Objective of the firm: make decisions so as to maximize profit.

Profit is defined as revenue, what it earns from selling the good, minus costs, what it costs to produce the good.

\[ \pi = R - C \]

or, in slightly different form

\[ \pi = p \cdot f(x) - w \cdot x \]

To orient our next few topics:
We will focus on \( f(x) \), the production function first.

Then we add in costs (the \( w \cdot x \)) component.

Then we add in selling the things we produced at price \( p \) to get at profit maximization.
Technologically efficient production: the firm cannot produce more output given the amount of inputs it is using, and the firm cannot produce the amount of output it is producing by using fewer inputs.

Production function.

A firm gathers together inputs, or factors of production.

The firm then applies a technology, or production process to these inputs.

The result is an output – can be a good or a service.

No costs of input are involved yet.

No selling of the product is going on yet.

Just (boring old) production!!!!
Define a broad definition of a production function

\[ Q = f(K, L, E, M) \]

\( Q \) is output of the good (in many studies, people use \( Y \) rather than \( Q \) – means the same thing)

\( K \) is capital

\( L \) is labor

\( E \) is energy

\( M \) is materials

\( f(.) \) defines the relationship between the quantities of inputs used and the maximum quantity of output that can be produced given current knowledge about technology and organization.

What is the nature of the \( f(.) \)? It can take on lots of different forms.

An important part of econometric work is to estimate the nature of the production function.

We don’t have to worry about that here.
We can treat inputs in the production function as fixed or variable inputs.

In the short run, factors of production that cannot easily be varied are viewed as fixed inputs.

A factor for which it is relatively easy to adjust the quantity quickly is a variable input.

The long run is the time span required to adjust all inputs.

There is no precise definition of time period applied to these terms. It is a relative relationship.

All inputs are variable in the long run (there are no fixed inputs in the long run).

Show production function, feasible zone, frontier, technologically inefficient zone, and progress.
An example:

A commonly made assumption is that labor is the most variable of inputs, so we define it to be the variable input, hold the others constant in the analysis. (No law here, but convention)

\[ Q = f(K, L, E, M) \]

K “bar”, E “bar”, M “bar” mean fixed in short run.

<table>
<thead>
<tr>
<th>Labor units</th>
<th>Total Output</th>
<th>Marginal Physical product</th>
<th>Average Physical product</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>NA</td>
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<tr>
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<td>12</td>
</tr>
<tr>
<td>…</td>
<td></td>
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</tr>
</tbody>
</table>
Marginal Physical Product of Labor (MPP\(_L\)): the change in total output resulting from the use of an additional unit of labor, all else constant.

\[
MPP_L = \frac{(\text{Change in output})}{(\text{Change in labor input that caused this change in output})}
\]

-or-

\[
\frac{\Delta Q}{\Delta L}
\]

Note that this contrasts with the average physical product of labor: the ratio of the output to the number of workers used to produce this labor.

\[
APP_L = \frac{(\text{Amount of output})}{(\text{Amount of labor input that is required to generate this level of output})}
\]

-or-

\[
\frac{Q}{L}
\]

Note further that there is also a marginal physical product of capital, of materials, of energy… Same story for average physical product.
We are focusing on labor, but other inputs also have marginal and average measures as we have just defined for labor.

Marginal physical product can be increasing: Increasing marginal returns. The total product curve is getting steeper and MPP is upward sloping.

Marginal physical product can be constant: Constant marginal returns. The total product curve is a straight line and the MPP is flat.

Marginal physical product can be decreasing: Decreasing marginal returns. The total product curve is getting less steep and MPP is downward sloping.
We can draw a graph of this information.

Why these shapes?
At low levels of labor, workers help each other do tasks that are hard for one person to do, or conversely, they specialize. This gives us initially increasing MPP.

Then, after we reach some critical level of labor,

They have to wait for each other to finish at a machine, or they get in each other’s way… then we get decreasing MPP.
If MPP curve is above APP curve, then APP is upward sloping. If MPP below APP, then APP downward sloping.

Think of heights for the intuition.

“The law of diminishing marginal returns”
If a firm keeps increasing an input, holding all other inputs and technology constant, the corresponding increases in output will become smaller eventually.

Not diminishing returns, but diminishing marginal returns.
Long run production.

That was a discussion of variation in output due to different levels of labor holding other things (K, E, M) constant.

Now, we are in the long run so all inputs are variable.

Note that the short run implies that at least one input is being held fixed. Do not come away from this with the impression that the difference between the short run and the long run is one versus two inputs. That is not right. In the short run, at least one input and potentially more than one input is held constant while one or more other inputs are allowed to vary. In the long run all inputs are allowed to vary.

However, to keep things simple, we are going to assume there are only two inputs used in production of our good.

We will call them capital and labor. More than two are possible in reality.

We focus on two because it is easier to draw and the logic carries through to higher dimensions.

We can combine different quantities of these inputs in a variety of ways to produce a given level of output. Define
a curve that traces out the minimum combinations of inputs required to produce a given level of output.

This is an isoquant. Again, if you want to think of this as a contour line, it is a contour line on the production function in 3-D space.

Properties:

1) The farther an isoquant is from the origin, the greater is the level of output. (remember more is better than less)
2) Isoquants do not cross, as that would imply inefficiency. (remember transitivity)
3) Isoquants slope downward, as they are efficient levels of production (remember there are tradeoffs).
<table>
<thead>
<tr>
<th>Labor</th>
<th>Machines</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
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<td>160</td>
</tr>
<tr>
<td>1000</td>
<td>200</td>
<td>160</td>
</tr>
<tr>
<td>1300</td>
<td>170</td>
<td>160</td>
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<tr>
<td>500</td>
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<td>180</td>
</tr>
<tr>
<td>1000</td>
<td>220</td>
<td>180</td>
</tr>
<tr>
<td>1300</td>
<td>190</td>
<td>180</td>
</tr>
</tbody>
</table>

Draw this (again noting frontier, inefficient, not feasible and progress):
What will influence the shape of the isoquant?

How substitutable are inputs?

Production function of processed pork.

Processed Pork = pigs bought in New York + pigs bought in Pennsylvania

Straight line graph

Production function of peanut butter sandwiches.

Peanut butter sandwiches = minimum (dollops of peanut butter, (slices of bread / 2))

I have 10 dollops of peanut butter, and 4 slices of bread, I can only make 2 sandwiches.

Leontief graph

Most lie intermediary to these two extreme cases.

Show contrast on single graph, note nature of subs, connect at upper and lower extreme.
The slope of the isoquant is called the marginal rate of technical substitution.

This tells us the tradeoff between inputs in production. It is measured as the number of units of one input that have to be given up while increasing the other input to continue to produce a given level of output.

The MRTS, like the MRS, is a negative number since it is implicitly a trade-off. Here, we define it for the capital to labor MRTS.

\[
MRTS_{KL} = \frac{\text{change capital}}{\text{change labor}} = \frac{\Delta K}{\Delta L}
\]

Unless goods are perfect substitutes (MRTS=-1 for example) or perfect complements (MRTS=-∞/ undefined, or 0), the MRTS varies as different points are considered on the isoquant.
Remember we are on an isoquant. The quantity of output stays the same. Therefore, we know that if we change labor and change capital on a given isoquant, the total output should not change.

Recall that $MPP_L = \frac{\Delta Q}{\Delta L}$

, and a similar expression exists for the marginal product of capital.

So if we know the change in total product is zero by definition, and we know the definition of the marginal product is what we just saw, we can make sense of the following:

If $\Delta Q = MPP_L \cdot \Delta L + MPP_K \cdot \Delta K$, and we are looking at a (L,K) change that keeps on a given isoquant such that change in Q is zero, $0 = MPP_L \cdot \Delta L + MPP_K \cdot \Delta K$

Rearrange, and find: $\frac{MPP_L}{MPP_K} = \frac{\Delta K}{\Delta L}$
Show math, note connection to calculus, and answer zero minus zero question if it comes up. Also note connection back to MRS and the Marginal Utility equations developed earlier.

While not overwhelming exciting, this allows us to gain the insight that the marginal rate of technical substitution is equal to the negative of the ratio of the marginal products (important: note the numerator – denominator relationship).

\[ MRTS_{KL} = -\frac{MPP_L}{MPP_K} = \frac{\Delta K}{\Delta L} \]

From an intuitive point of view, the movement along an isoquant is related to marginal changes.

I am getting this level of output using a specific mix of inputs, now I want to move over there to another mix of inputs holding output constant.

That is a marginal change.

And this is like the description of the slope of an indifference curve if you recall.
Returns to scale.

Up until now, we have been considering adjustments to our input bundles, holding output constant. That is how we have defined an isoquant. Or we have been changing one input at a time, holding others constant. That is how we thought about a production function.

Now, however, we want to turn to the question of how changes in the total input bundle are related to changes in output.

What can we learn by comparing different isoquants rather than looking at movement along a given isoquant?

We are going to look at a specific type of change to the input bundle – blowups. Equal percentage change applied to all inputs.

I use labor and capital to produce my good. Let’s say we can continue to ignore other inputs like materials and energy for production of our good.

What are the implications of different production functions for changing input levels?
Say I use 2 units of labor and 3 units of capital to produce 6 units of output. In this case, assume the production function is defined by capital times labor. If I double both units (4 units of labor and 6 units of capital) I get 24 units of output.

\[ 2 \times 3 = 6 \]
\[ \text{new output} = (2 \times 2) \times (3 \times 2) = 4 \times (2 \times 3) = 4 \times 6 = 24 \]

Doubling inputs gives a four-fold increase in output.
\[ \frac{24}{6} = 4 \]

Increasing Returns to scale. Doubling inputs leads to a more than double increase in outputs.

\[ f(2K, 2L) > 2f(K, L) \]
Say instead that the production function is capital plus labor (perfect substitutes in production).

\[ 2+3=5 \]

new output \( = (2\times 2) + (2\times 3) = 2\times (2+3) = 10 \)

Doubling inputs gives a doubling of output.
\( \frac{10}{5}=2 \)

Constant returns to scale. Case two (additive production function). Doubling inputs leads to a doubling of output.

\[ f(2\times K, 2\times L) = 2\times f(K, L) \]
Finally, assume we have a production function that defines output as \((\text{capital times labor})^{1/10}\) as a particular example.

Old output = \((2*3)^{1/10}=1.2\)
New output = \(((2*2)*(2*3))^{1/10} = (4*6)^{1/10}=1.4\)

Doubling inputs increases output by 17%  
\(\frac{1.4}{1.2}=1.167\)

Decreasing returns to scale. Doubling inputs leads to a less than double increase in output.

\(f(2*\text{K},2*\text{L}) < 2*f(\text{K},\text{L})\)
Returns to scale can depend on where you are in the production function.

A common pattern is IRS over low levels of input, CRS over moderate levels, and DRS with high levels of input.

If we think of isoquants as contour lines, they are close together near the origin, and spread further apart as we move away from the origin.