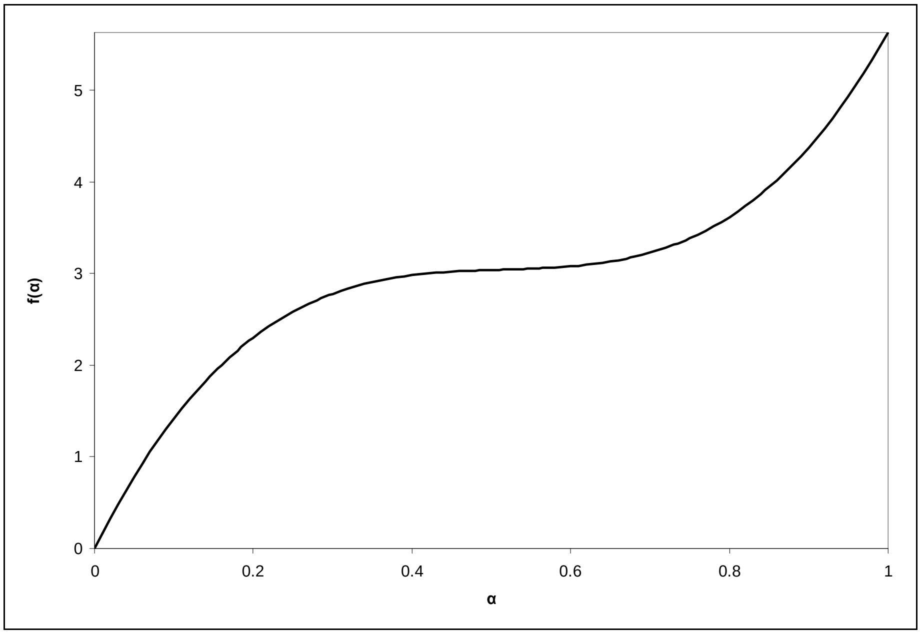
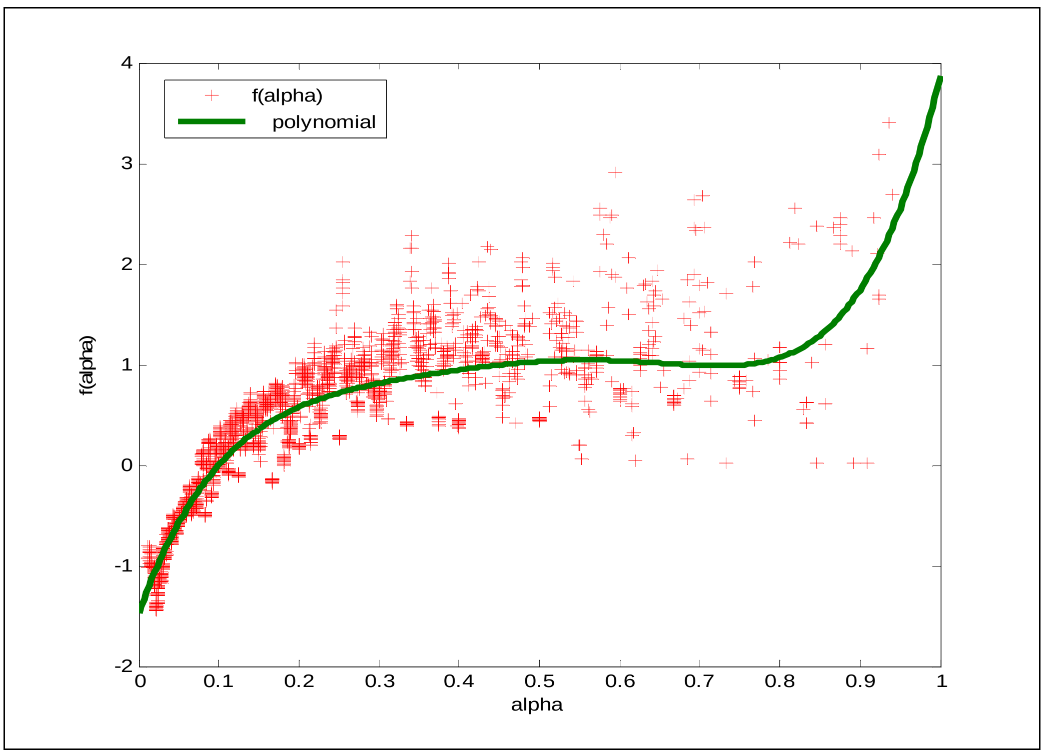



**Figure 3:** Semi-Parametric $f(\alpha_{st})$, Endogeneity Ignored.




Crosses are estimated values of *f*. Green curve is fit using MATLAB Basic Fitting algorithm.

**Figure 4:** Final Third-Order Polynomial Estimate of $f(\alpha)$.

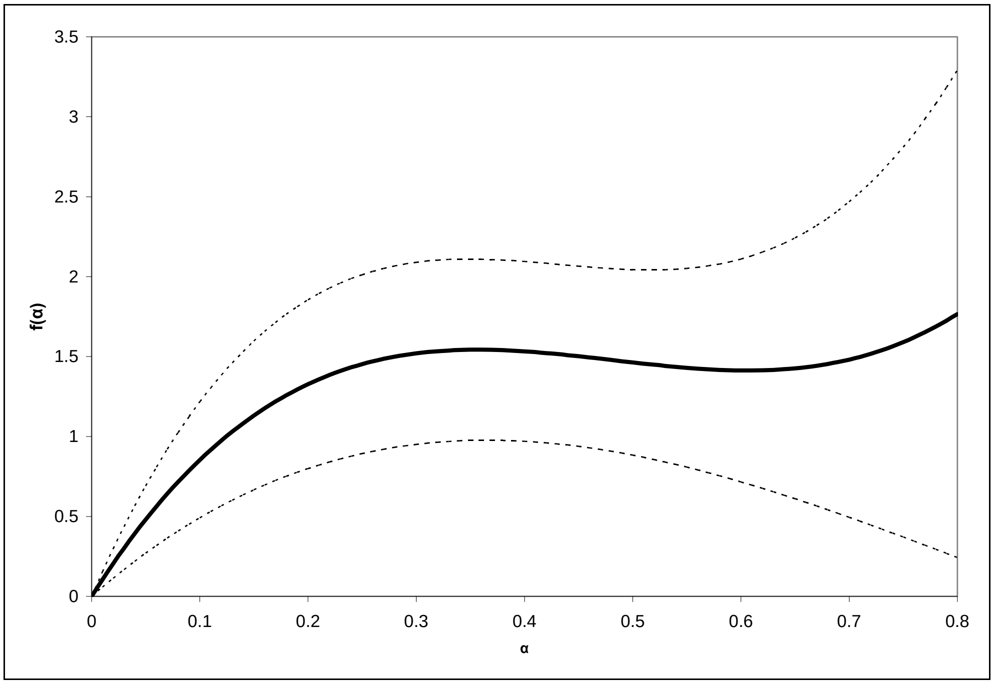



**Figure 5:** Histogram of Instrumented Shares $\tilde{\alpha}_{st}$.

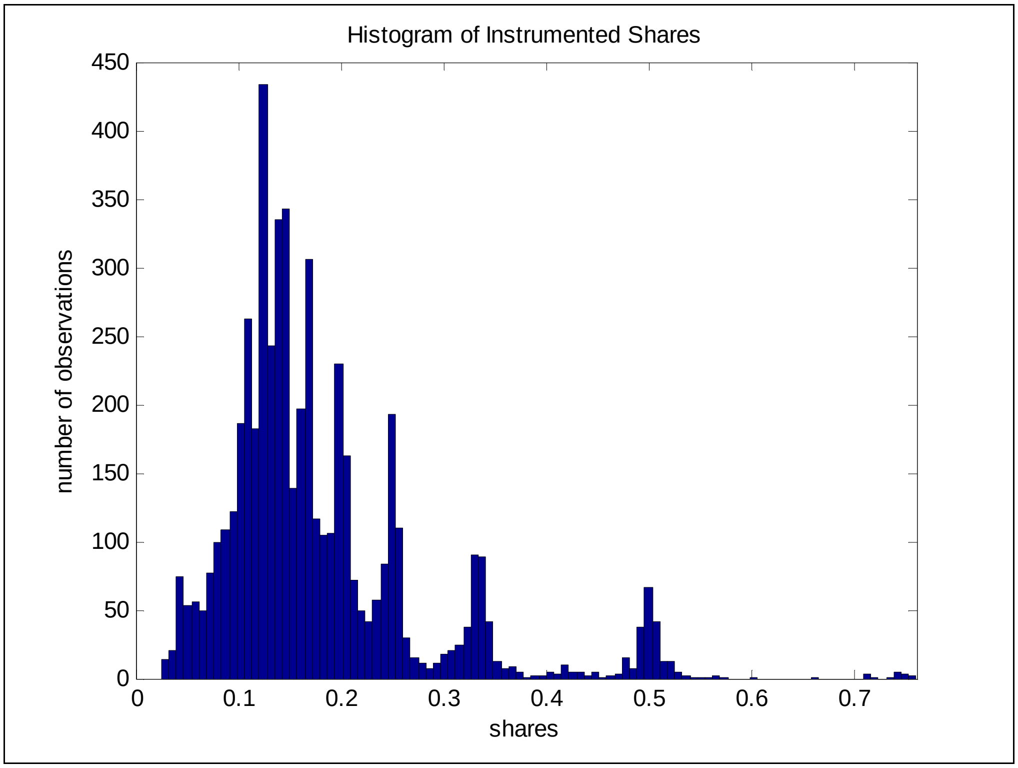


**Figure 1. Approximate Potential Estimated Equilibria.**